IJARSCT

ISSN (Online) 2581-9429

International Journal of Advanced Research in Science, Communication and Technology (IJARSCT)

Impact Factor: 7.301

Volume 3, Issue 2, March 2023

Certain Identities of C^h in Finsler Spaces

Abdalstar A. Saleem[1], Alaa A. Abdallah[2*]

Department of Mathematics, Faculty of Sciences, Aden University, Aden, Yemen[1]

Department of Mathematics, Dr. Babasaheb Ambedkar Marathwada University, Aurangabad, India[2]

Department of Mathematics, Abyan University, Abyan, Yemen

saleemabdalstar@gmail.com and maths.aab@bamu.ac.in

Abstract: *The C^h generalized birecurrent space and C^h special generalized birecurrent space have been introduced by Saleem [4]. Now, in this paper, we introduce and discuss two theorems related to the above mentioned spaces.*

Keywords: C^h generalized birecurrent space and C^h special generalized birecurrent space.

I. INTRODUCTION

The concepts of C^h −recurrent space and C^h −birecurrent space are introduced by Matsumoto [7] and Pandey and Verma [9], respectively. Also, Mishra and Lodhi [5] discussed the properties of C^h − recurrent and C^v −recurrent spaces for second order. Recently, various special forms of the $h(hv)$ −torsion tensor C_{jkh} in generalized recurrent and birecurrent space have been studied by [2, 3].

Let us consider an n −dimensional Finsler space F_n equipped with the line elements (x, y) and the fundamental metric function F is positively homogeneous of degree one in y^i. The metric tensor $g_{ij}(x, y)$ is positivly homogeneous of degree zero in y^i and symmetric in its lower indices which is defined by.

(1.1) $g_{ij}(x,y) = \frac{1}{2}\dot{\partial}_i\dot{\partial}_j F^2$.

By differentiating (1.1) partially with respect to y^k, we obtain the tensor C_{ijk} that is known as $(h)hv$ *−torsion tensor* which defined by [8]

(1.2) $C_{ijk} = \frac{1}{2}\dot{\partial}_k g_{ij} = \frac{1}{4}\dot{\partial}_k\dot{\partial}_i\dot{\partial}_j F^2$.

It is positively homogeneous of degree −1 in y^i and symmetric in all its indices. The above tensor C_{ijk} satisfies

(1.3) $C_{ijk} = g_{ih}C^h_{jk}$,

where C^i_{jk} is called *associate tensor of the tensor* C_{ijk}.

Cartan$h$ −covariant differentiation with respect to x^k is given by [1, 6]

(1.4) $X^i_{|k} = \partial_k X^i - (\dot{\partial}_r X^i)G^r_k + X^r\Gamma^{*i}_{rk}$.

The h −covariant derivative of the vector y^i and associate metric tensor g_{ij} are vanish identically, i.e.

(1.5) a) $y^j_{|l} = 0$ and b) $g_{ij|l} = 0$.

Taking the h–covariant derivative of (1.4) in the sense of Cartan, we have

(1.6) $X^i_{|k|h} = \partial_h(X^i_{|k}) - (X^i_{|r})\Gamma^{*r}_{kh} + (X^r_{|k})\Gamma^{*i}_{rh} - \dot{\partial}_r(X^i_{|k})\Gamma^{*r}_{h\ell}y^\ell$.

From (1.4) and (1.6), we get the commutation formula for h −covariant derivative of an arbitrary tensor X^i_j which is given by [1]

(1.7) $X^i_{j|m|l} - X^i_{j|l|m} = X^r_j R^i_{rlm} - X^i_r R^r_{jlm} - X^i_j|_r H^r_{lm}$,

where R^i_{rlm} is called h −curvature tensor and satisfies

(1.8) $R^i_{rlm}y^r = H^r_{lm}$

(1.9) $R^i_{rlm}\, g_{hi} = R_{rhlm}$,

where R_{rhlm} is associate curvature tensor of R^i_{rlm} .

II. PRELIMINARIES

In this section, we introduce some definitions that need it for the purpose of the study. The space which satisfies the recurrence property for R^i_{jkh} introduced by Verma [10] that is characterized by

(2.1) $R^i_{jkh|l} = \lambda_l \, R^i_{jkh}$,

where $\lambda_l$is non-zero covariant vector field. Transvecting (2.1) byy^j, using (1.5) and (1.8), we get

(2.2) $H^i_{kh|l} = \lambda_l H^i_{kh}$.

The concept of C −recurrent space in sense of Cartan studied by Matsumoto [7] that is characterized by

(2.3) $C^i_{jk|l} = \lambda_l C^i_{jk}$,

where $\lambda_l$is non-zero covariant vector field. Also, the concept of C −birecurrent space in sense of Cartan studied by Pandey and Varma [9] that is characterized by

(2.4) $C^i_{jk|l|m} = a_{lm} C^i_{jk}$,

where a_{lm} is non-zero covariant tensor field.

Saleem [4] introduced the C^h −generalized birecurrent space and C^h −special generalized birecurrent space which are characterized by

(2.5) $C^i_{jk|l|m} = \lambda_l C^i_{jk|m} + a_{lm} C^i_{jk}$

and

(2.6) $C^i_{jk|l|m} = \lambda_l C^i_{jk|m}$, respectively.

III. MAIN RESULTS

In this section, two theorems have been established and proved in C^h −generalized birecurrent space and C^h −special generalized birecurrent space. Differentiating (2.5) covariant with respect to x^n in sense of Cartan, we get

(3.1) $C^i_{jk|l|m|n} = \lambda_{l|n} C^i_{jk|m} + \lambda_l C^i_{jk|m|n} + a_{lm|n} C^i_{jk} + a_{lm} C^i_{jk|n}$.

Interchanging the indices l and m in (3.1) and by subtracting it from (3.1), using (2.3) and (2.4), we get

(3.2) $C^i_{jk|l|m|n} - C^i_{jk|m|l|n} = \{(\lambda_l a_{mn} - \lambda_m a_{ln}) + (\lambda_m \lambda_{l|n} - \lambda_l \lambda_{m|n}) + (a_{lm|n} - a_{ml|n}) + \lambda_n (a_{lm} - a_{ml})\} C^i_{jk}$.

Using the symmetric property of the tensor a_{lm} in (3.2), we get

(3.3) $C^i_{jk|l|m|n} - C^i_{jk|m|l|n} = \{(\lambda_l a_{mn} - \lambda_m a_{ln}) + (\lambda_m \lambda_{l|n} - \lambda_l \lambda_{m|n})\} C^i_{jk}$.

Transvecting (3.3) byg_{hi} using (1.3) and (1.5), we get

(3.4) $C_{hjk|l|m|n} - C_{hjk|m|l|n} = \{(\lambda_l a_{mn} - \lambda_m a_{ln}) + (\lambda_m \lambda_{l|n} - \lambda_l \lambda_{m|n})\} C_{hjk}$.

Thus, we conclude

Theorem 3.1. *In C^h −generalized birecurrentspace, the third derivative in sense of Cartan for the torsion tensor C_{hjk} and its associate tensor C^i_{jk} with symmetric property of the tensor a_{lm} satisfy the identities (3.4) and (3.3), respectively.*

By using same technique in above theorem for (2.6), we get the following corollay:

Corollary 3.1. *In C^h −special generalized birecurrentspace, the third derivative in sense of Cartan for the tensors C^i_{jk} and C_{hjk}satisfy the following identities*

$$C^i_{jk|l|m|n} - C^i_{jk|m|l|n} = \{(\lambda_l a_{mn} - \lambda_m a_{ln}) + (\lambda_m \lambda_{l|n} - \lambda_l \lambda_{m|n})\} C^i_{jk}$$

and

$C_{hjk|l|m|n} - C_{hjk|m|l|n} = \{(\lambda_l a_{mn} - \lambda_m a_{ln}) + (\lambda_m \lambda_{l|n} - \lambda_l \lambda_{m|n})\} C_{hjk}$, *respectively.*

Interchanging the indices l and m in (2.4), using the commutation formula (1.7) and with symmetric property of the tensor a_{lm}, we get

(3.5) $C^i_{jk|l|m} - C^i_{jk|m|l} = C^r_{jk} R^i_{rlm} - C^i_{rk} R^r_{jlm} - C^i_{jr} R^r_{klm} - C^i_{jk}|_r H^r_{lm}$.

Differentiating (3.5) covariant with respect to x^n in the sense of Cartan, we get

(3.6) $C^i_{jk|l|m|n} - C^i_{jk|m|l|n} = (C^r_{jk} R^i_{rlm} - C^i_{rk} R^r_{jlm} - C^i_{jr} R^r_{klm} - C^i_{jk}|_r H^r_{lm})_{|n}$.

Using (3.3) in (3.6), we get

(3.7) $(C^r_{jk}R^i_{rlm} - C^i_{rk}R^r_{jlm} - C^i_{jr}R^r_{klm} - C^i_{jk}|_r H^r_{lm})_{|n} = \{(\lambda_l a_{mn} - \lambda_m a_{ln}) + (\lambda_m \lambda_{l|n} - \lambda_l \lambda_{m|n})\}C^i_{jk}$.
Transvecting (3.7) by g_{hi} using (1.3), (1.5) and (1.9), we get
(3.8) $(C^r_{jk}R_{rhlm} - C_{rhk}R^r_{jlm} - C_{jrh}R^r_{klm} - C_{jhk}|_r H^r_{lm})_{|n} = \{(\lambda_l a_{mn} - \lambda_m a_{ln}) + (\lambda_m \lambda_{l|n} - \lambda_l \lambda_{m|n})\}C_{hjk}$.
Thus, we conclude

Theorem 3.2. *In C^h −generalized birecurrentspace, we have the identities (3.7) and (3.8).* By using same technique in above theorem for (2.6), we get the following corollay:
Corollary 3.2. *In C^h −special generalized birecurrentspace, we have the identities*

$$(C^r_{jk}R^i_{rlm} - C^i_{rk}R^r_{jlm} - C^i_{jr}R^r_{klm} - C^i_{jk}|_r H^r_{lm})_{|n} = \{(\lambda_l a_{mn} - \lambda_m a_{ln}) + (\lambda_m \lambda_{l|n} - \lambda_l \lambda_{m|n})\}C^i_{jk}$$

and

$$(C^r_{jk}R_{rhlm} - C_{rhk}R^r_{jlm} - C_{jrh}R^r_{klm} - C_{jhk}|_r H^r_{lm})_{|n} = \{(\lambda_l a_{mn} - \lambda_m a_{ln}) + (\lambda_m \lambda_{l|n} - \lambda_l \lambda_{m|n})\}C^i_{jk}.$$

IV. CONCLUSION

Certain identities belong to C^h −generalized birecurrent space and C^h −special generalized birecurrent space have been obtained.

REFERENCES

[1]. A. A. Abdallah, A. A. Navlekar, K. P. Ghadle and B. Hardan, Fundamentals and recent studies of Finsler geometry, International Journal of Advances in Applied Mathematics and Mechanics, 10(2), (2022), 27-38.
[2]. A. A. Abdallah, A. A. Hamoud, A. A. Navlekar and K. P. Ghadle, On special spaces of h(hv)-Torsion tensor C_jkh in generalized recurrent Finsler space, Bull. Pure Appl. Sci. Sect. E Math. Stat. 41E(1), (2022),74-80.
[3]. A. A. Abdallah, A. A. Navlekar, K. P. Ghadle and B. Hardan, Several forms of h(hv)-Torsion tensor C_jkh in generalized βP-birecurrent space, International Journal of Advanced Research in Science, Engineering and Technology, 9(7), (2022), 19505-19510.
[4]. A. A. Saleem, On certain generalized birecurrent and trirecurrentFinsler space, M. SC. Thesis, Univ. of Aden, (Aden) (Yemen), (2011).
[5]. C. K. Mishra and G. Lodhi, On C^h-recurrent and C^v-recurrent Finsler spaces of second order, Int. J. Contemp. Math. Sciences, Vol. 3, No. 17, (2008), 801 – 810.
[6]. É. Cartan, Les espaces de Finsler, Actualités, Paris, 79 (1934). 2nd edit. (1971).
[7]. M. Matsumoto, On h-isotropic and C^h-recurrent Finsler space, J. Math. Kyoto Univ., 11 (1971), 1-9.
[8]. M. Matsumoto, On Finsler spaces with curvature tensors of some special forms, Tensor, N.S., 22 (1971), 201-204.
[9]. P. N. Pande and R. Varma, C^h-birecurrent Finsler spaces, second conference of the Int. Acad. of Physical Sciences, (December 13 – 14, 1997).
[10]. R. Verma, Some transformation in Finsler spaces, D. Ph. Thesis, Univ. of Allahbad, (Allahabad) (India), (1991).