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A Fractional Treatment to Food-Borne Disease Modeling by q- Homotopy Analysis Transform Method (q-HATM)

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Abstract: *A non-linear mathematical model has been proposed and examined here portraying impact of biological control of the expansion of the fly population and the transmission of food-borne illnesses. In the model's design, we made the assumption that the human population becomes exposed to food borne illnesses through close contact between those who are vulnerable and those who are already afflicted. In the same way that flies contaminate human food by bringing infectious disease bacteria from the outside in, vulnerable people can also become infected by indirect transmission. Additionally, we employ the fractional approach in the well-known technique known as q-HATM to mathematically analyze suggested model. This technique can be used to acquire the analytical findings of suggested model have convergent series with necessary computation of several important components.*

Keywords: q-HATM; Food-borne disease; Fractional differential equations; Homotopy Analysis Transform Method; Non-linear; Biological control.

I. INTRODUCTION

Since 17th century, the research in the area of fractional calculus has been started. Due to its complex characteristics, still now, only mathematical framework has been marked by several researchers [1, 2]. In the past few decades, modern mathematicians have developed its theory and a very few applications of fractional calculus has been presented [3]. Nowadays, it is a well proven theory and exploited in many scientific disciplines, such as various branches of engineering [4]. It is very important to use fractional differential equations apart from the differential equations having order integer to understand any real world problem. Due to its nonlocal nature, various real phenomenon can be modeled using differential equations of order having fractions [5, 6]. For example, fractional differential equation is capable to well include memory and hereditary ectensions of several real world physics situations, various processes etc., [7, 8]. These benefit makes order models of fractions more real than the basic models involving integral order differential equations to understand the real world situations [9]. Also, the formulation of accurate and fast numerical schemes is a complex job in this scenario. In more general situations, accurate solution of many fraction equations are still tedious job to perform. That is why, finding the approximate solutions and development of different numerical schemes is necessary to recognize the behavior of solutions for various fraction equation models with their applications [10].

In previous decades, a potential tool use fractional order descipline has emerged likerosenau-hy man equation [11-23].

Therefore, in the next section, we make few assumptions to formulate model for spread of Food-borne controlling by introduction of the biocontrol agent parasitic wasps in the region under consideration.

II. PROPOSED MODEL AND ITS FORMULATION

Here, we have differential equation model for the spread and controlling Food-borne disease in a region under consideration.

Let us consider five dynamic variables namely; susceptible individuals$S(t)$, infected individuals$I(t)$, pupae of flies population$P(t)$, adult flies population$C(t)$, parasitic wasps population$W(t)$. The ordinary differential equation model forFood-borne diseases containing five dynamic variables is as follows:

$$\frac{dS}{dt} = A - \beta SI - \lambda SC - \mu S + \nu I,$$

$$\frac{dI}{dt} = \beta SI + \lambda SC - (m + \mu + \nu)I,$$

$$\frac{dP}{dt} = gC - \mu_1 P - \alpha_1 P^2 - \gamma P - \phi PW, \quad (1)$$

$$\frac{dC}{dt} = \gamma P - \theta_0 C,$$

$$\frac{dW}{dt} = k\phi PW - \theta_1 W,$$

It is assumed in the model formulation that the individuals who are coming in the region either through birth or immigration, this rate is denoted by the constant$A$. Further, it is assumed that the susceptible individuals get the disease with direct contact of susceptibles with infectives and infectives with rate β, and indirect transmission due to susceptible and adult flies population at a rateλ. The infected individuals who are severely infected with the disease, they experience disease induced death at a rate$m$ and the individuals who successfully clear the infection, they got the temporary immunity and then they shift towards susceptible class again with rateν. The natural death incorporated with rate μ.

Now we describe the last three equations involving the interaction of pupae of flies $P(t)$, adult flies population $C(t)$ and parasitic wasps population$W(t)$. To model the effective control measures, we use predator-prey dynamics to model the interaction between pupae of flies and parasitic wasps. As the development of flies occurs in the aquatic environment through four stages, like egg, larvae, pupae and adult fly population. Control measures applied at any stage of these instars will lead to control adult flies in a region under consideration. As the fly lay eggs in a dirty environment so the introduction of parasitic wasps in the environment may help in reducing the pupae of flies. In these dirt environment the flies develops into adult stage and free to move the nearby housing societies and transfers the bacteria of various infectious disease to the edibles of human population and then humans get the various infectious diseases. As the growth of pupae of flies will be directly proportional to the number of eggs laying per adult flies. Therefore, the constant g is the egg laying rate of an adult fly in the environment. The constant μ_1 denote the natural death of pupae in the environment. The constant $\alpha_1$denote the intraspecific competition experienced by pupae of flies. The constant γ is the rate at which pupae of flies acquire the adult stage. The constant θ_0 is death rate by natural of an adult fly.Constant $\phi$is the depletion coefficient due to the interaction between pupae of flies and parasitic wasps. The constant k is the proportionality constant which represents the replication factor of parasitic wasps due to predation of pupae of flies. The constant$\theta_1$denotes the natural death rate of parasitic wasps. Now the model system (1) is converted into a fractional differential equation model. We will apply q-HATM method to analyze and obtained results will be validated numerically by presenting a suitable numerical scheme. Thus, the model system involving fractional derivatives is expressed has following form

$$\frac{d^\alpha S}{dt^\alpha} = A - \beta SI - \lambda SC - \mu S + \nu I,$$

$$\frac{d^\alpha I}{dt^\alpha} = \beta SI + \lambda SC - (m + \mu + \nu)I,$$

$$\frac{d^\alpha P}{dt^\alpha} = gC - \mu_1 P - \alpha_1 P^2 - \gamma P - \phi PW, (2)$$
$$\frac{d^\alpha C}{dt^\alpha} = \gamma P - \theta_0 C,$$
$$\frac{d^\alpha W}{dt^\alpha} = k\phi PW - \theta_1 W,$$

From here, we present analysis and simulation of the model system (2).
Solution convergence is confirmed by auxiliary parameter. q–HAM is actually an improvement of the embedding parameter q ∈ [0, 1] arising in HAM to q ∈ [0, $\frac{1}{n}$], $n \geq 1$. Authority of FHATM is its potential of adjusting two strong computational methodologies for probing FDEs.
The objective of this paper is to obtain numerical solution of time–fractional model of food-borne disease by q-HATM. We have used Caputo fractional derivative because its main advantage is that with these derivatives, initial conditions for FDEs undertake the similar form as for the integer order differential equations.

III. PRELIMINARIES

Here, we proceed with some definitions, which we use to analyze the proposed model
Definition 3.1. Consider real function h (χ), χ > 0. It is called in space $C_\zeta, \zeta \in R$ if ∋ a real no. b (> ζ), s.t. h (χ) = χ^b $h_1(\chi)$, $h_1 \in C[0,\infty]$. It is clear that $C_\zeta \subset C_\gamma$ if $\gamma \leq \zeta$.

Definition 3.2. Consider a function h (χ), χ > 0. It is called in space $C_\zeta^m, m \in \mathbb{N} \cup \{0\}$ if $h^{(m)} \in C_\zeta$.

Definition 3.3. Left sided Caputo fractional derivative of h, $h \in C_{-1}^m, m \in \mathbb{N} \cup \{0\}$,

$$D_t^\beta h(t) = \begin{cases} I^{m-\beta} h^{(m)}(t), m-1 < \beta < m, m \in \mathbb{N}, \\ \frac{d^m}{dt^m} h(t), \ \beta = m, \end{cases}$$

a. $I_t^\zeta h(x,t) = \frac{1}{\Gamma\zeta} \int_0^t (t-s)^{\zeta-1} h(x,s) ds;\ \zeta, t > 0.$

b. $D_\tau^\nu V(x,\tau) = I_\tau^{m-\nu} \frac{\partial^m V(x,\tau)}{\partial t^m}, m-1 < \nu \leq m.$

c. $D_t^\zeta I_t^\zeta h(t) = h(t), m-1 < \zeta \leq m, m \in \mathbb{N}.$

d. $I_t^\zeta D_t^\zeta h(t) = h(t) - \sum_{k=1}^{m-1} h^k(0^+) \frac{t^k}{k!}, m-1 < \zeta \leq m,, m \in \mathbb{N}.$

e. $I^v t^\zeta = \frac{\Gamma(\zeta+1)}{\Gamma(v+\zeta+1)} t^{v+\zeta}.$

IV. Q-HATM FRACTIONAL FOOD-BORNE MODEL

$$\frac{d^\alpha S}{dt^\alpha} = A - \beta SI - \lambda SC - \mu S + \nu I,$$
$$\frac{d^\alpha I}{dt^\alpha} = \beta SI + \lambda SC - (m + \mu + \nu) I,$$
$$\frac{d^\alpha P}{dt^\alpha} = gC - \mu_1 P - P^2 \alpha_1 - \gamma P - \phi PW, (3)$$

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$$\frac{d^{\alpha}C}{dt^{\alpha}} = \gamma P - \theta_0 C,$$
$$\frac{d^{\alpha}W}{dt^{\alpha}} = k\phi PW - \theta_1 W$$

with initial conditions $S(0) = S_0, I(0) = I_{0,} P(0) = P_0, C(0) = C_0, W(0) = W_0$

By using initial conditions, $S(0) = S_0, I(0) = I_{0,} P(0) = P_0, C(0) = C_0, W(0) = W_0$ and using the Laplace transform to Eqns. (3), we get

$L[S] - \frac{S_0}{p} - \frac{1}{p^{\alpha}} L[A - \beta SI - \lambda SC - \mu S + \nu I] = 0,$

$L[I] - \frac{I_0}{p} - \frac{1}{p^{\alpha}} L[\beta SI + \lambda SC - (m + \mu + \nu)I] = 0,$

$L[P] - \frac{P_0}{p} - \frac{1}{p^{\alpha}} L[gC - \mu_1 P - P^2\alpha_1 - \gamma P - \phi PW] = 0,$(4)

$L[C] - \frac{C_0}{p} - \frac{1}{p^{\alpha}} L[\gamma P - \theta_0 C] = 0,$

$L[W] - \frac{W_0}{p} - \frac{1}{p^{\alpha}} L[k\phi PW - \theta_1 W] = 0.$

We define the nonlinear operator as

$$N[\phi(t;q)] = L[\phi(t;q)] - \left(1 - \frac{k_m}{n}\right)\frac{S_0}{p} - \frac{1}{p^{\alpha}} L[A - \beta\phi(t;q)\phi(t;q) - \lambda\phi(t;q)\phi(t;q) - \mu\phi(t;q) + \nu\phi(t;q)],$$

$$N[\phi(t;q)] = L[\phi(t;q)] - \left(1 - \frac{k_m}{n}\right)\frac{I_0}{p} - \frac{1}{p^{\alpha}} L[\beta\phi(t;q)\phi(t;q) + \lambda\phi(t;q)\phi(t;q) - (m + \mu + \nu)\phi(t;q)],$$

$$N[\phi(t;q)] = L[\phi(t;q)] - \left(1 - \frac{k_m}{n}\right)\frac{P_0}{p} - \frac{1}{p^{\alpha}} L[g\phi(t;q) - \mu_1\phi(t;q) - \phi(t;q)^2\alpha_1 - \gamma\phi(t;q) - \phi\,\phi(t;q)\phi(t;q)],$$

$$N[\phi(t;q)] = L[\phi(t;q)] - \left(1 - \frac{k_m}{n}\right)\frac{C_0}{p} - \frac{1}{p^{\alpha}} L[\gamma\phi(t;q) - \theta_0\phi(t;q)],$$

$$N[\phi(t;q)] = L[\phi(t;q)] - \left(1 - \frac{k_m}{n}\right)\frac{W_0}{p} - \frac{1}{p^{\alpha}} L[k\phi\,\phi(t;q)\phi(t;q) - \theta_1\phi(t;q)]. \quad (5)$$

By using the aforesaid procedure of proposed numerical scheme, we obtain the m^{th} order deformation equation for H(x, t) = 1 as

$$L[S_m(t) - k_m S_{m-1}(t)] = \hbar\, æ_m(S_{m-1}),$$
$$L[I_m(t) - k_m I_{m-1}(t)] = \hbar\, æ_m(I_{m-1}),$$
$$L[P_m(t) - k_m P_{m-1}(t)] = \hbar\, æ_m(P_{m-1}),(6)$$
$$L[C_m(t) - k_m C_{m-1}(t)] = \hbar\, æ_m(C_{m-1}),$$
$$L[W_m(t) - k_m W_{m-1}(t)] = \hbar\, æ_m(W_{m-1}).$$

On employing the inversion of the Laplace transform, we get

$$S_m(t) = k_m S_{m-1}(t) + \hbar L^{-1}[æ_m(S_{m-1})],$$
$$I_m(t) = k_m I_{m-1}(t) + \hbar L^{-1}[æ_m(I_{m-1})],$$
$$P_m(t) = k_m P_{m-1}(t) + \hbar L^{-1}[æ_m(P_{m-1})],$$
$$C_m(t) = k_m C_{m-1}(t) + \hbar L^{-1}[æ_m(C_{m-1})],$$
$$W_m(t) = k_m W_{m-1}(t) + \hbar L^{-1}[æ_m(W_{m-1})], \quad (7)$$

In Eq. (7), we express $æ_m(\vec{u}_{m-1})$ in a new manner as:

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$$æ_m(S_{m-1}) = L[S_{m-1}(t)] - \left(1-\frac{k_m}{n}\right)\frac{S_0}{p} - \frac{1}{p^\alpha} L[A - \beta S_{m-1} I_{m-1} - \lambda S_{m-1} C_{m-1} - \mu S_{m-1} + \nu I_{m-1}],$$

$$æ_m(I_{m-1}) = L[I_{m-1}(t)] - \left(1-\frac{k_m}{n}\right)\frac{I_0}{p} - \frac{1}{p^\alpha} L[\beta S_{m-1} I_{m-1} + \lambda S_{m-1} C_{m-1} - (m+\mu+\nu) I_{m-1}],$$

$$æ_m(P_{m-1}) = L[I_{m-1}(t)] - \left(1-\frac{k_m}{n}\right)\frac{P_0}{p} - \frac{1}{p^\alpha} L[g C_{m-1} - \mu_1 P_{m-1} - (P_{m-1})^2\alpha_1 - \gamma P_{m-1} - \phi P_{m-1} W_{m-1}],$$

$$æ_m(C_{m-1}) = L[C_{m-1}(t)] - \left(1-\frac{k_m}{n}\right)\frac{C_0}{p} - \frac{1}{p^\alpha} L[\gamma P_{m-1} - \theta_0 C_{m-1}],$$

$$æ_m(W_{m-1}) = L[W_{m-1}(t)] - \left(1-\frac{k_m}{n}\right)\frac{W_0}{p} - \frac{1}{p^\alpha} L[k\phi P_{m-1} W_{m-1} - \theta_1 W_{m-1}]. \quad (8)$$

and k_m is presented as $k_m = \begin{cases} 0, m \le 1, \\ n, m > 1. \end{cases}$ (9)

In Eq. (8), P_m is homotopy polynomial and expressed as

$$P_m = \frac{1}{m!}\left[\frac{\partial^m \phi(x,y;q)}{\partial q^m}\right]\Big|_{q=0} \quad (10)$$

And $\phi = \phi_0 + q\,\phi_1 + q^2\,\phi_2 + \dots$ (11)

The q-HATM solution is presented in subsequent form

$S(t) = \sum_{m=1}^{\infty} S_m(t)\left(\frac{1}{n}\right)^m,$

$I(t) = \sum_{m=1}^{\infty} I_m(t)\left(\frac{1}{n}\right)^m,$

$P(t) = \sum_{m=1}^{\infty} P_m(t)\left(\frac{1}{n}\right)^m,$

$C(t) = \sum_{m=1}^{\infty} C_m(t)\left(\frac{1}{n}\right)^m,$

$W(t) = \sum_{m=1}^{\infty} W_m(t)\left(\frac{1}{n}\right)^m.$ (12)

V. NUMERICAL EXPERIMENT AND DISCUSSION

Here in numerical investigation, we present numerical simulation of model system (3), in which the fractional order derivatives have been considered.

Now, we take the initial approximation $S(0) = S_0, I(0) = I_0, P(0) = P_0, C(0) = C_0, W(0) = W_0$ and iterative scheme (3); we have the following approximations of the q-HATM solution:

$$S_1 = \frac{-\hbar\, t^\alpha (A - \beta S_0 I_0 - \lambda S_0 C_0 - \mu S_0 + \nu I_0)}{\Gamma(1+\alpha)},$$

$$I_1 = \frac{-\hbar\, t^\alpha (\beta S_0 I_0 + \lambda S_0 C_0 - (m+\mu+\nu) I_0)}{\Gamma(1+\alpha)},$$

$$P_1 = \frac{-\hbar\, t^\alpha (g C_0 - \mu_1 P_0 - (P_0)^2\alpha_1 - \gamma P_0 - \phi P_0 W_0)}{\Gamma(1+\alpha)},$$

$$C_1 = \frac{-\hbar\, t^\alpha (\gamma P_0 - \theta_0 C_0)}{\Gamma(1+\alpha)},$$

$$W_1 = \frac{-\hbar\, t^{\alpha}(k\phi \mathrm{P_0 W_0} - \theta_1 \mathrm{W_0})}{\Gamma(1+\alpha)}, S_2$$

$$= \hbar\, t^{\alpha}\left(\frac{-A\,(1+\hbar+n)-(\hbar+n)\nu \mathrm{I_0}+(\hbar+n)(\mu+\beta \mathrm{I_0}+\lambda\, \mathrm{C_0})\mathrm{S_0}}{\Gamma(1+\alpha)}\right.$$

$$-\frac{\hbar\, t^{\alpha}(A\mu+\nu\,(m+2\mu+\nu)\mathrm{I_0}-(\mu^2+\beta\,(\mu+\nu)\mathrm{I_0}+\lambda\,(\mu+\nu)\mathrm{C_0})\mathrm{S_0})}{\Gamma(1+2\alpha)}$$

$$\left.-\frac{\hbar^2 t^{2\alpha}\Gamma(1+2\alpha)(A+\nu\, \mathrm{I_0}-(\mu+\beta \mathrm{I_0}+\lambda\, \mathrm{C_0})\mathrm{S_0})\left(\beta \mathrm{I_0}(m+\mu+\nu-\beta \mathrm{S_0})-\lambda\left(\gamma \mathrm{P_0}+\mathrm{C_0}(\beta\, \mathrm{S_0}-\theta_0)\right)\right)}{\Gamma(1+\alpha)^2\,\Gamma(1+3\alpha)}\right),$$

$$I_2 = \hbar\, t^{\alpha}\left(\frac{(\hbar+n)((m+\mu+\nu)\mathrm{I_0}-(\beta \mathrm{I_0}+\lambda\, \mathrm{C_0})\mathrm{S_0})}{\Gamma(1+\alpha)}+\frac{\hbar\, t^{\alpha}(m+\mu+\nu)\,((m+\mu+\nu)\mathrm{I_0}-(\beta \mathrm{I_0}+\lambda\, \mathrm{C_0})\mathrm{S_0})}{\Gamma(1+2\alpha)}+\right.$$
$$\left.\frac{\hbar^2 t^{2\alpha}\Gamma(1+2\alpha)(A+\nu\, \mathrm{I_0}-(\mu+\beta \mathrm{I_0}+\lambda\, \mathrm{C_0})\mathrm{S_0})\left(\beta \mathrm{I_0}(m+\mu+\nu-\beta \mathrm{S_0})-\lambda\left(\gamma \mathrm{P_0}+\mathrm{C_0}(\beta\, \mathrm{S_0}-\theta_0)\right)\right)}{\Gamma(1+\alpha)^2\,\Gamma(1+3\alpha)}\right),$$

$$P_2 = \hbar\, t^{\alpha}\left(-\frac{(\hbar+n)(g\, \mathrm{C_0}-\mathrm{P_0}\,(\gamma+\phi \mathrm{W_0}+\mathrm{P_0}\alpha_1+\mu_1)\,)}{\Gamma(1+\alpha)}-\frac{(g\, \mathrm{C_0}(\gamma+\theta_0+\mu_1)-\mathrm{P_0}\,(\gamma(g+\gamma)+2\gamma\mu_1+\mu_1^2+\phi \mathrm{W_0}(\gamma+\mu_1)+\mathrm{P_0}\alpha_1\,(\gamma+\mu_1)))}{\Gamma(1+2\alpha)}+\right.$$
$$\left.\frac{\hbar^2 t^{2\alpha}\Gamma(1+2\alpha)(g\, \mathrm{C_0}-\mathrm{P_0}(\gamma+\phi \mathrm{W_0}+\mathrm{P_0}\alpha_1+\mu_1))(g\, \mathrm{C_0}\alpha_1-p_0^2\alpha_1^2-\phi \mathrm{W_0}\theta_1+\mathrm{P_0}(\phi \mathrm{W_0}(k\phi-\alpha_1)-\alpha_1(\gamma+\mu_1)\,))}{\Gamma(1+\alpha)^2\,\Gamma(1+3\alpha)}\right),$$

$$C_2 = \hbar\, t^{\alpha}\left(-\frac{(\hbar+n)(\gamma \mathrm{P_0}-\mathrm{C_0}\theta_0)}{\Gamma(1+\alpha)}+\frac{\hbar\, t^{\alpha}\,(\,\mathrm{C_0}(g\gamma+\theta_0^2)-\gamma \mathrm{P_0}(\gamma+\phi \mathrm{W_0}+\mathrm{P_0}\alpha_1+\theta_0+\mu_1\,))}{\Gamma(1+2\alpha)}\right),$$

$$W_2 = \hbar\, t^{\alpha}\mathrm{W_0}(k\phi \mathrm{P_0}-\theta_1)\left(-\frac{(\hbar+n)}{\Gamma(1+\alpha)}\right.$$
$$\left.+\hbar\, t^{\alpha}\left(-\frac{\theta_1}{\Gamma(1+2\alpha)}+\frac{4^{\alpha}\hbar\, k t^{\alpha}\phi\Gamma\left(\frac{1}{2}+\alpha\right)\left(-g\, \mathrm{C_0}+\mathrm{P_0}(\gamma+\phi \mathrm{W_0}+\mathrm{P_0}\alpha_1+\mu_1)\right)}{\sqrt{\pi}\Gamma(1+\alpha)\,\Gamma(1+3\alpha)}\right)\right).$$

Making use, compute components $\mathrm{S_m}$, $\mathrm{I_m}$, $\mathrm{P_m}$, $\mathrm{C_m}$ *and* $\mathrm{W_m}$, m bigger than 1 of the *q*- HATM and solution is denoted as

$$\mathrm{S(t)} = \sum_{m=0}^{\infty} \mathrm{S_m(t)}\left(\frac{1}{n}\right)^{\mathrm{m}}$$

$$\mathrm{I(t)} = \sum_{m=0}^{\infty} \mathrm{I_m(t)}\left(\frac{1}{n}\right)^{\mathrm{m}},$$

$$\mathrm{P(t)} = \sum_{m=0}^{\infty} \mathrm{P_m(t)}\left(\frac{1}{n}\right)^{\mathrm{m}},$$

$$\mathrm{C(t)} = \sum_{m=0}^{\infty} \mathrm{C_m(t)}\left(\frac{1}{n}\right)^{\mathrm{m}},$$

$$\mathrm{W(t)} = \sum_{m=0}^{\infty} \mathrm{W_m(t)}\left(\frac{1}{n}\right)^{\mathrm{m}}.$$

This section portrays numerical simulation of solutions of the system in terms distinctα. Numerical invetigate are found q- HATM. We utilize parameters. The initial conditions in conducting numericals are given by

$$S(0) = 100000, I(0) = 3700, P(0) = 400, C(0) = 10000, W(0) = 1000.$$

Further, all populations of system (1) are assumed in numbers.

Parameter	Value	Parameter	Value
A	1	μ_1	0.09
μ	0.00005	γ	0.33
υ	0.2	α_1	0.001
m	0.02	ϕ	0.01
β	0.005	k	1600
λ	0.09	θ_0	0.1
g	5	θ_1	0.25

Figures 1-5, S, I, P, C, W plotted with different α. From Figure 1, observe that number of susceptible individuals (S) rise with time whenα decreases the number of susceptible individuals (S) declines. From Figure 2, seethat number of infected individuals (I) decreases with time α decreases the number of susceptible individuals (S) increases. From Fig. 3, we can seethat the number of pupae of flies population (P) decreases with timeα decreases number of pupae of flies population (P) increases. From Fig. 4, observe that number of adult flies population (C) rises with time α decreases adult fly (C) decline. From Fig. 5, observe that number of parasitic wasps (W) depletes with time as α decreases parasitic wasps (W) rises. Fig. 6 indicates that the susceptible individuals get the disease due to direct interact between susceptible, infected individuals withtime. Fig. 7 indicates that effect of rate at which infected transmission due to susceptible and adult flies population into the number of infected individuals with respect to time. Fig. 8 indicates that the effect of the rate at which pupae of flies acquire adult stage with time. Fig. 9 indicates replication coefficient due to interact between parasitic wasps and pupae with time. Fig. 10 presents effect of pupae of flies and interaction between parasitic wasps and pupae with time. Fig. 11 expresses that the effect of the proportionality constant at which replication factor of parasitic wasps due to predation of pupae of flies with respect to time.

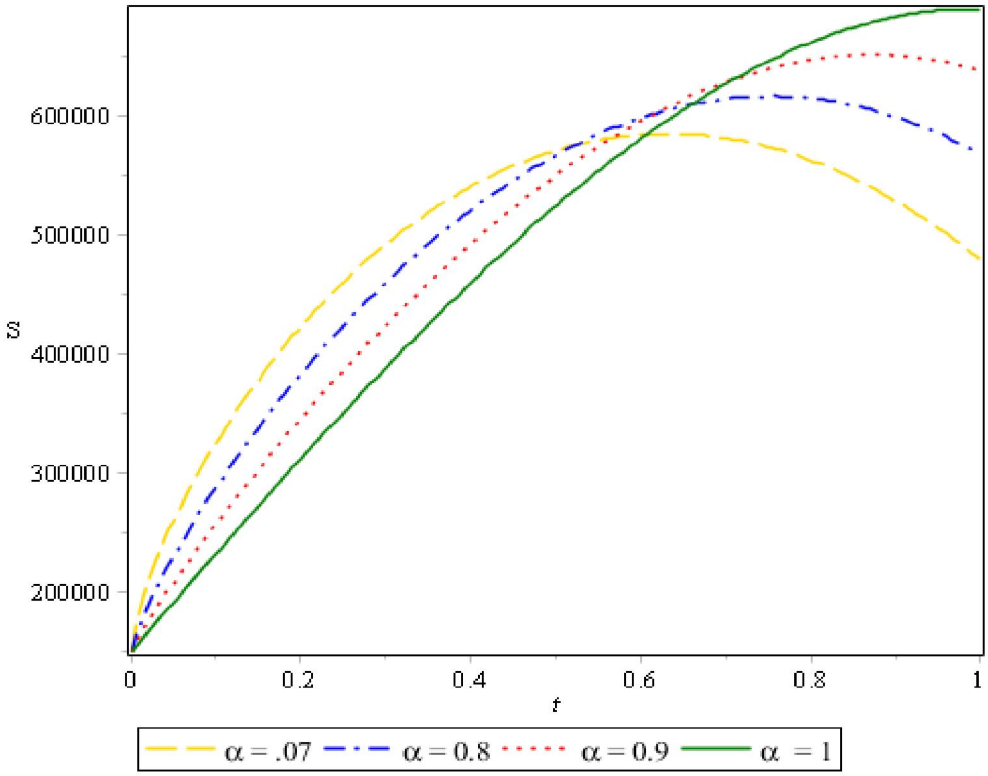

Fig. 1. Behaviour of S(t) with time t for distinct values of α

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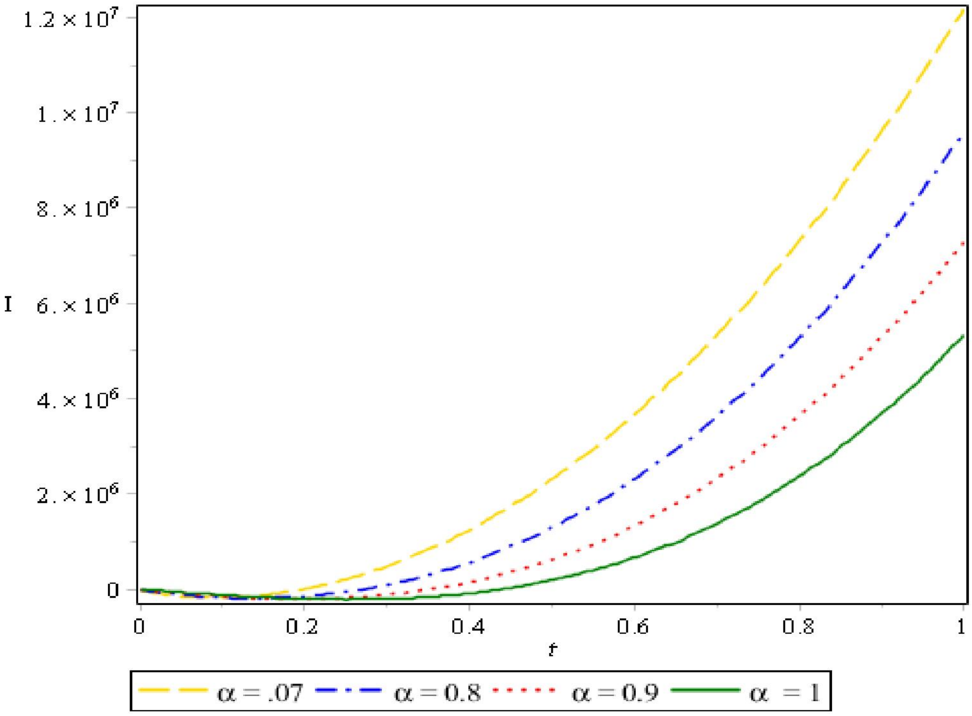

Fig. 2. Behaviour of I(t) with time for different values of α

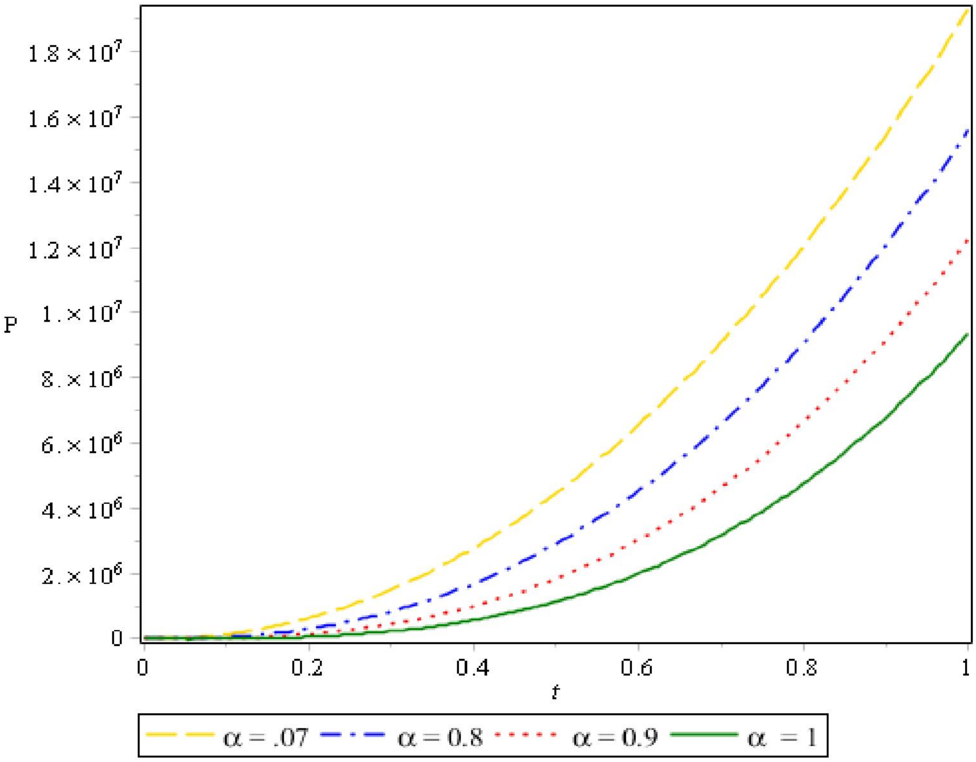

Fig. 3. Behaviour of P(t) with time for different values of α

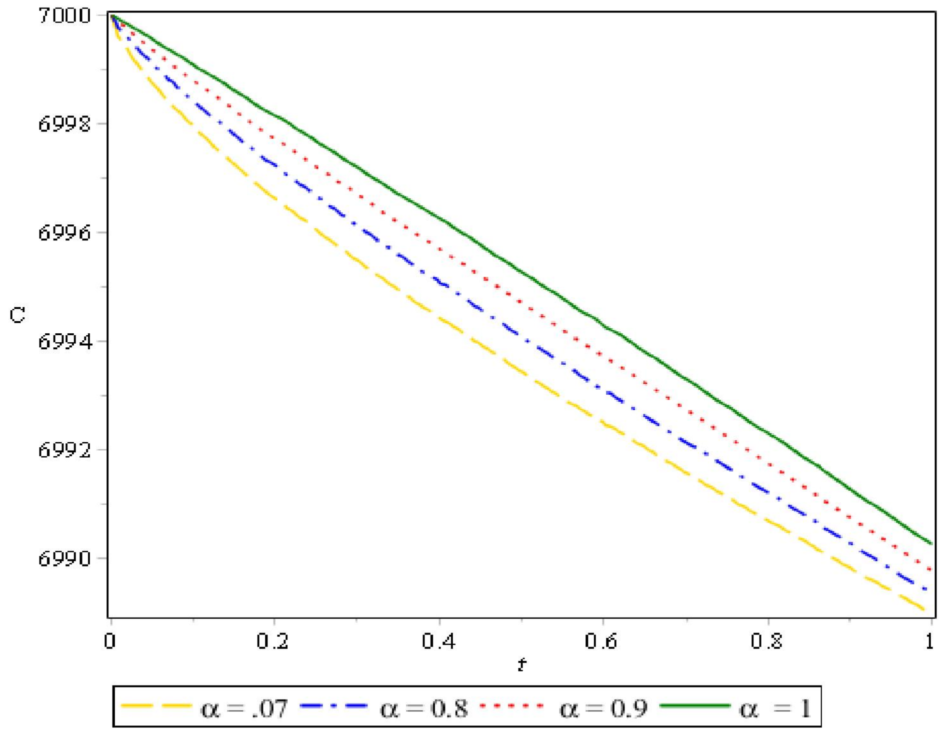

Fig. 4. Behaviour of C(t) with time for different values of α

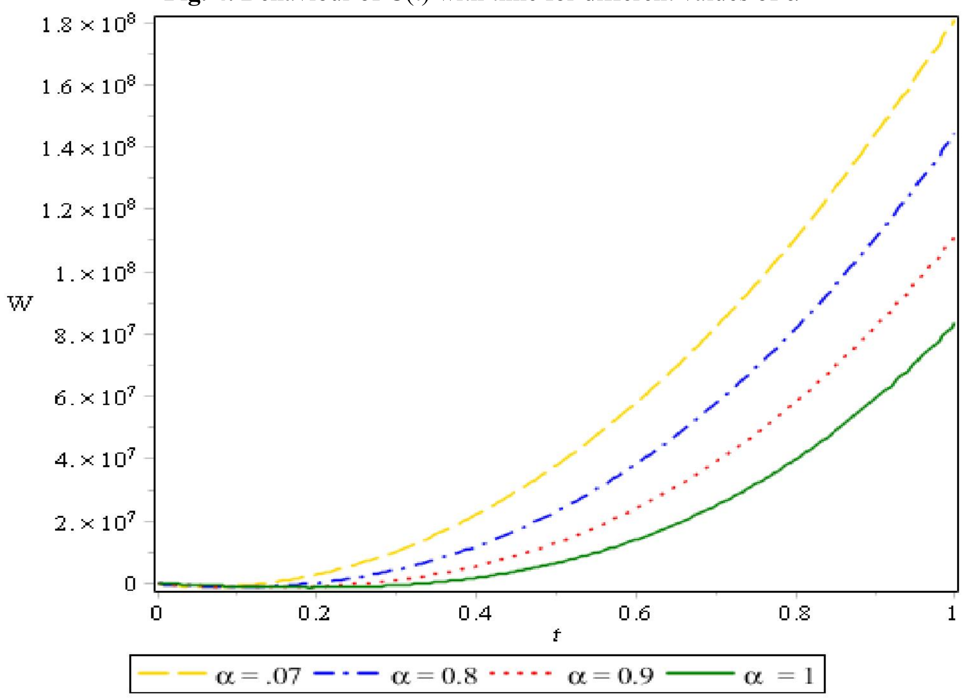

Fig. 5. Behaviour of W(t) with time for different values of α

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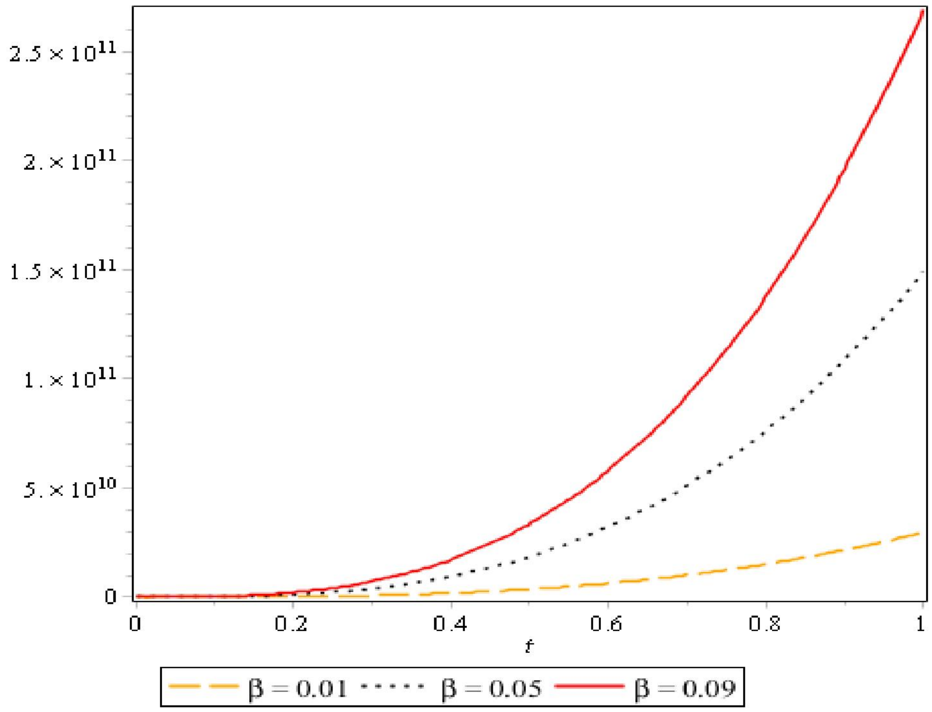

Fig. 6. Nature of I(t) with time t for different values of β

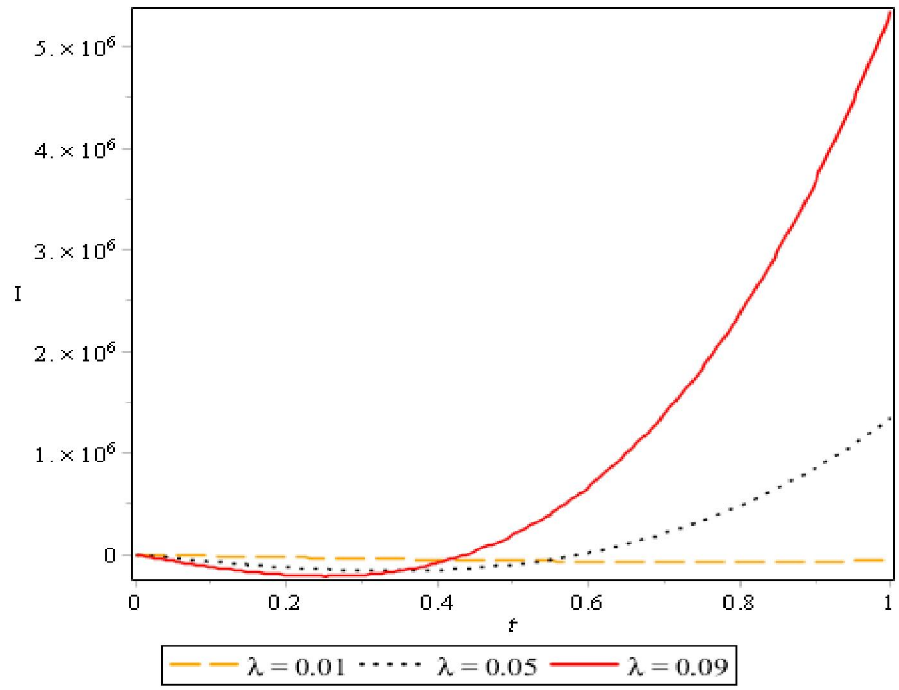

Fig. 7. Nature of I(t) with time t for different values of λ

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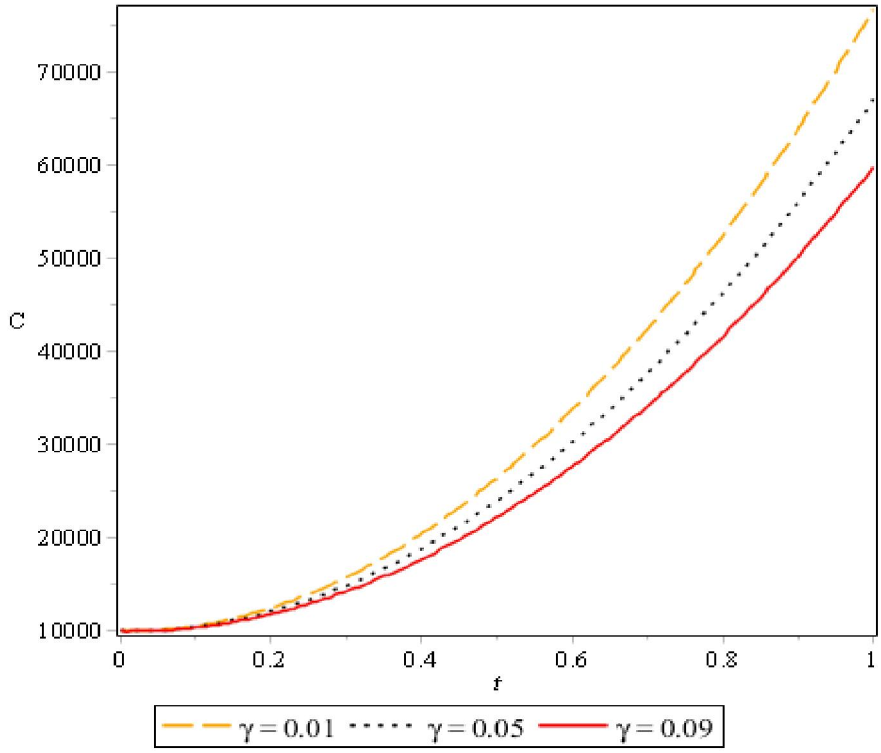

Fig. 8. Nature of C(t) w.r.t. time t for different values of γ

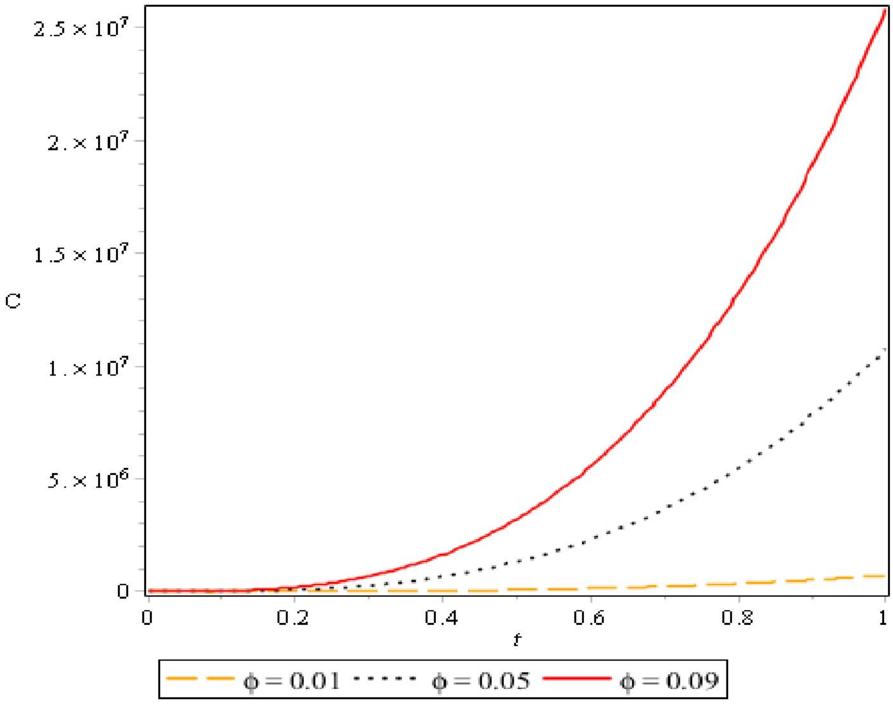

Fig. 9. Nature of C(t) with time t for different values of ϕ

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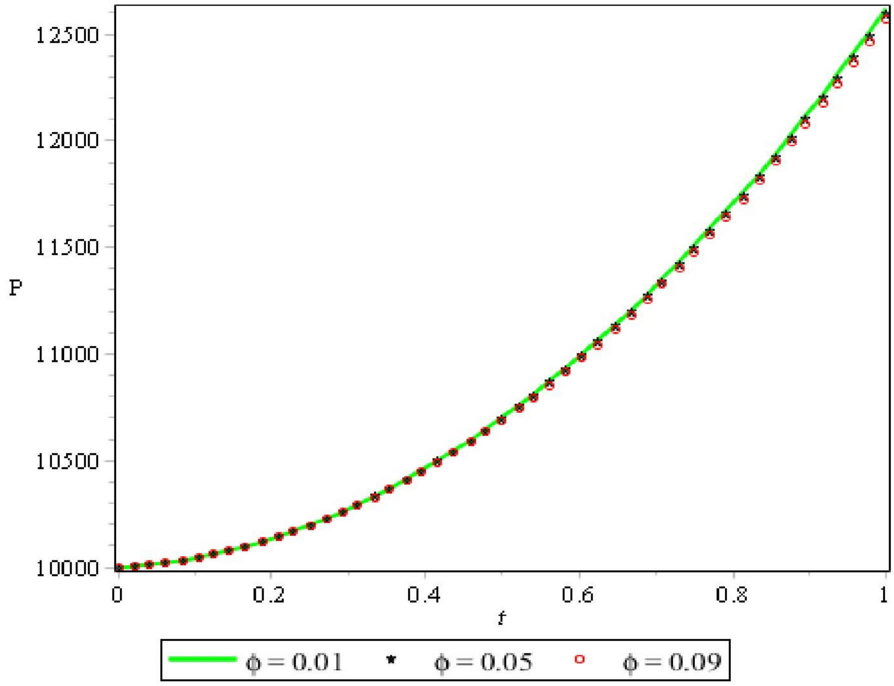

Fig. 10. Nature of P(t) with time t for different values of ϕ

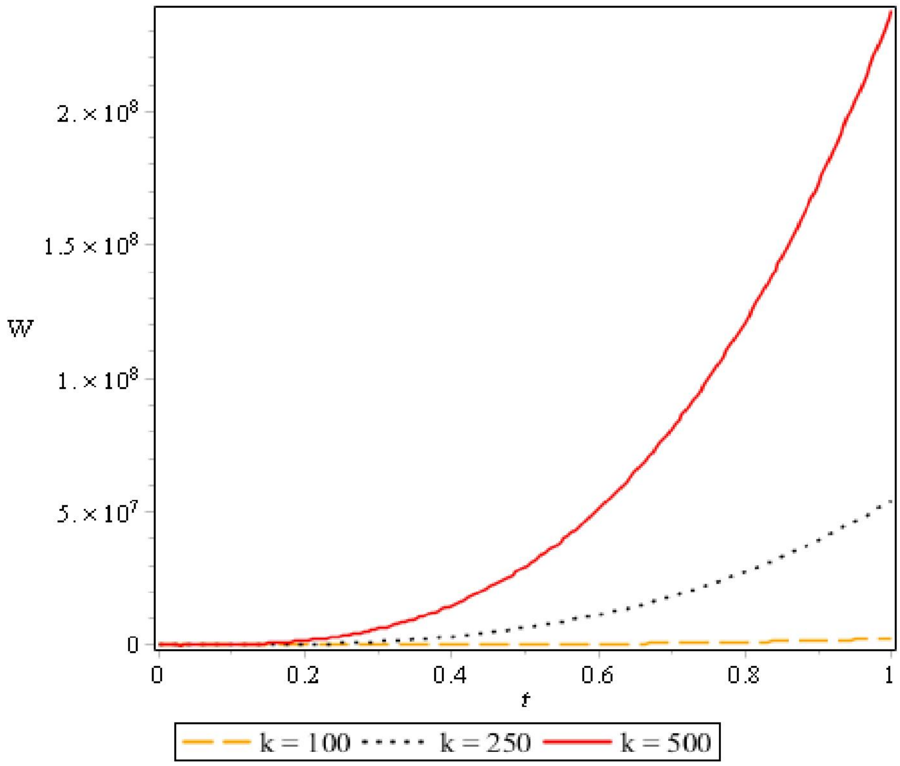

Fig. 11. Nature of W(t) with time t for different values of ϕ

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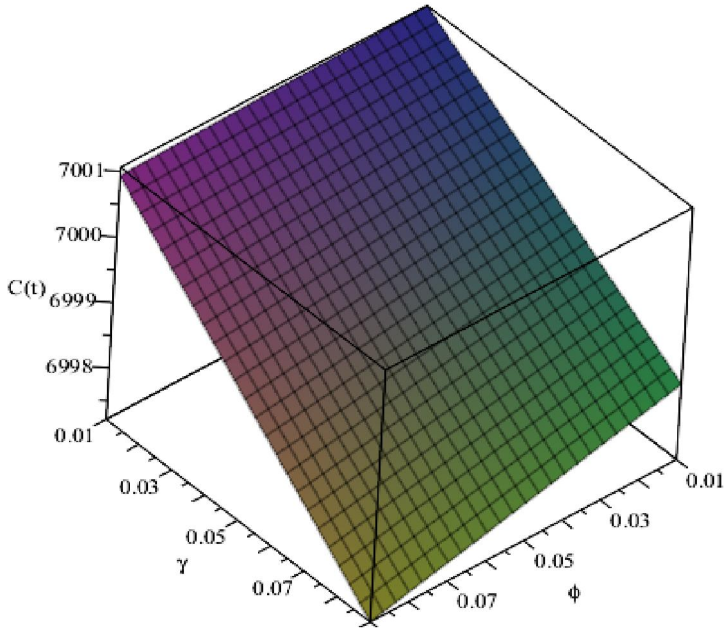

Fig. 12. Response of C(t) with ϕ and γ when $\alpha = 1$.

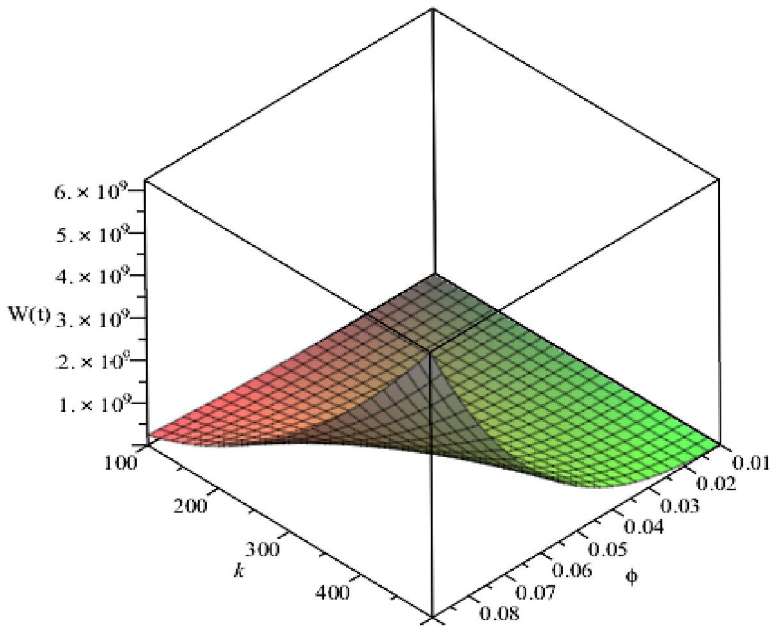

Fig. 13. Response of W(t) with ϕ and k when $\alpha = 1$.

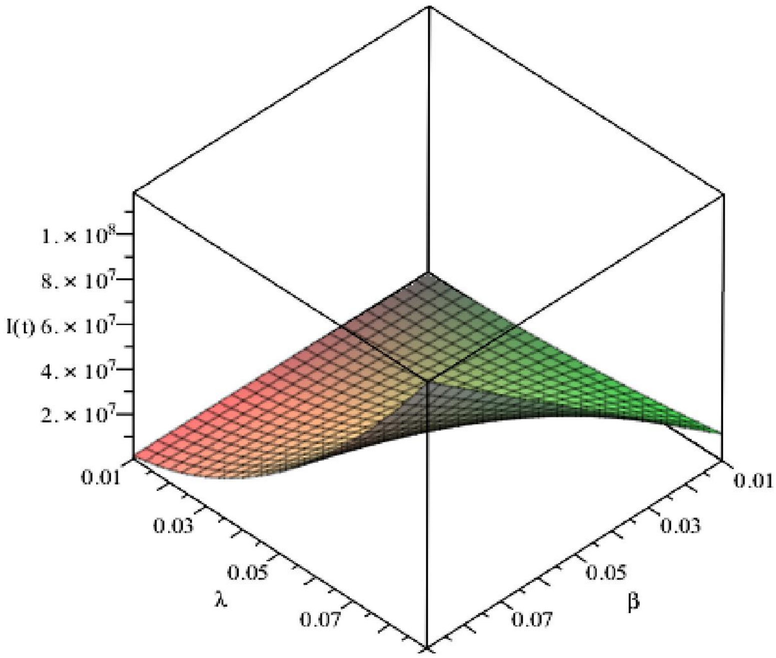

Fig. 14. Response of I(t) with λ and β when α = 1.

VI. CONCLUSION

q-HATM is more efficient, convenient and easier than other existing methods. Here, we summaries our results obtained from the analysis of the model system (2). The proposed model has been analyzed using fractional calculus involving very popular method q-HATM. In this investigation, we have proposed and analyzed a non-linear mathematical model for the spread of food-borne diseases and control by biocontrol agent parasitic wasps. The numerical experiments have been performed by using the biologically feasible parameter values, few of which are taken from the existing literature. The numerical experiment reveals that the flies control is possible with the use of parasitic wasps in the regions where flies are actively contributing in the spread of the food-borne diseases. Our model is applicable to any food-borne or water-borne diseases where the carriers like flies are likely to contaminate the food or water. Food and water are two very important things for the survival of nay species. Therefore, clean water and food must be provided so that the diseases may not harm human population.

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