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On the Iman Transform and Systems of Ordinary Differential Equations

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Abstract: *In this paper we introduce some properties and definition of the new integral transform, called Iman transform. Farther, we use Iman transform to solve systems of ordinary differential equations.*

Keywords: Iman transform-system of differential equations.

I. INTRODUCTION

In the literature there are numerous integral transforms and widely used in physics, astronomy as well as in engineering. The integral transform method is also an efficient method to solve the system of ordinary differential equations. Recently, Iman Almardy introduceda new transform and named as Iman transform which is defined by the following formula.

$$I\{f(t)\} = \frac{1}{v^2}\int_0^\infty e^{-v^2 t} f(t)dt = I(v)$$

$$k_1 \le v \le k_2$$

And applied this new transform to the solution of system of ordinary differential equations.

In this study, our purpose is to show the applicability of this interesting new transform and its efficiency in solving the linear system of ordinary differential equations.

Iman Almardy introduced the Iman transform defined by formula

$$I\{f(t), v\} = \frac{1}{v^4}\int_0^\infty f\left(\frac{t}{v^2}\right) e^{-v^2 t} dt = I(v)$$

$$k_1 \le v \le k_2 \ (1)$$

Theorem:

Let $I(v)$ is Iman transform of [I (f (t))=I (v)] . then:

(1) $I\left[\frac{df(t)}{dt}\right] = v^2\, I(v) - \frac{1}{v^2} f(0)$

(2) $I\left[\frac{d^2 f(t)}{dt^2}\right] = v^4\, I(v) - f(0) - \frac{1}{v^2} f'(0)$

(3) $I\left[\frac{d^n f(t)}{dt^n}\right] = v^n\, I(v) - \sum_{k=0}^{n-1} \frac{1}{v^{4-2n+2k}} f^{(k)}(0)$

Proof:

(1) $I\{f'(t)\} = \frac{1}{v^2}\int_0^\infty e^{-v^2 t} f^{'(t)} dt$ Integrating by parts to find that

$$I\left[\frac{df(t)}{dt}\right] = v^2\, I(v) - \frac{1}{v^2} f(0)$$

(2) $let\ g(t) = f'(t), then\ I[g'(t)] = v^2\, I[g(t)] - \frac{1}{v^2} g(0)$

By using (1) we find that:

$$I\left[\frac{d^2 f(t)}{dt^2}\right] = v^4\, I(v) - f(0) - \frac{1}{v^2} f'(0)$$

(3) Can be proof by mathematical induction.

II. SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS

Iman transform method is very effective for the solution of the response of a linear system governed by an ordinary differential equation to the initial data and | or to an external disturbance (or external input function). More precisely, we seek the solution of a linear system for its state at subsequent time $t>0$ due to the initial state at $t = 0$ and / or to the disturbance applied for $t>0$.

Example I: (System of First Order Ordinary Differential Equations)

Consider the system

$$\begin{cases} \dfrac{dx_1}{dt} = a_{11}x_1 + a_{12}x_2 + b_1(t) \\ \dfrac{dx_2}{dt} = a_{21}x_1 + a_{22}x_2 + b_2(t) \end{cases} \quad (2)$$

With the initial data. $x_1(0) = x_{10}$ and . $x_2(0) = x_{20}$

Where $a_{11}, a_{12}, a_{21}, a_{22}$ are constants

Introducing the matrices

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \frac{dx}{dt} = \begin{bmatrix} \dfrac{dx_1}{dt} \\ \dfrac{dx_2}{dt} \end{bmatrix}, A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$b(t)\begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix} and\ x_0 = \begin{bmatrix} x_{10} \\ x_{20} \end{bmatrix}$$

We can write the above system in a matrix differential system as

$$\frac{dx}{dt} = Ax + b(t), x(0) = x_0 \ (3)$$

We take Iman transform of the system with the initial conditions to get.

$$\begin{cases} \left(1 - \dfrac{a_{11}}{v^2}\right)\overline{x}_1 - \dfrac{a_{12}}{v^2}\overline{x}_2 = \dfrac{\overline{b}_1(v)}{v^2} + \dfrac{x_{10}}{v^4} \\ \\ \left(1 - \dfrac{a_{22}}{v^2}\right)\overline{x}_2 - \dfrac{a_{21}}{v^2}\overline{x}_1 = \dfrac{\overline{b}_2(v)}{v^2} + \dfrac{x_{10}}{v^4} \end{cases}$$

Where $\overline{x}_1$, $\overline{x}_2$, $\overline{b}_1$, $\overline{b}_2$ are Iman transform of x_1 , x_2 , b_1 , b_2 respectively.

The solution of these algebraic systems is**,**

$$\overline{x}_1(v) = \frac{\begin{vmatrix} \frac{\overline{b}_1(v)}{v^2} + \frac{x_{10}}{v^4} & -\frac{a_{12}}{v^2} \\ \frac{\overline{b}_2(v)}{v^2} + \frac{x_{20}}{v^4} & -\frac{a_{21}}{v^2} \end{vmatrix}}{\begin{vmatrix} 1 - \frac{a_{11}}{v^2} & -\frac{a_{12}}{v^2} \\ -\frac{a_{21}}{v^2} & 1 - \frac{a_{22}}{v^2} \end{vmatrix}}, \overline{x}_2(v) = \frac{\begin{vmatrix} 1 - \frac{a_{11}}{v^2} & \frac{\overline{b}_1(v)}{v^2} + \frac{x_{10}}{v^4} \\ -\frac{a_{21}}{v^2} & \frac{\overline{b}_2(v)}{v^2} + \frac{x_{20}}{v^4} \end{vmatrix}}{\begin{vmatrix} 1 - \frac{a_{11}}{v^2} & -\frac{a_{12}}{v^2} \\ -\frac{a_{21}}{v^2} & 1 - \frac{a_{22}}{v^2} \end{vmatrix}}$$

Expanding these determinants, results for $\overline{x}_1(v)$ and $\overline{x}_1(v)$ can readily be inverted, and the solutions for x_1(t) and x_2 (t) can be found in closed forms.

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Examples II

Solve the system of differential equations

$$\begin{cases} \frac{dx}{dt} = 2x - 3y, t > 0 \\ \frac{dy}{dt} = y - 2x \end{cases} \quad (2)$$

With the initial data

$$x(0) = 8, y(0) = 3$$

Take Iman transform of the system with the initial conditions to get

$$\begin{cases} \overline{x} - \frac{8}{v^4} = \frac{2}{v^2}\overline{x} - \frac{3}{v^2}\overline{y} \\ \overline{y} - \frac{3}{v^4} = \frac{1}{v^2}\overline{y} - \frac{2}{v^2}\overline{x} \end{cases}$$

Solve these equations for $\overline{x}$and $\overline{y}$we find:

$$\overline{x} = \frac{5}{v^2(v^2+1)} + \frac{3}{v^2(v^2-4)}$$

$$\overline{y} = \frac{5}{v^2(v^2+1)} - \frac{3}{v^2(v^2-4)}$$

By taking inverse Iman transform we have:

$$x = 5e^{-t} + 3e^{4t}, y = 5e^{-t} - 3e^{4t}$$

Examples III

Solve the second order couplet differential system

$$\begin{cases} \frac{d^2x}{dt^2} - 3x - 4y = 0 \\ \frac{d^2y}{dt^2} + x + y = 0 \end{cases} \quad t > 0 \ (4)$$

With initial conditions.

$$x(0) = y(0) = 0, \frac{dx}{dt}(0) = 2, \frac{dy}{dt}(0) = 0 \ (5)$$

The use of the Iman transform into (4) with (5) gives

$$\begin{cases} \left(1 - \frac{3}{v^4}\right)\overline{x} - \frac{4}{v^4}\overline{y} = \frac{2}{v^6} \\ \frac{1}{v^4}\overline{x} + \left(1 + \frac{1}{v^4}\right)\overline{y} = 0 \end{cases}$$

Using the same method in example (II) to find the solution in the form:

$$x(t) = t[e^t + e^{-t}], y(t) = \frac{1}{2}[e^t - e^{-t} - te^t - te^{-t}]$$

III. CONCLUSION

Iman transform provides powerful method for analyzing derivatives. It is heavily used to solve ordinary differential equations, Partial differential equations and system of ordinary differential equations.

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