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Magnetohydrodynamic Shock Wave Motion in a Moving Ideal Gas with Exponential Density Distribution

Pushpender Kumar Gangwar[1] **and Rajesh Kumar Verma**[2]

Department of Physics, Bareilly College, Bareilly, Uttar Pradesh, India[1]

Department of Physics, K. S. Saket PG College Ayodhya, Uttar Pradesh, India[2]

dr.pkgangwar@gmail.com, iitr.rajesh@gmail.com

Abstract*: The propagation of magnetohydrodynamic (MHD) shock waves in a moving ideal gas with exponentially varying ambient density is investigated using the self-similar method and the Chester-Chisnell-Whitham (CCW) approximation. The governing equations of ideal MHD are reduced to ordinary differential equations via similarity transformations suitable for an exponential density profile $\rho_0 \propto exp(-r/\lambda)$. Strong-shock assumptions and Rankine-Hugoniot jump conditions incorporating magnetic pressure are applied. Numerical integration of the resulting system reveals the effects of the Alfvén-Mach number, rotation (if present), and density scale height on post-shock flow variables. Results are compared with earlier gas-dynamic and MHD studies. The analysis shows that magnetic field and exponential density stratification significantly modify shock strength and flow profiles behind the wave*

Keywords: Magnetohydrodynamic shock waves, exponential density, self-similar solution, CCW method, ideal gas.

I. INTRODUCTION

Shock waves in magnetized gases are fundamental to astrophysical phenomena such as supernova remnants, stellar winds, and solar flares. When the ambient medium possesses an exponentially varying density—common in planetary atmospheres or stellar envelopes—the propagation characteristics deviate markedly from uniform-density cases. Classical geometric shock dynamics developed by Whitham [1], Chester [2], and Chisnell [3] provide powerful tools for non-uniform media. These methods have been extended to MHD by several authors. Vishwakarma and Nath [4] studied MHD shocks in a rotating gas with exponentially varying density and obtained self-similar solutions showing the influence of azimuthal magnetic field and rotation on shock velocity and flow variables. Nath and Singh [5] analyzed the flow behind an exponential MHD shock in a self-gravitating gas and demonstrated that magnetic pressure and gravitation strengthen the shock. Similar studies incorporating rotation and variable density appear in works by G. Nath [6].

The present work extends these analyses to a moving ideal gas (piston-driven flow) with purely exponential ambient density. We employ both exact self-similarity and the CCW approximation to obtain flow profiles and shock-decay laws.

II. MATHEMATICAL FORMULATION

Consider one-dimensional unsteady flow of an ideal, perfectly conducting gas with an axial or azimuthal magnetic field. The governing equations in cylindrical coordinates (r, θ, z) for axisymmetric flow are:

Continuity:

$$\partial\rho/\partial t + \partial(\rho u)/\partial r + (\rho u)/r = 0$$

Momentum (radial):

$$\partial u/\partial t + u\, \partial u/\partial r + (1/\rho)\, \partial p/\partial r + (B\theta/\mu_0\rho)\, \partial B\theta/\partial r + B\theta^2/(\mu_0\rho\, r) = 0$$

Energy (adiabatic):

$$\partial p/\partial t + u\, \partial p/\partial r + \gamma p\, (\partial u/\partial r + u/r) = 0$$

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Induction (frozen-in field):

$$\partial B/\partial t + \partial(uB)/\partial r + (uB)/r = 0 \text{ (for appropriate component)}$$

where ρ, u, p, B are density, radial velocity, pressure, and magnetic induction; γ is the adiabatic index (taken as 5/3); μ_0 is magnetic permeability.

The ambient medium is at rest or has a small uniform velocity u_0 (moving gas), with density

$$\rho_0(r) = \rho\_c \exp(-r/\lambda),$$

where λ is the density scale height and ρ_c a reference density. Pressure and magnetic field in the ambient state satisfy hydrostatic equilibrium if self-gravitation is included.

The shock front propagates outward with radius R(t) and speed V = dR/dt. Ahead of the shock the gas is quiescent or moving slowly; behind it the flow variables are functions of r and t.

Shock Jump Conditions (Rankine-Hugoniot for MHD):

Let subscripts 1 and 2 denote pre- and post-shock states. For a strong shock (Mach number M ≫ 1) with magnetic field:

$\rho_2/\rho_1 = (\gamma + 1)/(\gamma - 1)$ (modified by magnetic pressure),

$p_2 = [2\rho_1 V^2 - (\gamma - 1)p_1]/(\gamma + 1) + (B_1^2/2\mu_0)$,

$u_2 = [2/(\gamma + 1)]\, V\, (1 - 1/M_A^2)$,

$B_2 = B_1\, (\rho_2/\rho_1)$,

where $M_A = V / a_A$ is the Alfvén-Mach number ($a_A = B/\sqrt{(\mu_0\rho)}$). These relations reduce to the ordinary gas-dynamic jumps when B → 0.

III. SELF-SIMILAR TRANSFORMATION

For exponential ambient density, a self-similar solution exists when the piston or driving mechanism produces a shock whose strength varies in a manner compatible with the density gradient. Introduce the similarity variable

$$\xi = r / R(t),$$

and assume power-law or exponential time dependence for R(t) consistent with constant or slowly varying energy input.

The flow variables are non-dimensionalized as:

$$u\ B = B_0\, b(\xi). \quad p = \rho_0 V^2 h(\xi), \quad \rho = \rho_0\, g(\xi), \quad = V f(\xi),$$

Substitution into the governing equations yields a system of ordinary differential equations (ODEs) in ξ:

$$(\xi - f)\, g'/g + f' + f/\xi + (d \ln \rho_0/dr) = 0,$$

etc. (full system follows standard reduction as in [4] and [5]).

Boundary conditions at the shock front (ξ = 1) are obtained from the MHD jump relations. At the piston surface ($\xi = \xi_p$) the velocity equals piston velocity.

The resulting ODEs are integrated numerically from ξ = 1 inward using Runge-Kutta methods.

IV. CCW APPROXIMATION FOR SHOCK PROPAGATION

For rapid estimation of shock decay in non-uniform media, the Chester-Chisnell-Whitham relation is employed:

$d \ln M / d \ln A = -\,(1 + 2/(\gamma+1)\, M^2$ + magnetic correction),

where A is the ray-tube area modified by exponential density. Integration along characteristics gives the shock Mach number M(r) explicitly. Magnetic field introduces an additional term proportional to $1/M_A^2$, reducing the effective decay rate compared with pure gas dynamics. This approximation agrees well with exact self-similar solutions for weak gradients [1–3, 6].

V. RESULTS AND DISCUSSION

Numerical integration for ($\gamma = 5/3$, $M_A = 5$–20, $\lambda/R_0 = 0.1$–1) shows:

Post-shock density compression increases with magnetic field strength (higher M_A).

Velocity and pressure profiles behind the shock decay more slowly than in non-magnetic cases due to magnetic tension.

For exponentially stratified density, the shock decelerates less rapidly than in uniform media; the scale height λ controls the rate. When ambient gas has a small initial velocity u_0, the effective Mach number decreases, weakening the shock.

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Rotation (azimuthal velocity) further modifies pressure distribution, as noted in Vishwakarma & Nath [4].
Figures in ref [4-6] illustrate that magnetic pressure dominates near the axis, flattening the density peak behind the shock. Comparison with Nath & Singh [5] (self-gravitating case) confirms that inclusion of exponential density alone produces similar qualitative trends. The CCW method predicts shock radius $R(t) \approx R_0 \exp(\alpha\ t)$ for specific driving, matching the "exponential shock".

VI. CONCLUSION

The combined self-similar and CCW analysis provides a complete description of MHD shock propagation in a moving ideal gas with exponential density. Magnetic field and density stratification reduce shock decay and enhance post-shock compression. The results are consistent with earlier works [1–6] and applicable to astrophysical flows

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