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A Comparative Study of Aboodh and Laplace Transforms to Solve Ordinary Differential Equations of First and Second Order

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Abstract: *Real life problems which can be formulated in differential equations can be solved by different integral transform. In here, we will discuss some formulas and properties of two transform namely Aboodh transform and Laplace transform and will be used to solve the same set of Differential equation and will be compared with each other.*

Keywords: Ordinary Differential equation, Aboodh transform and Laplace transform

I. INTRODUCTION

Differential equation have played a significant role in mathematics and its relevance has increased day by day. There are so many techniques to study and solve differential equation. Mainly many integral transforms namely Hankel, Laplace, Fourier etc. were used to solve the given differential equation. In addition to these transform, one more transform was added by Abood which he called after his name "Aboodh Transform" which can be used to solve ordinary and partial differential equation.

"Aboodh Transform" is not widely known and used . Hence this integral transform will be used to solve some problems of ordinary differential equation and will be compared with the mostly used integral transform "Laplace transform" by discussing some formulas and properties of both the transforms.

Here, Laplace transform is denoted by LT, Aboodh transform is denoted by AT and differential equation by D.E

II. DEFINITION OF LAPLACE AND ABOODH TRANSFORM

Laplace Transform: For a given function $f(t) \forall t \geq 0$ LT is given by $$L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt = F(s)$$ L- Laplace operator , s =parameter	Aboodh Transform: For a given function $f(t)$ $\forall t\ > 0$ AT is given by $$A\{f(t)\} = \frac{1}{v}\int_0^\infty e^{-vt} f(t)dt = K(v)$$ $$k_1 \leq v \leq k_2$$ v=variable factor of t, $k_1, k_2 - finite\ or\ in\ finite$ A- Aboo dh operator

For both the transform the sufficient condition for the function to exist is:-

1) Function should be piecewise continuous
2) Function should be of exponential order

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III. AT AND LT OF SOME STANDARD FUNCTIONS

S. No	$f(t)$	$A\{f(t)\}$	$L\{f(t)\}$
1	1	$\frac{1}{v^2}$	$\frac{1}{s}$
2	t	$\frac{1}{v^3}$	$\frac{1}{s^2}$
3	e^{at}	$\frac{1}{v^2 - av}$	$\frac{1}{s-a}$
4	$\sin(at)$	$\frac{a}{v^3 + a^2 v}$	$\frac{a}{s^2 + a^2}$
5	$\cos(at)$	$\frac{1}{v^2 + a^2}$	$\frac{s}{s^2 + a^2}$

IV. DEFINITION OF INVERSE LAPLACE AND ABOODH TRANSFORM

Inverse Laplace Transform: If $F(s)$ *is LT of* $f(t)$, then $f(t) = L^{-1}\{F(s)\}$is the inverse of $F(s)$ $L^{-1}$is the inverse Laplace operator	Inverse Aboodh Transform: If $K(v)$ *is AT of* $f(t)$, then $f(t) = L^{-1}\{k(v)\}$is the inverse of $K(v)$ $A^{-1}$is the inverse Aboodh operator

V. INVERSE AT AND INVERSE LT OF SOME STANDARD FUNCTIONS

S. N	$f(t)$	$A\{f(t)\}$	$L\{f(t)\}$
1	1	$\frac{1}{v^2}$	$\frac{1}{s}$
2	t	$\frac{1}{v^3}$	$\frac{1}{s^2}$
3	e^{at}	$\frac{1}{v^2 - av}$	$\frac{1}{s-a}$
4	$\sin(at)$	$\frac{a}{v^3 + a^2 v}$	$\frac{a}{s^2 + a^2}$
5	$\cos(at)$	$\frac{1}{v^2 + a^2}$	$\frac{s}{s^2 + a^2}$

VI. ABOODH AND LAPLACE TRANSFORM OF DERIVATIVES OF FUNCTION $f(t)$

6.1) $AT\ of\ \frac{d(f(t))}{dt} : if\ A\{f(t)\} = K(v), then$

$$A\left[\frac{df(t)}{dt}\right] = v\,K(v) - \frac{1}{v}f(0)$$

$$A\left[\frac{d^2f(t)}{dt^2}\right] = v^2\,K(v) - f(0) - \frac{1}{v}f'(0)$$

6.1) $LT\ of\ \frac{d(f(t))}{dt} : if\ L\{f(t)\} = F(S), then$

$$L\left[\frac{df(t)}{dt}\right] = sF(s) - f(0)$$

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$$L\left[\frac{d^2f(t)}{dt^2}\right] = s^2\,F(s) - sf(0) - f'(0)$$

VII. SOLUTION OF ORDINARY DE OF 1ST ORDER BY LT AND AT

Consider the given linear ordinary $DE\ \frac{df}{dt} + k\,f = g(t)$ (1)
with initial condition $f(0) = a$: where function of 't'is denoted by $g(t)$ and k , a are constants

7.1 Solving the given DE by AT:

Apply AT on both side in equation (1), we get

$$A\{\frac{df}{dt}\} + kA\{\,f\,\} = A\{g(t)\}$$
$$v\,F(v) - \frac{1}{v}f(0) + kF(v) = G(v)$$
$$v\,F(v) + kF(v) = G(v) + \frac{a}{v}$$
$$(v\ + k)F(v) = G(v) + \frac{a}{v}$$
$$F(v) = \frac{G(v)}{(v\ + k)} + \frac{a}{v(v\ + k)}$$

With applying the inverse in the above step, we acquire the solution.

7.2 Solving the given DE by LT:

Apply LT on both side in equation (1), we get

$$L\{\frac{df}{dt}\} + kL\{\,f\,\} = L\{g(t)\}$$
$$sF(s) - f(0) + kF(s) = G(s)$$
$$s\,F(s) + kF(s) = G(s) + a$$
$$(s\ + k)F(s) = G(s) + a$$
$$F(s) = \frac{G(s)}{(s + k)} + \frac{a}{(s + k)}$$

With applying the inverse in the above step, we acquire the solution.

8. Examples of Ordinary of 1st order DE by LT and AT

1. $\boldsymbol{solve}\ \frac{df}{dt} + 13\,f = e^{11t}$ **,** $\boldsymbol{give\ f(0) = 1}$

LT	AT
$\frac{df}{dt} + 13\,f = e^{11t}$ Apply LT on both side in equation, we get $L\{\frac{df}{dt}\} + 13L\{\,f\} = L\{e^{11t}\}$ $sF(s) - f(0) + 13F(s) = \frac{1}{(s-11)}$ $s\,F(s) + 13F(s) - 1 = \frac{1}{(s-11)}$ $(s\ + 13)F(s) = \frac{1}{(s-11)} + 1$	$\frac{df}{dt} + 13\,f = e^{11t}$ Apply AT on both side in equation, we get $A\{\frac{df}{dt}\} + 13A\{\,f\} = A\{e^{11t}\}$ $vF(v) - vf(0) + 13F(v) = \frac{1}{v(v-11)}$ $v\,F(v) + 13F(v) - \frac{1}{v} = \frac{1}{v(v-11)}$ $(v\ + 13)F(v) = \frac{1}{v(v-11)} + \frac{1}{v}$

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$$F(s) = \frac{s-10}{(s-11)(s+13)}$$ Applying Inverse LT, we get $$\boldsymbol{f(t) = \frac{23}{24}e^{-13t} + \frac{1}{24}e^{-13t}}$$	$$F(v) = \frac{v-10}{v(v-11)(v+13)}$$ Applying Inverse AT, we get $$\boldsymbol{f(t) = \frac{23}{24}e^{-13t} + \frac{1}{24}e^{-13t}}$$

$$2.\ \boldsymbol{solve}\ \frac{df}{dt} + 27\,f = \cos 9t\ ,\boldsymbol{give}\ \boldsymbol{f(0) = 0}$$

LT	AT
$\frac{df}{dt} + 27\,f = cos9t$ Apply LT on both side in equation, we get $L\{\frac{df}{dt}\} + 27L\{f\} = L\{cos9t\}$ $sF(s) - f(0) + 27F(s) = \frac{s}{(s^2+81)}$ $s\,F(s) + 27F(s) = \frac{s}{(s^2+81)}$ $(s+27)F(s) = \frac{s}{(s^2+81)}$ $F(s) = \frac{s}{(s^2+81)(s+27)}$ Applying Inverse LT, we get $\boldsymbol{f(t) = \frac{1}{30}cos9t + \frac{1}{90}sin9t - \frac{1}{30}e^{-27t}}$	$\frac{df}{dt} + 27\,f = cos9t$ Apply AT on both side in equation, we get $A\{\frac{df}{dt}\} + 27A\{f\} = A\{cos9t\}$ $vF(v) - \frac{1}{v}f(0) + 27F(v) = \frac{1}{(v^2+81)}$ $v\,F(v) + 27F(v) = \frac{1}{(v^2+81)}$ $(v+27)F(v) = \frac{1}{(v^2+81)}$ $F(v) = \frac{1}{(v^2+81)(v+27)}$ Applying Inverse AT, we get $\boldsymbol{f(t) = \frac{1}{30}cos9t + \frac{1}{90}sin9t - \frac{1}{30}e^{-27t}}$

IX. SOLUTION OF ORDINARY DE OF 2ND ORDER by LT AND AT

Consider the given linear ordinary DE $\frac{d^2f}{dt^2} + k_1\frac{df}{dt} + k_2 f = g(t)$ (1) with initial condition f(0) =a and f'(0) = b where function of 't' is denoted by g(t) and k_1,k_2, a, b are constants

9.1 Solving the given DE by *AT*:

Apply ***AT*** on both side in equation (1), we get

$$A\{\frac{d^2f}{dt^2}\} + k_1A\{\frac{df}{dt}\} + k_2A\{f\} = A\{g(t)\}$$

$$\left[v^2\,F(v) - f(0) - \frac{1}{v}f'(0)\right] + k_1\left[v\,F(v) - \frac{1}{v}f(0)\right] + k_2\boldsymbol{F(v)} = \boldsymbol{G(v)}$$

$$\left[v^2\,F(v) - a - \frac{b}{v}\right] + k_1\left[v\,F(v) - \frac{a}{v}\right] + k_2\boldsymbol{F(v)} = \boldsymbol{G(v)}$$

$$v^2\,F(v) + k_1v\,F(v) + k_2\boldsymbol{F(v)} = \boldsymbol{G(v)} + \boldsymbol{a} + \frac{b}{v} + \frac{ak_1}{v}$$

$$[v^2 + k_1v + k_2]F(v) = \boldsymbol{G(v)} + \boldsymbol{a} + \frac{b}{v} + \frac{ak_1}{v}$$

$$[v^2 + k_1 v + k_2]F(v) = \boldsymbol{G(v)} + \boldsymbol{a} + \frac{b}{v} + \frac{ak_1}{v}$$
$$[v^2 + k_1 v + k_2]F(v) = \boldsymbol{G(v)} + \boldsymbol{a} + \frac{(ak_1 + b)}{v}$$

$$F(v) = \frac{\boldsymbol{G(v)}}{[v^2 + k_1 v + k_2]} + \frac{a}{[v^2 + k_1 v + k_2]} + \frac{(ak_1 + b)}{v[v^2 + k_1 v + k_2]}$$

With applying the inverse in the above step, we acquire the solution.

9.2 Solving the given DE by LT:

Apply LT on both side in equation (1), we get

$$L\{\frac{d^2 f}{dt^2}\} + k_1 L\{\frac{df}{dt}\} + k_2 L\{f\} = L\{g(t)\}$$
$$[s^2\, F(s) - sf(0) - f'(0)] + k_1[s\, F(s) - f(0)] + k_2\boldsymbol{F(s)} = \boldsymbol{G(s)}$$
$$[s^2\, F(s) - a - b] + k_1[sF(s)] + k_2\boldsymbol{F(v)} = \boldsymbol{G(v)}$$
$$s^2\, F(s) + k_1 sF(s) + k_2\boldsymbol{F(s)} = \boldsymbol{G(s)} + \boldsymbol{as} + b + ak_1$$
$$[s^2 + k_1 s + k_2]F(s) = \boldsymbol{G(v)} + \boldsymbol{as} + b + ak_1$$

$$F(s) = \frac{\boldsymbol{G(s)}}{[s^2 + k_1 s + k_2]} + \frac{as}{[s^2 + k_1 s + k_2]} + \frac{(ak_1 + b)}{[s^2 + k_1 s + k_2]}$$

With applying the inverse in the above step, we acquire the solution

X. EXAMPLE OF ORDINARY OF 2ND ORDER DE BY LT AND AT

$$\boldsymbol{solve}\ \frac{d^2 f}{dt^2} + f = \cos t\ , \boldsymbol{give}\ \boldsymbol{f(0) = f'(0) = 0}$$

LT	***AT***
$\frac{d^2 f}{dt^2} + f = 3\cos 2t$ Apply LT on both side in equation, we get $L\left\{\frac{d^2 f}{dt^2}\right\} + L\{f\} = 3L\{cos2t\}$ $s^2\, F(s) - sf(0) - f'(0) + F(s) = \frac{3s}{(s^2+4)}$ $s^2\, F(s) + F(s) = \frac{3s}{(s^2+4)}$ $(s^2+1)F(s) = \frac{3s}{(s^2+4)}$ $F(s) = \frac{3s}{(s^2+4)(s^2+1)}$ Applying Inverse LT, we get $\boldsymbol{f(t) = cost - cos2t}$	$\frac{d^2 f}{dt^2} + f = 3\cos 2t$ Apply AT on both side in equation, we get $A\left\{\frac{d^2 f}{dt^2}\right\} + A\{f\} = 3A\{cos2t\}$ $v^2\, F(v) - f(0) - \frac{1}{v}f'(0) + F(v) = \frac{3}{(v^2+4)}$ $(v^2+1)F(v) = \frac{3}{(v^2+4)}$ $F(v) = \frac{3}{(v^2+4)(v^2+1)}$ Applying Inverse AT, we get $\boldsymbol{f(t) = cost - cos2t}$

XI. CONCLUSION

As compared by some examples both the methods that is Aboodh transform and Laplace Transform, both methods works almost the same way and give the exact solution to ordinary D.E. As AT is newly found, many properties are yet to be found, hence it restrict to solve some of ordinary D.E.

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