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New Methodology for Assignment Problems in Trapezoidal Fuzzy Numbers

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Abstract: *In order to solve a fuzzy Assignment Problem (FAP), a new idea was adopted. In this problem, C symbolises the cost of assigning n jobs to n workers, and it has been regarded trapezoidal fuzzy numbers. The appropriate way for solving FAP is to employ a ranking function for fuzzy costs. By introducing a fuzzy number into the costs, a numerical example is analysed.*

Keywords: Assignment problem, Trapezoidal Fuzzy numbers, Ranking function.

I. INTRODUCTION

An assignment problem is a type of linear programming problem that involves assigning various activities (jobs, tasks, or sources) to an equal number of service facilities (men, machines, labourers, etc.) on a one-to-one basis in such a way that the total time or total cost involved is minimised, and the total sale or total profit is maximised, or the group's total satisfaction is maximised. Lin and Wen (2004) used a labelling technique to solve the assignment issue with fuzzy interval number costs. Fuzzy Assignment Problem, Mohamed Muamer (2020), Journal of Science, Vol. 10, August 2020.

Srinivasan and Getharamani (2013) used Reuben's ranking strategy to solve a fuzzy assignment problem using one's own assignment method. A multi-objective assignment issue with interval parameters is presented by Saleh (2014).

In FAP issues, the coefficients are considered to be precise. In practise, the coefficients (some or all) aren't accurate due to measurement errors, or they fluctuate with market conditions, etc.

The proposed fuzzy assignment problem FAP is solved using the Hungarian approach and the ranking function for fuzzy costs in this study. Trapezoidal fuzzy numbers are a type of fuzzy number. Programming with coding has been implemented. For the sake of illustration, an illustrative example is provided.

II. PRELIMINARIES

In this section, some basic definitions are presented.

2.1 Definition of a Fuzzy set

Assume X is a universe of discourse , definition of fuzzy set is as follows;

$$A = \{< x, \mu_A(x) > / x \in X\}$$

Where μ_A: X→[0,1], μ_A(x) denotes the membership degree of the element x to the set A.

2.2 Normality of a Fuzzy set

A fuzzy set A of the universe of discourse of X is called a normal fuzzy set implying that there exists

$$x \in X : \mu_A(x) = 1.$$

2.3 Convexity of a Fuzzy set

A fuzzy set A on the universe of discourse of X is convex if and only if for all x_1 and x_2 in X :

$$\mu_A(\lambda x_1 + (1-\lambda)x_2 \geq \min\big(\mu_A(x_1), \mu_A(x_2)\big), where\ \lambda \in [0,1]$$

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2.4 $\alpha - cuts$

The most important fuzzy set concept is an $\alpha - cut$ and its variant, a strong $\alpha - cut$. Assume a fuzzy set A defined on X and any number $\alpha \in [0,1]$, *the* $\alpha - cut$, ${}^{\alpha}A$ *and the strong* $\alpha - cut$ ${}^{\alpha+}A$ *are the crisp sets*

$${}^{\alpha}A = \{x/A(x) \geq \alpha\}$$

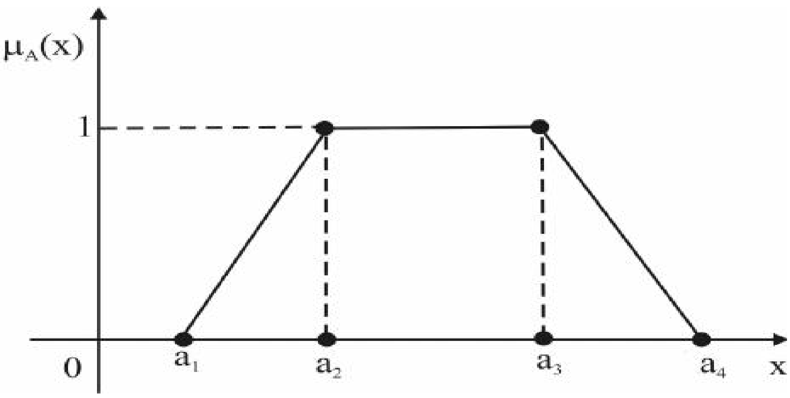
$${}^{\alpha+}A = \{x/A(x) > \alpha\}$$

Thus, the $\alpha - cut$(or the strong $\alpha - cut$) *of a fuzzy set A* is the crisp set ${}^{\alpha}A$(or the crisp set ${}^{\alpha+}A$) which contains most of the elements of X (universal set) whose membership grades in A are larger than or equal to (or only greater than) the specified value of α. α-cut can hence, be viewed as a bridge by which crisp sets and fuzzy sets were connected. Zadeh explained α-cut and strong α-cut in the form of first decomposition theorem in 1971. Special fuzzy set αAare defined as the decomposition of A which is the representation of arbitrary fuzzy set A in the α-cuts of A

2.5 Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $\tilde{a}$ quadruplet (a_1, a_2 a_3 a_4) is defined as follows. Its membership function is defined.]
$\mu_{\tilde{a}}(x)$A trapezoidal fuzzy number $\tilde{a}$ quadruplet (a_1, a_2 a_3 a_4) is defined as follows. Its membership function is defined as in [figure 1.4]

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{x - a_1}{a_2 - a_1} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \frac{a_4 - x}{a_4 - a_3} & a_3 \leq x \leq a_4 \\ 0 & a_4 > x \end{cases}$$



2.6 Ranking Function

A ranking function $\Re:(R)\dashrightarrow R$ where $F(R)$ set of fuzzy numbers defined on set of real numbers, maps each fuzzy number into real number, where a natural order exists.

2.6.1 Ranking function for trapezoidal fuzzy numbers

The value of a Trapezoidal fuzzy number is obtained by the ranking formula

$A = (a, b, c, d)$ is given by $\frac{a}{6} + \frac{b}{3} + \frac{c}{3} + \frac{d}{6}$

III. ASSIGNMENT PROBLEM

An assignment problem is a form of linear programming problem in which the goal is to reduce the cost or time it takes for a group of people to complete a set of tasks. Furthermore, an assignment problem has the same form as a transportation problem. There are two sorts of assignment problems: balanced assignment and unbalanced assignment.

The basic goal of the assignment problem is to reduce the overall time it takes to complete a series of activities, optimise skill ratings, or reduce the cost of the assignments. There must be as many facilities as tasks in the assignment problem, say n of each.

Solution:

Step 1: Prepare a Square Matrix.

Step 2: Subtract the minimum element of each row from the all the elements of the row. Do the same for all remaining rows.

Step 3: Now subtract the minimum element of each column from all the elements of the column. Do the same for all remaining columns.

Step 4: Draw the minimum number of horizontal and vertical lines to cover all the zeros in the resulting matrix. As a general rule, first cover the row/column which have more zeros.

If the number of minimum lines is equal to the number of rows (columns) then optimum solution can be made. So, go to step 7.

If the number of minimum lines is not .equal to the number of rows (columns) then optimum solution cannot be made. Go to step 5.

Step 5: Determine the smallest element in the matrix, not covered by the minimum number of lines. Subtract this minimum element from all uncovered elements and add the same element at the intersection of horizontal and vertical lines.

Step 6: Repeat steps 4 and 5 until the minimum number of lines is equal to the number rows (column).

Step 7: Examine the rows successively, until a row with a single zero is found, mark this zero by '□' to make the assignment. Then, mark a cross (×) over all zeros if lying in the row and column of the marked '□' zero, showing that they cannot be considered for future assignment. Continue in this manner until all the rows have been examined. Repeat the same procedure for columns also.

Step 8: Repeat the step 7 until one of the following situation arise:

(i) If no unmarked zero is left, then the process ends; or

(ii) If there are more than one of the unmarked zeros in any column or row, then mark '□'. One of the unmarked zeros arbitrarily and mark a cross in the cells of remaining zeros in its row and column. Repeat the process until no unmarked zero is left in the matrix.

Step 9: Thus exactly one marked '□' zero in each row and each column of the matrix is obtained. The assignment corresponding to these marked '□' zeros will give the optimal assignment.

Write a mathematical model of assignment problem.

Solution:

Consider an assignment problem of assigning n jobs to m machines. Let C_{ij} be the unit cost of assigning i^{th} machine to the j^{th} job and

$$\text{Let } x_0 = \begin{cases} 0, \text{ if the } i^{th} \text{ facility is not assigned to } j^{th} \text{ job.} \\ 1, \text{ if the } i^{th} \text{ facility is assigned to } j^{th} \text{ job.} \end{cases}$$

Then, the mathematical model is,

Minimize

$$z = \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to constraints,

$$\sum_{i=1}^{n} X_{ij} = 1,\ j = 1, 2, \ldots n$$

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And $\quad X_{ij} = 0(or)1$

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IV. FUZZY ASSIGNMENT PROBLEM

Fuzzy Assignment Cost				
Job→ Person↓	1	2	k	n
1	C_{11}	C_{12}	C_{1k}	C_{1n}
2	C_{21}	C_{22}	C_{2k}	C_{2n}
k	C_{k1}	C_{k2}	C_{k3}	C_{kn}
n	C_{n1}	C_{n2}	C_{nk}	C_{nn}

Consider an fuzzy assignment problem of assigning n jobs to m machines. Let C_{ij} be the unit cost of assigning i^{th} machine to the j^{th} job and

$$\text{Let } x_0 = \begin{cases} 0, \text{ if the } i^{th} \text{ facility is not assigned to } j^{th} \text{ job.} \\ 1, \text{ if the } i^{th} \text{ facility is assigned to } j^{th} \text{ job.} \end{cases}$$

Then, the mathematical model is,

Minimize

$$z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to constraints,

$$\sum_{i=1}^{n} X_{ij} = 1, \ j = 1, 2, \ldots n$$

$$\sum_{j=1}^{n} X_{ij} = 1, \ j = 1, 2, \ldots n$$

And $\quad X_{ij} = 0(or)1$

Definition 4.1. Optimal Solution of Fuzzy Assignment Problems

The optimal solution of the Fuzzy Assignment Problem is the set of non-negative integers { } which satisfies the following characteristics:

$\sum_{i=1}^{i=n} x_{ij} = 1, j = 1,2,3, \ldots .n$ and$\sum_{i=1}^{i=n} x_{ij} = 1, i = 1,2,3, \ldots .n$

4.2 Assignment Algorithm

An approach is proposed for transformation Fuzzy Assignment Problem (FAP) into a crisp Assignment Problem (AP) using the ranking function for fuzzy assignment cost matrix.

Step 1. For the entire fuzzy assignment cost matrix $(\tilde{C}ij)\times n$, find ranking function for all fuzzy assignment cost matrix $(\Re(\tilde{C}ij))n\times n$

Step 2. Replace the fuzzy cost $\tilde{C}ij$ by their respective $\Re(\tilde{C}ij)$, by the ranking function $\frac{a}{6}+\frac{b}{3}+\frac{c}{3}+\frac{d}{6}$then the fuzzy assignment is given below

Fuzzy Assignment Cost				
Job→ Person↓	1	2	k	n
1	$R(C_{11})$	$R(C_{12})$	$R(C_{1k})$	$R(C_{1n})$
2	$R(C_{21})$	$R(C_{22})$	$R(C_{2k})$	$R(C_{2n})$

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k	$R(C_{k1})$	$R(C_{k2})$	$R(C_{k3})$	$R(C_{kn})$
n	$R(C_{n1})$	$R(C_{n2})$	$R(C_{nk})$	$R(C_{nn})$

V. NUMERICAL EXAMPLE

Illustrations 1 :Solve the following fuzzy assignment problem which is shown below. The cell entries represent the processing time in hours (C_{ij}) of the job i if it is assigned to the operator j.Consider a fuzzy assignment problem with rows representing four jobs and columns representing the four operators with Assignment cost. The cost matrix $(\tilde{C}ij)\times n$ is given whose elements are trapezoidal fuzzy numbers

		Operator (j)			
		1	2	3	4
Job(i)	1	(18,21,24,30)	(12,18,21,30)	(12,15,27,30)	(18,21,24,36)
	2	(12,18,21,24)	(12,15,27,30)	(12,18,21,30)	(12,18,21,30)
	3	(12,18,21,30)	(12,15,21,24)	(18,21,24,36)	(12,18,24,36)
	4	(12,18,24,36)	(12,15,21,24)	(12,15,27,30)	(18,21,24,30)

		Operator (j)			
		1	2	3	4
Job(i)	1	23	20	21	24
	2	19	21	20	20
	3	20	18	24	22
	4	22	18	21	23

Solution:

Here, No. of rows = 4

No. of columns = 4

Hence, the given problem is a balanced assignment problem. Fuzzy assignment problem is consider and use the ranking function $\frac{a}{6}+\frac{b}{3}+\frac{c}{3}+\frac{d}{6}$

Step 1: Select the smallest element from first row and subtract it from all other elements in the first row. Do the same for all remaining rows. we get,

	1	2	3	4
1	3	0	1	4
2	0	2	1	1
3	2	0	6	4
4	4	0	3	5

Step 2: Select the smallest element from first column and subtract it from all other elements from the first column. Do the same for all remaining columns we get,

	1	2	3	4
1	3	0	0	3
2	0	2	0	0
3	2	0	5	3
4	4	0	2	4

Step 3: Draw the minimum number of lines covering all zeros. As a rule generally, we should first cover those rows/columns which contains larger number of zeros.

	1	2	3	4
1	3	0	0	3
2	0	2	0	0
3	2	0	5	3
4	4	0	2	4

Step 4: Select the smallest uncovered (by the lines) cost element (here it is 2). Subtract this element from all uncovered elements including itself and add this element to each element located at the intersection of any two lines. The cost elements through which only one line passes remain unaltered. The reduced table is

	1	**2**	**3**	**4**
1	3	0	0	3
2	0	4	0	0
3	0	0	3	1
4	2	0	0	2

Step 5: Draw the minimum number of lines covering all zeros. As a rule generally, we should first cover those rows/columns which contains larger number of zeros.

	1	**2**	**3**	**4**
1	3	2	0	3
2	0	4	0	0
3	0	0	3	1
4	2	0	0	2

Step 6: Minimum no. of lines drawn in equal to 5 (no of rows/column). Hence optimal solution is reached. Check all the rows until a row with exactly one zero is found. Mark this zero, indicating that an assignment will be made there, cross out all zeroes in the column corresponding to the marked zero, repeat the same procedure until each row has a marked zero.

	1	**2**	**3**	**4**
1	3	2	[0]	3
2	~~0~~	4	~~0~~	[0]
3	[0]	~~0~~	3	1
4	2	[0]	~~0~~	1

Optimal assignment is,

Job 1 → Operator 3 → 21

Job 2 → Operator 4 → 20

Job 3 → Operator 1 → 20

Job 4 → Operator 2 → <u>18</u>

The fuzzy optimal time is(48,66,90,114)

Minimum Processing Time is 79 hours.

VI. CONCLUSION

Hungarian approach was used to solve a fuzzy assignment problem with a Trapezoidal fuzzy number using a new ranking function. As an example, four jobs are assigned to four operators with little processing time.

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