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On Non-homogeneous Ternary Cubic Equation

$$5(x^2+y^2)-6xy=z^3$$

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Abstract: *This paper aims at determining non-zero distinct integer solutions to the non-homogeneous ternary cubic equation* $5(x^2+y^2)-6xy=z^3$.*Various choices of integer solutions are exhibited.*

Keywords: Ternary Cubic, Non-Homogeneous Cubic, Integer Solutions

I. INTRODUCTION

The Diophantine equations are rich in variety and offer an unlimited field for research [1-4]. In particular refer [5-27] for a few problems on ternary cubic equation with 3 unknowns. This paper concerns with yet another interesting ternary cubic diophantine equation with three variables given by $5(x^2+y^2)-6xy=z^3$ for determining its infinitely many non-zero distinct integral solutions.

II. METHOD OF ANALYSIS

The non-homogeneous ternary cubic equation to be solved is

$$5(x^2+y^2)-6xy=z^3 \qquad (1)$$

To start with, observe that (1) is satisfied by the triples given by

$$(x,y,z)=(12\alpha^{3k},4\alpha^{3k},8\alpha^{2k}),(a(5a^2+5b^2-6ab)\alpha^{3t},b(5a^2+5b^2-6ab)\alpha^{3t},(5a^2+5b^2-6ab)\alpha^{2t})$$

$$((2s-1)(2s^2-4s+4),(2s-3)(2s^2-4s+4),2(2s^2-4s+4))$$

However, there are other choices of integer solutions to (1) that are illustrated below:

Illustration 1:

Introduction of the linear transformations

$$x=4u+2v,y=4u-2v,z=4w \qquad (2)$$

in (1) leads to

$$u^2+v^2=w^3 \qquad (3)$$

which is satisfied by

$$u=m(m^2+n^2),v=n(m^2+n^2),w=(m^2+n^2) \qquad (4)$$

In view of (2) , the corresponding integer solutions to (1) are as follows:

$$x=(4m+2n)(m^2+n^2),y=(4m-2n)(m^2+n^2),z=4(m^2+n^2)$$

Note 1:

Observe that (3) is also satisfied by

$$u=m(m^2-3n^2),v=n(3m^2-n^2),w=(m^2+n^2)$$

The corresponding integer solutions to (1) are given by

$$x=4m(m^2-3n^2)+2n(3m^2-n^2),\ y=4m(m^2-3n^2)-2n(3m^2-n^2),z=4(m^2+n^2)$$

Note 2:

(3) is written as

$$u^2 + v^2 = w^3 * 1 \tag{5}$$

Consider the integer 1 on the R.H.S. of (5) as

$$1 = \frac{(p^2 - q^2 + i2pq)(p^2 - q^2 - i2pq)}{(p^2 + q^2)^2}, p \geq q \geq 0 \tag{6}$$

Substitute (6) and the value of w given by (4) in (5). Employing the method of factorization & equating the real and imaginary parts ,the values of u, v are obtained. Replacing

$m = (p^2 + q^2)M, n = (p^2 + q^2)N$ in the values of u, v, w and using (2) , the corresponding integer solutions to (1) are given by

$$x = (p^2 + q^2)^2[(M^3 - 3MN^2)(4p^2 - 4q^2 + 4pq) + (3M^2N - N^3)(2p^2 - 2q^2 - 8pq)],$$
$$y = (p^2 + q^2)^2[(M^3 - 3MN^2)(4p^2 - 4q^2 - 4pq) + (3M^2N - N^3)(-2p^2 + 2q^2 - 8pq)],$$
$$z = 4(p^2 + q^2)^2(M^2 + N^2)$$

Illustration 2:

Introduction of the linear transformations

$$x = u + v, y = u - v, z = 2w, u \neq v \neq 0 \tag{7}$$

in (1) leads to

$$u^2 + 4v^2 = 2w^3 \tag{8}$$

Take w as

$$w = 4(a^2 + b^2) \tag{9}$$

The integer 2 on the R.H.S. of (8) is taken as

$$2 = (1+i)(1-i) \tag{10}$$

Substituting (9) & (10) in (8) and employing the method of factorization , consider

$$u + i2v = (1+i)(2a + i2b)^3$$

Equating the real and imaginary parts in the above equation , the values of u, v are obtained.

In view of (7), the corresponding integer solutions to (1) are as follows:

$$x = 12a^3 - 36ab^2 - 12a^2b + 4b^3,$$
$$y = 4a^3 - 12ab^2 - 36a^2b + 12b^3,$$
$$z = 8(a^2 + b^2)$$

Note 3:

In addition to (10) , the integer 2 on the R.H.S. of (8) is expressed as exhibited below:

$$2 = \frac{(7+i)(7-i)}{25}, \frac{(1+7i)(1-7i)}{25}$$

Following the above procedure , two more sets of integer solutions to (1) are obtained.

Note 4:

Consider (8) as

$$u^2 + 4v^2 = 2w^3 * 1$$

The repetition of the process as in Note 2 of Illustration 1, one obtains a different set of integer solutions to (1).

III. CONCLUSION

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous cubic diophantine equation with three unknowns given by $5(x^2+y^2)-6xy=z^3$. One may search for other sets of integer solutions to the considered equation as well as other choices of the third degree diophantine equations with multi-variables

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