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On Finding Integer Solutions to Non-homogeneous Ternary Cubic Equation

$$(x^2+y^2)-xy-k(x+y)+k^2=z^3$$

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Abstract: *This paper concerns with the problem of obtaining non-zero distinct integer solutions to the non-homogeneous ternary cubic equation* $(x^2+y^2)-xy-k(x+y)+k^2=z^3$. *Different sets of integer solutions are illustrated.*

Keywords: Non-Homogeneous Cubic, Ternary Cubic, Integer Solutions

I. INTRODUCTION

The Diophantine equations are rich in variety and offer an unlimited field for research [1-4]. In particular refer [5-13] for a few problems on ternary cubic equation with 3 unknowns. This paper concerns with yet another interesting ternary cubic diophantine equation with three variables given by $(x^2+y^2)-xy-k(x+y)+k^2=z^3$ for determining its infinitely many non-zero distinct integral solutions

II. METHOD OF ANALYSIS

The non-homogeneous ternary cubic equation under consideration is

$$(x^2+y^2)-xy-k(x+y)+k^2=z^3 \quad (1)$$

Introduction of the linear transformations

$$x=u+v,\ y=u-v,\ u\neq v\neq 0 \quad (2)$$

in (1) leads to

$$U^2+3v^2=z^3 \quad (3)$$

where $U=u-k$ (4)

We solve (3) through different ways and using (2), one obtains different sets of

Integer solutions to (1).

Way 1:

It is observed that (3) is satisfied by

$$U=m(m^2+3n^2)\alpha^{3t},\ v=n(m^2+3n^2)\alpha^{3t},\ z=(m^2+3n^2)\alpha^{2t} \quad (5)$$

Substituting the value of U from (5) in (4),note that

$$u=m(m^2+3n^2)\alpha^{3t}+k$$

In view of (2),it is seen that

$$x=(m+n)(m^2+3n^2)\alpha^{3t}+k,\ y=(m-n)(m^2+3n^2)\alpha^{3t}+k \quad (6)$$

Thus the values of x, y, z given by (6) and (5) represent the integer solutions to (1).

Way 2:

Let $z=a^2+3b^2$ (7)

Substituting (7) in (3) and employing the method of factorization ,we have

$$U = a^3 - 9ab^2, v = 3a^2 b - 3b^3 \quad (8)$$

Using (8) in (4) and in view of (2) ,one has

$$x = a^3 - 9ab^2 + 3a^2 b - 3b^3 + k, y = a^3 - 9ab^2 - 3a^2 b + 3b^3 + k \quad (9)$$

Thus the values of x, y, z given by (9) and (7) represent the integer solutions to (1).

Way 3:

Rewrite (3) as

$$U^2 + 3v^2 = z^3 * 1 \quad (10)$$

Consider 1 on the R.H.S. of (10) as

$$1 = \frac{(1 + i4\sqrt{3})(1 - i4\sqrt{3})}{49} \quad (11)$$

Following the analysis similar to Way2 , and replacing a by 7A ,b by 7B, the values of x, y, z satisfying (1) are given by

$$x = 7^2[5A^3 - 45AB^2 - 33A^2B + 33B^3] + k$$

$$y = 7^2[-3A^3 + 27AB^2 - 39A^2B + 39B^3] + k$$

$$z = 49(A^2 + 3B^2)$$

Note 1:

The integer 1 on the R.H.S. of (10) is also expressed as

$$1 = \frac{(3r^2 - s^2 + i\sqrt{3}2rs)(3r^2 - s^2 - i\sqrt{3}2rs)}{(3r^2 + s^2)^2},$$

$$1 = \frac{(1 + i\sqrt{3})(1 - i\sqrt{3})}{2^2},$$

$$1 = \frac{(11 + i5\sqrt{3})(11 - i5\sqrt{3})}{14^2}$$

Repeating the above process, different sets of solutions to (1) are obtained.

Way 4:

Introduction of the linear transformations

$$U = X + 3T, v = X - T, z = 2w \quad (12)$$

in (3) leads to

$$X^2 + 3T^2 = 2w^3 \quad (13)$$

The choice

$$w = X \quad (14)$$

gives

$$3T^2 = X^2(2X - 1)$$

which is satisfied by

$$T = (2s - 1)(6s^2 - 6s + 2), X = 6s^2 - 6s + 2 \quad (15)$$

From (14) ,one has

$$w = 6s^2 - 6s + 2 \quad (16)$$

Substituting (15) ,(16) in (12) and employing (4) &(2) , the corresponding integer solutions to (1) are given by

$$x = 4s(6s^2 - 6s + 2) + k, y = (8s - 4)(6s^2 - 6s + 2) + k, z = 2(6s^2 - 6s + 2)$$

Note 2:

Instead of (12) , if we consider the transformations as

$$U = X - 3T, v = X + T, z = 2w$$

then , the corresponding integer solutions to (1) are given by

$$x = (4 - 4s)(6s^2 - 6s + 2) + k, y = (4 - 8s)(6s^2 - 6s + 2) + k, z = 2(6s^2 - 6s + 2)$$

The choice

$$w = X \quad (17)$$

in (13) gives

$$X^2 = T^2(2T - 3)$$

which is satisfied by

$$T = (2s^2 - 2s + 2), X = (2s^2 - 2s + 2)(2s - 1) \quad (18)$$

From (17) ,one has

$$w = (2s^2 - 2s + 2)(2s - 1) \quad (19)$$

Substituting (18) ,(19) in (12) and employing (4) &(2) , the corresponding integer solutions to (1) are given by

$$x = 4s(2s^2 - 2s + 2) + k, y = 4(2s^2 - 2s + 2) + k, z = 2(2s - 1)(2s^2 - 2s + 2)$$

Note 3:

Instead of (12) , if we consider the transformations as

$$U = X - 3T, v = X + T, z = 2w$$

then , the corresponding integer solutions to (1) are given by

$$x = (4s - 4)(2s^2 - 2s + 2) + k, y = -4(2s^2 - 2s + 2) + k, z = (4s - 2)(2s^2 - 2s + 2)$$

III. CONCLUSION

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous cubic diophantine equation with three unknowns given by $(x^2 + y^2) - xy - k(x + y) + k^2 = z^3$ .One may search for other sets of integer solutions to the considered equation as well as other choices of the third degree diophantine equations with multi-variables

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