

The Planckian Power Spectrum of Black Holes

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Abstract: *There are no "totally" black holes. The paradox proposed by Stephen Hawking highlighted a new correlation among quantum-mechanical, thermodynamic, and gravitational systems, known as black hole thermodynamics, in the early 1970s. One of the five predictions made by current theories of black hole thermodynamics is that all black holes emit thermal radiation (now known as Hawking radiation) with what can be viewed as a Planckian spectral distribution. In this paper, we outline the theoretical basis of this thermal radiation, its dependence upon either mass or spin, the effect of greybody corrections to the Planckian radiation, and current indirect observational results supporting Hawking's prediction. We describe several notable theoretical issues related to the discovery of black hole thermodynamics, including the information loss paradox and the scrambling of quantum information at a black hole's event horizon. Although we cannot yet directly detect Hawking radiation with current technology, the prediction that black holes emit Planckian radiation remains a highly stimulating area of research in high-energy particle physics and quantum gravity*

Keywords: Black Hole Thermodynamics; Hawking Radiation; Planckian Spectral Distribution; Greybody Corrections; Black Hole Mass and Spin; Information Loss Paradox; Quantum Information Scrambling; Indirect Observational Evidence

I. INTRODUCTION

The notion that the most extreme objects in space-time - black holes - produce energy by virtue of their mass (which is being decreased by allowing 'virtual particles' from their vacuum to escape) is a deeply philosophical one. Nevertheless, this outcome was demonstrated taking into consideration the quantum nature of the interactions of the vacuum with black holes, by Stephen Hawking, in his historic 1974 calculation showing that the event horizon around a black hole is full of virtual particles that have an equal probability for fluctuation and, when one member of the particle pair escapes to infinity, the other one falls into the black hole. Since the escaping particle has positive energy and the particle that is falling into the black hole has negative energy, this effect causes the mass of the black hole to decrease. This process will form a steady thermal radiation glow from the black hole.

Furthermore, this thermal radiation from black holes has a Planckian spectrum identical in functional form to that of an ideal black body emitting radiation at H_T (the Hawking temperature); it is not a coincidence. All derivations yielding the Hawking radiation spectrum using Bogoliubov transformations, tunnelling, or Euclidean path integrals will necessarily yield either a Bose-Einstein or Fermi-Dirac type of energy distribution that is at the Hawking temperature.

II. THE PLANCKIAN SPECTRUM AND HAWKING TEMPERATURE

Origin of the Thermal Radiation

The concept of a vacuum state is well defined in quantum field theory on flat spacetime, but it becomes ambiguous in curved spacetime. This ambiguity leads to different observers, moving in a variety of states, having different particle contents for a quantum field. The primary mathematical tool that relates these different particle contents in both the far past and far future for a black hole is the Bogoliubov transformation between the two modes of the quantum field. Hawking [1, 2] demonstrated that the future observer will see a thermal state of photons and all other modes of the

quantum field, even though that observer started off observing the mode content in the vacuum state (as the photon had been emitted from the horizon of the black hole).

For a Schwarzschild black hole with mass M , the characteristic temperature of Hawking radiation (the black body radiation emitted from a black hole) is inversely related to the mass of the black hole. The inversion of this relationship implies that stellar- and super-massive black holes are extremely cold in comparison to the Cosmic microwave background (CMB), while microscopic black holes will be extremely hot compared to the CMB.

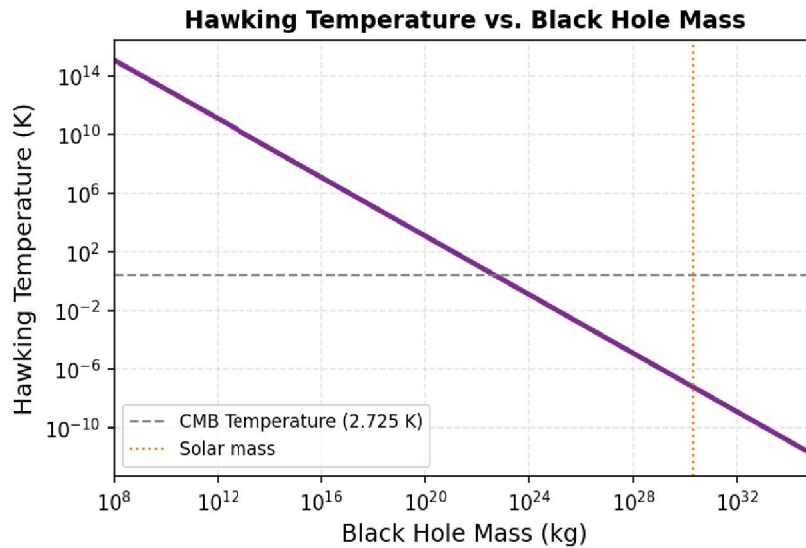


Figure 1. Hawking temperature as a function of black hole mass. The dashed horizontal line marks the CMB temperature of 2.725 K; only black holes lighter than roughly 10^{22} kg are hot enough to evaporate in isolation. The vertical dotted line indicates one solar mass.

The Planck Distribution

The radiation which is produced by a Schwarzschild black hole in the (l,m) mode also has the same Planckian structure, but it is modified by a grey body factor that depends on frequency. For a body which perfectly absorbs radiation (i.e. a point-like body), the grey body factor is equal to one (1), thus resulting in the black-body spectrum being a pure black-body spectrum. The total luminescence, obtained by integrating the total energy over all possible frequencies, follows T^4 , which is consistent with the Stefan-Boltzmann (S-B) law and shows a pretty amazing convergence between two significant discoveries in both classical physics and modern physics [3].

According to Wien’s law of displacement, a change in temperature will cause a change in frequency (specifically, the frequency at which maximum luminescence occurs). For example, a solar mass black hole has an average Hawking temperature of about 60nK and therefore would have a peak in the microwave region. Conversely, the spectrum for a Planck mass black hole would have a peak in the gamma ray region ($T \approx 10^{32}$ K). Figure II provides information on three specific mass categories, where the y-axis indicates the frequency of the respective spectrum (in MHz) and the x-axis denotes the mass of the corresponding black hole.

**Planckian Emission Spectra of Black Holes
(Hawking Radiation)**

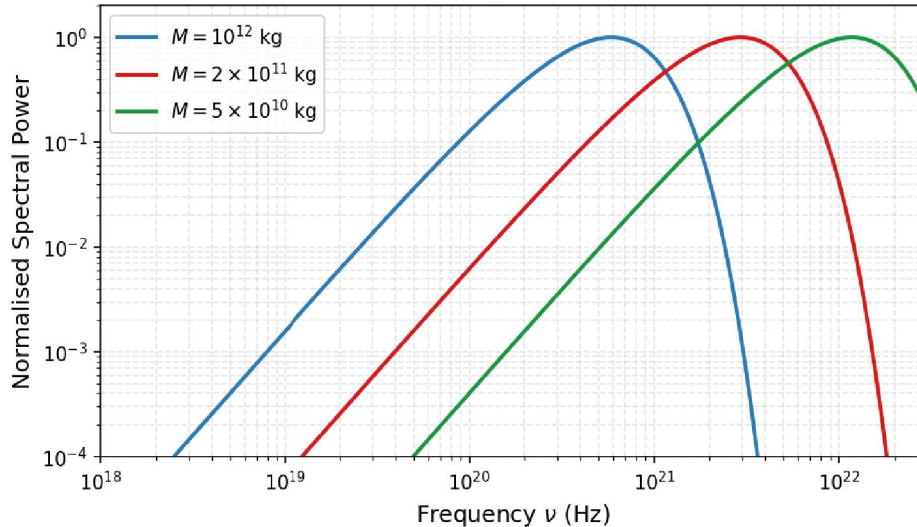


Figure 2. Normalised Planckian emission spectra for black holes of three representative masses. More massive (cooler) black holes peak at lower frequencies, consistent with Wien's displacement law. The curves are computed using the exact Planck function with no greybody correction.

III. GREYBODY FACTORS AND SPECTRAL CORRECTIONS

The perfect Planck spectrum is not realistic. In reality, some of the photons produced in the vicinity of the event horizon will not be able to escape to infinity because a portion of them gets reflected back towards the black hole due to the black hole's surrounding gravitational field. The greybody factor is the name given to the transmission coefficient for a photon of frequency ν to escape from the exterior to the interior of the black hole horizon; it reduces the intensity of photons emitted at lower frequencies and near the peak due to the significant effective potential [4].

The greybody factor is very sensitive to the particle's spin, the orbital angular momentum of the mode, and (for spinning black holes) the black hole's angular momentum. For scalar particles, greybody factors have been derived analytically in limiting cases and numerically for all frequencies. For gravitons and gauge bosons, greybody factors are more complicated to calculate, but have been determined with high accuracy using Leaver's continued fraction technique. Consequently, the greybody factor-corrected spectrum is different from the perfect Planck spectrum in a manner that contains detailed information about the black hole spacetime geometry [5].

When considering the Kerr black hole that has angular momentum, there is an extra aspect to the occurrence of superradiance imposed by the law of superradiance that is at low frequencies where the black hole can emit more than it receives from incoming radiation in the form of a 'greybody' factor that will exceed unity which means instead of absorbing incoming waves, the black hole will amplify them. This is referred to as superradiant emission and causes a modified power spectrum in the co-moving modes and therefore results in an actual quantum-gravitational deviation from what would otherwise be expected, namely, a purely thermal distribution.[4]

IV. BLACK HOLE THERMODYNAMICS

Within the framework of black hole thermodynamics, the concept of Hawking radiation makes sense, as four fundamental laws analogous to the classical laws of thermodynamics govern this area of physics. The zeroth law (the law of thermodynamic equilibrium) states that the surface gravity of a stationary black hole is constant across the entire event horizon, similar to the equivalent temperature of all bodies in thermal equilibrium. The first law connects mass, angular momentum, and charge to any subsequent changes in the area of the areas surrounding a black hole. The second

law the area theorem, states that the area of a black hole's event horizon will always increase, just like the entropy of a substance increases in all thermodynamic processes.

The Bekenstein-Hawking entropy provides the theoretical framework for the identification of entropy with the event horizon in units of Planck length and has been the most significant single contribution to theoretical physics since the establishment of general relativity. This suggests that information regarding the interior of the black hole, i.e., the holographic nature of the black hole, is stored within the event horizon at a density of 1 bit of information per unit Planck area. The emission of Hawking radiation is the thermal process required for the thermodynamic framework of black holes to be self-consistent: without the existence of emitted radiation, a black hole cannot reach thermal equilibrium through its interaction with surrounding bodies, and therefore, the thermodynamic analogy would remain a formal construct. The existence of emitted Hawking radiation results in the analogy becoming a description of physics[6,7].

V. THE INFORMATION PARADOX

One of the most profound implications of Hawking radiation being of Planckian nature is that it leads to the issue regarding the paradox of information. When a black hole is created from a pure quantum state and subsequently evaporates so that it disappears completely, releasing radiation that is completely thermal in nature, then that radiation appears to carry no information related to the black hole's original state. Thus, after the evaporation, the black hole is said to be in a mixed state. This then implies that the unitarity of quantum mechanics has been violated since the unitary evolution of a closed quantum system requires that the evolution of the system maintains the information of the states [8].

The debate regarding the firewall hypothesis has drawn more attention to the tension between three statements in regards to black holes: Unitarity, local quantum field theory at the black hole horizon, and the equivalence principle, cannot all coexist without the addition of new physical ideas and/or a substantial change in our concepts of local nature and space/time at the Planck scale.

VI. OBSERVATIONAL PROSPECTS

An observational program's most obvious challenge is its scale. The Hawking temperature of astrophysical black holes, including those of several solar masses, is on the order of tens of nanokelvin, or about 10^{-10} Kelvin, which is over ten orders of magnitude less than that of the cosmic microwave background. As a result, it will not be possible to directly observe astrophysical Hawking radiation with any currently conceivable technology [1].

Primordial black holes are a potentially useful proxy for indirectly measuring dark matter due to their origin from density fluctuations following inflation during the early universe. Black holes with a mass of less than roughly 5×10^{11} kg would not have survived until now and should produce a diffuse background of gamma-ray, gravitino and other particle emissions when they evaporate. The Fermi-LAT gamma-ray observatory has searched for evaporation signatures from these black holes that will be used to establish upper limits for their contribution to the overall dark matter mass budget [10].

One possibility of verifying the Planck nature of Hawking radiation is through the use of a mathematical equivalence between the equations that describe phonons in a supersonic flow of fluid and those that describe photons close to an event horizon. These sonic analogues have been referred to as 'dumb holes' and have been created using Bose-Einstein condensates. Recent results show that thermal phonons were observed in the vicinity of the sonic horizon, which agrees with the expected equivalent analogue Hawking temperature; therefore, this study provides not only the most convincing theoretical demonstration of the Planckian radiation mechanism but also represents the most compelling experimental evidence of this phenomenon to date [11].

VII. CONCLUSION

The Planckian spectrum of black holes is one of the most remarkable results of physics. It represents the intersection of 3 distinct areas of study: General Relativity, Quantum Field Theory, and Thermodynamics. For many decades, theoretical development has been fueled by this result. The thermal nature of Hawking radiation, i.e., at leading order, a black hole behaves as an ordinary blackbody with a temperature inversely proportional to its gravitational mass, is fully

established by several independent derivations. Hence, it enjoys the same level of confidence from experiment as do all other comparably successful theoretical predictions. The deviation from a perfect Planckian curve due to greybody factors encodes very rich information concerning the geometry of the black hole. There are also a lot of remaining puzzles associated with the radiation, the information paradox; structure of quantum states near the horizon, and the holographic nature of entropy that remain unresolved and continue to push forward the most ambitious work currently being done in Quantum Gravity. The continuing improvement in sensitivity for both analogue experiments and searches for primordial black holes creates an increasing opportunity for empirical contact with these ideas (even if indirectly).

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