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Study of Elastic Properties of Mantle Solids & Variations of Density (ρ) of Earth with Depth (r)

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Abstract: *In the present study, there is now some knowledge on the elastic characteristics of the key rock-forming minerals in the mantle. Gruneisen parameter is reasonably constant from material to material and temperature independent. The majority of measurements are taken under settings that are very different from the pressure and temperature conditions in the deep mantle. This is useful because density fluctuations with temperature, pressure, composition, and phase are pretty well understood. At high temperatures, the elastic characteristics are mostly determined by volume due to thermal expansion. Because the lower mantle is under simultaneous high pressure and high temperature, it is unclear if the standard high-temperature limit simplifications are necessarily accurate.*

Keywords: Anderson-Gruneisen parameter, isothermal bulk modulus, elastic properties

I. INTRODUCTION

The crust, mantle, outer core, and inner core are the four principal parts of the Earth's structure, which determines the life of the planet's surface. Each layer has a unique chemical makeup as well as a unique physical state The Earth's mantle is a solid-liquid combination. The upper and lower mantles, respectively, are solid and liquid because there is more heat and pressure in deeper parts of the earth.

The mantle region is the thickest layer of the geosphere and is made up mostly of silicate rock, which contains more iron and magnesium than the crust. About 84% of the earth's total volume, which is largely made up of solid stuff, is made up of the mantle. Additionally, the mantle contains a geothermal gradient that varies in temperature from just below the earth's crust, where it is about 200 degrees Celsius, to just above the outer core, where it is around 4000 degrees Celsius. The mantle is mainly solid despite being hot enough for molten rock because of the pressure gradient.

Condensed matter's physical characteristics are more significantly influenced by pressure than by other variables like temperature. In order to comprehend the structure, dynamics, and genesis of the earth, it is essential to understand how earth elements behave under great pressure. Throughout the variety of circumstances that exist in the earth's mantle [1–5].

II. METHOD AND ANALYSIS

Understanding the relationship between elastic constants and bulk modulus from the analysis of the elastic characteristics of lower mantle solids aids in our comprehension of the behavior of the substance under the impact of high temperature and high pressure.

2.1 Composition of Mantle

The elements that are most prevalent in mantle rock are not included in the mantle's chemical makeup. Understanding the composition requires research and examination of the material found along mid-ocean ridges, where rising mantle material undergoes a significant enough reduction in pressure to undergo decompression and liquefy. The topmost mantle and these locations have been the source of all samples and analyses of the mantle's composition. Thus, the samples only partially reveal the rest of the mantle's makeup[6-8].

Sr. No.	Material	Percentage of mass
1	Potassium oxide	0.006
2	Phosphorous pent oxide	0.019
3	Titanium oxide	0.13
4	Sodium oxide	0.13
5.	Manganese oxide	0.13
6.	Nickel oxide	0.24
7.	Chromium oxide	0.57
8.	Calcium oxide	3.17
9.	Aluminum oxide	3.98
10.	Iron oxide	8.18
11.	Magnesium oxide	38.73
12.	Silicon dioxide	44.71

2.2 Temperature Dependent Elastic Properties of Solids

The Anderson-Gruneisen parameter δ_T play a central role also in describing thermal expansivityα and temperature dependence of isothermal bulk modulus B_T

$$\delta_T = -\frac{1}{\alpha B_T}\left(\frac{dB_T}{\partial T}\right)_P \text{ ----- (1)}$$

This relation can be used in following approximation

$$\left(\frac{dB_T}{dT}\right)_P = -\alpha B_T \delta_T = -\alpha_0 B^0{}_T \delta^0{}_T \text{ ----- (2)}$$

Where $\alpha_0 B^0{}_T, \delta^0{}_T$ are the values on $T=T_0$ at room temperature (300K) and integration of equation (2) then we get-

$$B_T = B^0{}_T - \alpha_0 B^0{}_T \delta^0{}_T\ (T-T_0) \text{ ------ (3)}$$

Equation (3) may be rewritten as follows, where B represents any of elastic moduli.

$$B_T = B^0{}_T - \alpha_0 B^0{}_T \delta^0{}_T\ (T-T_0) \text{ ------ (4)}$$

The total formulas for the temperature dependence of elastic constants can be stated as

$$\frac{C_{ij}}{C^0{}_{ij}} = C^0{}_{ij} - \alpha_0 C^0{}_{ij} \delta_{ij}\ (T-T_0) \text{ ------- (5)}$$

This relation is obtained with help of Anderson-Gruneisen-parameter which remains constant with change in temperature.

Kumar [2] reported a relation for the volume expression V/V0 as function of temperature T using the theory of high pressure and high temperature.

$$\frac{V}{V_0} = 1 - \frac{1}{A}\mathrm{In}\left[1 + \frac{A}{B_0}\{P - \alpha_0 B_0 (T - T_0)\}\right] \text{ --------- (6)}$$

Using relation the isothermal bulk modulus B then, we get

$$B = -V\left(\frac{\partial P}{\partial V}\right)_T$$

$$\frac{B}{B_0} = \left[1 - \frac{1}{A}\left\{1 + \frac{AP}{B_0} - A\alpha_0(T - T_0)\right\}\right] \times \left[1 + \frac{AP}{B_0} - A\alpha_0(T - T_0)\right] \ldots\ldots\ldots(7)$$

Now if pressure P =0, putting the value of pressure in equation (7) then we get

$$\frac{B}{B_0} = \left[1 - \frac{1}{A}\{1 - A\alpha_0(T - T_0)\}\right] \times [1 - A\alpha_0(T - T_0)] \text{ ------- (8)}$$

Equation (8) may be rewritten as follows, where B represents any of elastic moduli.

$$\frac{B}{B_0} = \left[1 - \frac{1}{A}\{1 - A\alpha_0(T - T_0)\}\right] \times [1 - A\alpha_0(T - T_0)] \text{ ------- (9)}$$

The temperature dependence of elastic constants can be expressed together as

$$\frac{C_{ij}}{C^0{}_{ij}} = \left[1 - \frac{1}{A_{ij}}\{1 - A_{ij}\alpha_0(T - T_0)\}\right] \times [1 - A_{ij}\alpha_0(T - T_0)] \text{ ---- (10)}$$

The third order an-harmonic factor from Kumar's formulation [2] was added in the current work by Kumar and Gupta [3], who also proposed an expression to show how the isothermal bulk modulus varies as a function of temperature.

A_{ij} is a parameter where $A_{ij} = [\delta^0{}_{ij} + 1]$ which discussed in detail by kumar and Bedi [4]

According to Kumar approximation [2], Volume expression V/V_0 as the function of temperature T at high pressure and temperature is represented by equation (6). At P=0 equation (6) can be written as

$$\frac{V}{V_0} - 1 = -\frac{1}{B'_0+1}\text{In}[1 - \alpha_0(B'_0 + 1)(T - T_0)] \text{ --------- } (11)$$

Where α_0 is the thermal expansion coefficient at T_0 and B'_0 is the first pressure derivative of bulk modulus which can be assumed nearly equal to δ as mentioned [5-9].

Thus on including the third order an-harmonic term in equation (11) takes as follows,

$$\frac{V}{V_0} - 1 = \frac{1}{A}\left[e^{\alpha_0 A(T-T_0)} - 1 + \frac{\{\alpha_0 A(T-T_0)\}^3}{6}\right] \text{ ---------- } (12)$$

Using the definition of the coefficient of volume thermal expansion α and equation (12) gives following relation.

$\alpha = \left(\frac{1}{V}\right)\left(\frac{dV}{dT}\right)_P$ and $\alpha_0 = \left(\frac{1}{V_0}\right)\left(\frac{dV}{dT}\right)_P$ then we get

$$\frac{\alpha}{\alpha_0} = \frac{e^{\alpha_0 A(T-T_0)} + \frac{\alpha^2{}_0 A^2(T-T_0)}{2}}{1+\left\{e^{a_0 A(T-T_0)} - 1 + \frac{a^3{}_0 A^3(T-T_0)}{6}\right\}} \text{ ---------- } (13)$$

Now we use the approximation (αβ) is independent of temperature T. this gives relation

$$\alpha B = \alpha_0 B_0$$

So therefore equation (13) can be written in B/B_0 as follows

$$\frac{B}{B_0} = \frac{1+\frac{1}{A}\left[e^{\alpha_0 A(T-T_0)} - 1 + \frac{\{\alpha_0 A(T-T_0)\}^3}{6}\right]}{e^{\alpha_0 A(T-T_0)} + \frac{\alpha^2{}_0 A^2(T-T_0)^2}{2}} \text{ ----------- } (14)$$

Equation (14) can be arranged as follows, where B represents any of elastic moduli.

$$\frac{B}{B_0} = \frac{1+\frac{1}{A}\left[e^{\alpha_0 A(T-T_0)} - 1 + \frac{\{\alpha_0 A(T-T_0)\}^3}{6}\right]}{e^{\alpha_0 A(T-T_0)} + \frac{\alpha^2{}_0 A^2(T-T_0)^2}{2}} \text{ ------------ } (15)$$

The total formulations for the elastic constants' temperature dependence are as follows:

$$\frac{C_{ij}}{C^0{}_{ij}} = \frac{1+\frac{1}{A}\left[e^{\alpha_0 A(T-T_0)} - 1 + \frac{\{\alpha_0 A(T-T_0)\}^3}{6}\right]}{e^{\alpha_0 A(T-T_0)} + \frac{\alpha^2{}_0 A^2(T-T_0)^2}{2}} \text{ ----------- } (16)$$

2.3 Pressure Dependent Elastic Properties of Solids

For determining how elastic constants are affected by pressure, a basic and easy method is suggested. The Anderson-Gruneisen δ_T parameter is defined

$$\delta_T = -\frac{1}{\alpha B}\left(\frac{\partial B}{\partial T}\right)_P \text{ ---------- } (17)$$

In the present study where α is the coefficient of volume thermal expantion and B is isothermal bulk modulus.

If we use the definition of thermal expansivity α

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \text{ -------- } (18)$$

We get equation (17) as

$$\delta_T = -\frac{V}{B}\left(\frac{\partial B}{\partial V}\right)_P \text{ -------- } (19)$$

If δ_T is independent on temperature, then

$$\frac{B_T(T,P)}{B_T(T_0,P)} = \left(\frac{V(T,P)}{V_0(T_0,P)}\right)^{-\delta_T} \text{ ------ } (20)$$

However, there is evidence that δ_T does not remain strictly constant but changes with the variation in volume[6-8].

Any elastic modulus or constants represented by B, then

$$B' = \frac{dB}{dP}$$

Considering, Kumar [3] approximation

$$B' = (\text{Constant})\frac{V}{V_0} - 1 \text{ --------- } (21)$$

$$= (A)\frac{V}{V_0} - 1$$

If, $V = V_0, B' = B'_0$ Then

$$A = B'_0 + 1$$

$$\frac{dB}{dP} = (B'_0 + 1)\frac{V}{V_0} - 1 \text{ ------(22)}$$

$$-\frac{V}{B}\frac{dB}{dV} = (B'_0 + 1)\frac{V}{V_0} - 1 \text{ ---------- (23)}$$

$$-\frac{dB}{B} = (B'_0 + 1)\frac{dV}{V_0} - \frac{dV}{V} \text{ -------- (24)}$$

$$-\text{InB} = (B'_0 + 1)\frac{V}{V_0} - \text{In V ------- (25)}$$

At P=0, $V=V_0$ and $B=B_0$

$$-\text{InB}_0 = (B'_0 + 1) - \text{In V}_0 \text{ -------- (26)}$$

Subtracting Eq. (25) from Eq. (26) then obtained Equation

$$\text{InB} - \text{InB}_0 = ((B'_0 + 1)\left(1 - \frac{V}{V_0}\right) + \text{InV} - \text{InV}_0 \text{ -------- (27)}$$

$$\text{In}\frac{B}{B_0} = (B'_0 + 1)\left(1 - \frac{V}{V_0}\right) + \text{In}\frac{V}{V_0} \text{ ------- (28)}$$

$$\text{In}\frac{B.V_0}{B_0.V} = (B'_0 + 1)\left(1 - \frac{V}{V_0}\right) \text{ ---------- (29)}$$

$$\frac{B}{B_0} = \left(\frac{V}{V_0}\right)\exp(B'_0 + 1)\left(1 - \frac{V}{V_0}\right) \text{ ---------- (30)}$$

The isothermal bulk modulus B is defined as, $= -V\frac{dP}{dV}$, the value put in equation (30).

$$\frac{1}{B_0}\left(-V\frac{dP}{dV}\right) = \left(\frac{V}{V_0}\right)\exp(B'_0 + 1)\left(1 - \frac{V}{V_0}\right) \text{ --------- (31)}$$

Equation (31) solved by the integration

$$-\frac{1}{B_0}P = \frac{1(-V_0)}{(B'_0+1)(V_0)}\exp(B'_0 + 1)\left(1 - \frac{V}{V_0}\right) + C \text{ ----------- (32)}$$

$$\frac{1}{B_0}P = \frac{1}{(B'_0+1)}\exp(B'_0 + 1)\left(1 - \frac{V}{V_0}\right) + C \text{ ------------ (33)}$$

Tap=0,V=V0,than,obtained $C = -\frac{1}{K'_0+1}$

$$\frac{1}{B'_0+1} + \frac{P}{B_0} = \frac{1}{B'_0+1}\exp(B'_0 + 1)\left(1 - \frac{V}{V_0}\right) \text{--------(34)}$$

$$1 + \frac{(B'_0+1)P}{B_0} = \exp(B'_0 + 1)\left(1 - \frac{V}{V_0}\right) \text{ ------------ (35)}$$

A is parameter defined as [12]

$$A = B'_0 + 1$$

$$\left[1 + \frac{AP}{B_0}\right] = \exp A\left(1 - \frac{V}{V_0}\right) \text{ ----------- (36)}$$

$$\frac{V}{V_0} = 1 - \frac{1}{A}\text{In}\left[1 + \frac{A\,P}{B_0}\right] \text{ ---------- (37)}$$

Using relation B then obtained Equation (38)

$$B = -V\left(\frac{\partial P}{\partial V}\right)_T$$

$$\frac{B}{B_0} = \left[1 - \frac{1}{A}\text{In}\left\{1 + \frac{A\,P}{B_0}\right\}\right]\left\{1 + \frac{A\,P}{B_0}\right\} \text{ ---------- (38)}$$

Equation (38) may be rewritten as follows, where M represents any of elastic moduli.

$$\frac{M}{M_o} = \left[1 - \frac{1}{A}In\left\{1 + \frac{AP}{M_o}\right\}\right]\left\{1 + \frac{AP}{M_o}\right\} \text{----------- (39)}$$

The collective expressions for the pressure dependence of elastic constants may be written as

$$\frac{C_{ij}}{C^o{}_{ij}} = \left[1 - \frac{1}{A_{ij}}In\left\{1 + \frac{A_{ij}P}{C^o{}_{ij}}\right\}\right]\left\{1 + \frac{A_{ij}P}{C^o{}_{ij}}\right\} \text{---------- (40)}$$

The interpretation of the elastic characteristics of mantle minerals depends on the structure and composition of the lower mantle. It is crucial to estimate how pressure will affect the minerals using elastic constants and their pressure derivatives of the mantle materials. Regarding the composition and interpretation of the lower mantle's structure. The study of the mantle minerals' elastic characteristics is crucial.

2.4 Variation of Density in the Earth:

There were offered a number of models for the density distributions. But a precise account of the density change in the upper mantle would offer a more conclusive test of compositional models.

As you descend further beneath the surface of the earth, pressure and density rise in tandem with the depth [11–15].

The variation of density ρ with radius in the earth r can be written

$$\frac{d\rho}{dr} = \left(\frac{\partial\rho}{\partial p}\right)\frac{dp}{dr} + \left(\frac{\partial p}{\partial T}\right)\frac{dT}{dr} + \left(\frac{\partial\rho}{\partial\emptyset}\right)\frac{d\emptyset}{dr} + \left(\frac{\partial\rho}{\partial C}\right)\frac{dC}{dr} \text{ ....(41)}$$

That is, the variables pressure, temperature, phase (4), and composition all affect density.We obtain the following results for a homogeneous adiabatic self-compressed region [16].

$$\frac{d\emptyset}{dr} = 0, \frac{dC}{dr} = 0$$

$$\frac{dp}{dr} = -g\rho \text{ , Where } g = \frac{GM(r)}{r2}$$

Away from thermal boundary layers, the mean temperature gradient is closing adiabatic. Physically, it would not be possible to achieve the temperature gradients required by a homogenous compositional model.

$$\frac{dT}{dp} = \left(\frac{\partial T}{\partial p}\right)_S = \frac{T\alpha}{\rho Cp} \text{ ....(42)}$$

Therefore, writing the temperature gradient is convenient.

$$\frac{dT}{dr} = \left(\frac{\partial T}{\partial p}\right)_S = \frac{T\alpha}{\rho Cp}\frac{dp}{dr} - r \text{ ....(43)}$$

Where r is the sub-adiabatic gradient. Adiabatic compression of a material is given by the adiabatic bulk modulus, B_S

$$B_S = \rho\left(\frac{\partial p}{\partial\rho}\right)_S$$

Anderson &Sammis have provided a more incisive demonstration of this conclusion (1969). If the seismic models depicted accurately approximate the geometry of the LVZ, then compositional differences or phase change must be used to explain this anomalous region[17].Given that seismic waves are also adiabatic, we can use

$$V_p^2 - \left(\frac{4}{3}\right)V_S^2 = \frac{B_S}{\rho} = \left(\frac{\partial p}{\partial\rho}\right)_S = \Phi$$

To determine the density fluctuation with depth in an adiabatic, homogenous region for which we have seismic data. Using the alternate terms above

$$\frac{d\rho}{dr} = -\frac{g\rho}{\Phi} + a\rho r \text{ ....(44)}$$

The radius of the earth affects the density of the planet. The higher proportion of iron in the lower mantle is what causes this increase in density.

We can say that as the depth of the earth increases, so does the temperature of the earth. On average, the temperature rises by around 1 degree Fahrenheit for every 70 feet of depth.

III. RESULTS AND DISCUSSION

The effects of high temperature and pressure on the mantle's characteristics and many geophysical issues are resolved. There are extrinsic and intrinsic components to the temperature derivatives of the elastic moduli. Since the resolution of geothermal models depends on both an accurate description of heat conduction and source distributions inside the Earth's interior and on events related to the early stages of planetary creation, they are challenging to estimate.For this reason, it's crucial to have a solid theoretical grasp of how temperature, composition, and pressure affect minerals' elastic and thermal properties. The effects of temperature can be used to explain a number of geophysical issues. The density and flexibility of the upper mantle are significantly influenced by temperature. As an illustration, the coefficient of thermal expansion, which regulates a number of properties, rises with temperature but falls with pressure. High temperature gradients can be seen in the upper mantle, which decrease from a high conductive gradient at the top to a convective gradient in the deeper interior.

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