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Finite Difference Solution of Rotatory Flow of Dissipative Fluid Past an Impulsively Started Infinite Vertical Plate

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Abstract: *A flow of dissipative fluid flow past an impulsively started infinite vertical plate in a rotating fluid by using finite difference method. It is observed that due to more rotation rate of heat transfer decreases, axial and transverse skin friction increases for all Pr. Also there is rise in the temperature for low density fluid (Pr < 0.71) but when Pr is large temperature increase due to more rotation of the system near the plate and decreases far away from the plate. And rotating speed increase axial velocity decrease and the transverse velocity is also decrease for all Prandlt number.*

Keywords: Dissipative fluid , Skin friction, Heat transfer, Prandlt number

I. INTRODUCTION

Exact solution of Navier-stokes equation was presented by Stokes (1851) which was concerned for the flow of viscous incompressible fluid past an infinite horizontal impulsively started plate, in a stationary mass of fluid. However Hall (1969) was presented a finite difference solution to the flow past an impulsively started semi- infinite horizontal plate. How the flow is affected by free convection currents ?. Instead of horizontal plate ,if an impulsive motion is given to an infinite vertical plate which is surrounded by an infinite mass of viscous incompressible fluid, first studied by Soundalgekar (1977) presented an exact solution to coupled partial differential equations by the Laplace-transfer technique. The effect of heating or cooling of the plate by the free convection currents was studied by neglecting viscous dissipative heat .If the impulsive motion given to the plate is such that the velocity is rather high or the surrounding liquid is of high Prandtl number or the situation considered at high gravitational field, then Gabhart (1962) has been shown the viscous dissipative heat cannot be neglected. Soundalgekar(1979) considered this problem by taking the effect of viscous dissipative heat on the motion past an impulsively started infinite vertical isothermal plate. Now during last few years the flow around the bodies in rotating fluid is receiving good attention from researcher as it has many application in the engineering fields. Lahurikar R.M., (2010) who gave exact solution by Laplace transform technique of the effect of rotation and free convection currents on the motion of the fluid near an impulsively started infinite vertical plate studied. Now it has been proposed to study the rotatory flow of dissipative fluid past an impulsively started infinite vertical plate. As the problem is governed now by coupled non-linear equations, exact solutions are not possible so we employ explicit finite difference method.

II. MATHEMATICAL ANALYSIS

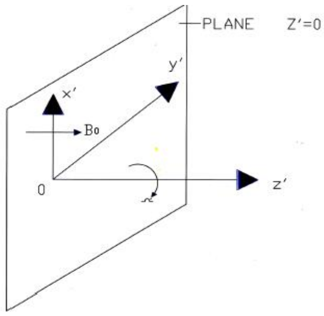

Figure 1: Schematic Diagram

As shown in Fig1 we assume that an infinite vertical plate surrounded by an infinite mass of stationary viscous incompressible fluid .Let the x' -axis be in the plate in vertically upward direction and the y'-axis be in horizontal direction assumed to be at right angle to the x' -axis. Then z'- axis is taken normal to the $x'y'$ –plane.

Initially we assumed the temperature T_∞' of plate and the fluid are same .Then at time $t'>0$ the plate is given an impulsive motion in vertically upward direction with a velocity U_0, The dissipative fluid starts rotating about the Z' -axis with the angular speed $\Omega'$and the plate temperature raised to T'_{ω}. Then the physical variables are functions of z' and t' . Then the problem is governed under usual Boussinesq's approximation by the coupled partial differential equation in non dimensional form

$$\frac{\partial u}{\partial t} - 2E_K v = \theta + \frac{\partial^2 u}{\partial z^2} \qquad (1)$$

$$\frac{\partial v}{\partial t} + 2E_K u = \frac{\partial^2 v}{\partial z^2} \qquad (2)$$

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} + \mathrm{Pr}\, E_C\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] \qquad (3)$$

and the initial boundary conditions are

$$\begin{array}{llll} u=0, & v=0 & \theta=0, & for\ all\ z,t\le 0 \\ u=1, & v=0 & \theta=1 & at\ z=0 \quad t>0 \\ u=0, & v=0 & \theta=0 & as\ \ z\to\infty\ ,\ t>0 \end{array} \qquad (4)$$

here the non-dimensional quantities are defined as follows.

$$u=\frac{u'}{U_0} \quad v=\frac{v'}{U_0}, \quad t=\frac{t'GrU_0{}^2}{v}, \quad z=\frac{z'\sqrt{Gr}U_0}{v} \ , \ Pr=\frac{\mu Cp}{k}$$

$$Gr=\frac{vg\beta(T'w-T'\infty)}{U_0{}^3}, \qquad \theta=\frac{T'-T'\infty}{T'w-T'\infty,}, \quad E_K=\frac{\Omega' v}{GrU^2{}_0},$$

$$Ec=\frac{\mathrm{U_0}^2}{Cp(T'_w-T'_\infty)} \qquad (5)$$

An exact solutions of equations (1)-(3) are not possible, so we now solve them by explicit finite difference method. Then the set of finite difference equations corresponding to equation (1) to (3) gives.

$$u_{i,j+1} = u_{i,j} + \Delta t\{2E_K v_{i,j} + \theta_{i,j}\} + \frac{\Delta t}{(\Delta z)^2}\{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\} \qquad (6)$$

$$v_{i,j+1} = v_{i,j} + \frac{\Delta t}{(\Delta z)^2}\{v_{i+1,j} - 2v_{i,j} + v_{i+1,j}\} - 2\Delta t E_k u_{i,j} \qquad (7)$$

$$\theta_{i,j+1} = \theta_{i,j} + \frac{\Delta t}{Pr(\Delta z)^2}\{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}\} + E_c\frac{\Delta t}{(\Delta z)^2}\{(u_{i+1,j}-u_{i,j})^2 + (v_{i+1,j}-v_{i,j})^2\} \qquad (8)$$

with following initial and boundary conditions in the finite difference form.

Initial conditions.

$$\begin{array}{llll} u(0,0)=0\,, & \theta(0,0)=1, & v(0,0)=0 & \\ u(i,0)=0, & \theta(i,0)=0, & v(i,0)=0 & for\ all\ i\ except\ i=0 \end{array} \qquad (9)$$

Boundary conditions.

$$\begin{array}{lll} u(0,j)=1, & \theta(0,j)=1, & v(0,j)=0 \\ u(41,j)=0, & \theta(41,j)=0, & v(41,j)=0 \end{array} \qquad (10)$$

here i corresponds to z and j corresponds to t

Here infinity is taken as z=4.1 because from the exact solutions of equations (1) to(3) for Ec=0 , it has been observed that u,v,θ tends to zero around $z\sim 4$ for all values of Pr. Velocity and temperature distribution computed from

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equation (6) to (10) by chosenΔt=0.00125. To Judge the accuracy of the convergence and stability of the finite difference scheme the same program was run with smaller values of Δt i.e.Δt=0.0009 or 0.001 and no significant changes in the results hence, the scheme is stable and convergent.With the help of Newton five point interpolation formula we calculate the skin friction and the rate of heat transfer entered in the following table.

$$\frac{\tau_{ax}}{\sqrt{Gr}} = -\frac{du}{dz}|_{z=0}, \quad \frac{\tau_{tr}}{\sqrt{Gr}} = -\frac{dv}{dz}|_{z=0}, \quad q = -\frac{d\theta}{dz}|_{z=0} \qquad (11)$$

TABLE--I

t	Pr	Ec	E_k	-τ_{ax}	-τ_{tr}	q
0.2	0.2	0.1	0.1	0.9186502	0.2680884	0.551297
			1.0	0.946446	0.533974	0.550885
			0.2	1.055331	1.050617	0.549277
		0.05	0.5	0.919091	0.268061	0.557029
0.4	0.2	0.1	0.5	0.424913	0.400266	0.391399
		0.05	0.5	0.425558	0.400179	0.395114

TABLE--II

t	Pr	Ec	E_k	-τ_{ax}	-τ_{tr}	q
0.2	0.71	0.1	0.1	0.9795061	0.05256951	1.020297
-	-	-	0.5	0.9881411	0.2625081	1.0198971
-	-	-	1.0	1.014979	0.5229021	1.01186572
-	-	-	2.0	1.1201229	1.0291639	1.0137781
-	-	0.05	0.5	0.9892391	0.2624489	1.0388609
0.4	0.71	0.1	0.5	0.525372	0.383581	0.723700
0.2	7	0.1	0.5	1.119609	0.255202	3.076963
-	100	0.1	0.5	1.2199475	0.252926	13.157520
0.2	7	0.1	0.1	1.111306	0.0510594	3.079280
-	-	-	2.0	1.246502	0.000680	3.041466
-	-	0.05	0.5	1.123003	0.255080	3.209859
0.4	7	-	0.5	0.7087229	0.36315529	2.1651631

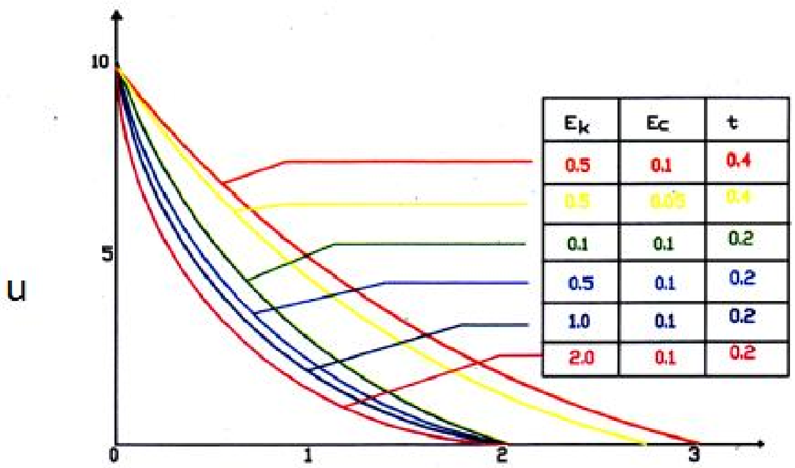

Figure 2: Axailvelocity Profiles, Pr=0.2

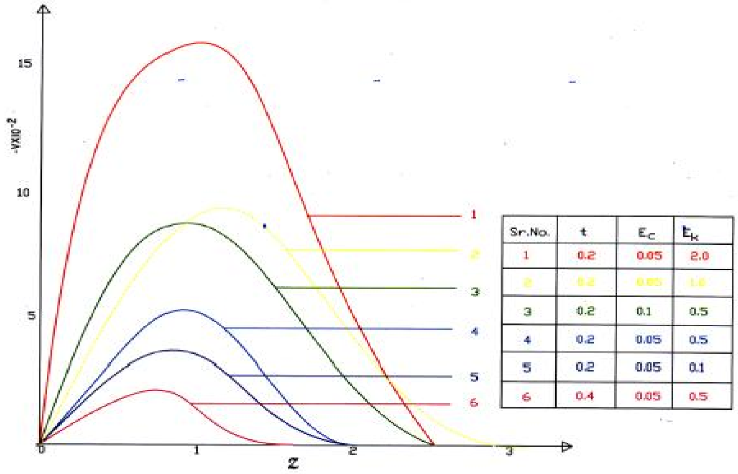

Figure 3: Transverse Velocity Profiles Pr=0.71

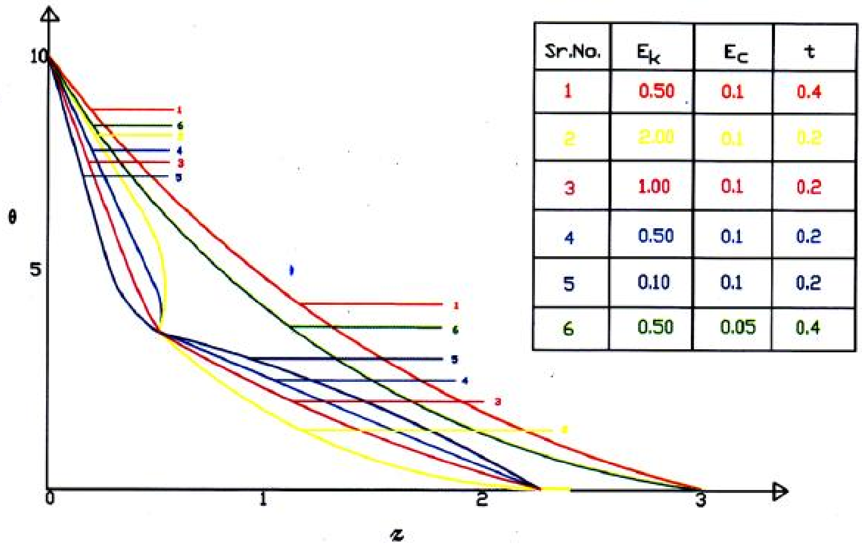

Figure 4: Temperature profile

III. NOMENCLATURE

Cp : Specific heat at constant pressure.
Ec : Eckert number.
Ek : Ekman number.
g : Acceleration due to gravity.
K : Thermal conductivity.
Pr : Prandtl number.
T' : Temperature of the plate.
$T'w$: Temperature of the plate.
$T'\infty$: Temperature of the fluid for away from the plate.
t' : Time.
U_0 : Impulsive Velocity of the plate.
(u', v') : Velocity components in $x'$and y' axis respectively.

Greek Symbols

τ : skin friction.
μ : Viscosity.

β : Coefficient of volume expansion.

ρ : Density.

Ω' : Angular velocity.

IV. CONCLUSION

From Table (I) and (II) and fig (2)-(4)
We observed that

1. There is rise in axial velocity, transverse velocity, and temperature for Pr=0.2 and 0.71 As time t increases. But there is fall in transverse velocity for Pr = 7.0. Axial skin friction and rate of heat transfer decreases and transverse skin friction increases as t increases for all Pr.
2. There is rise in temperature when Rotation speed increases for Pr= 0.2
3. But when Pr=0.71 and 7 temperature increases due to more rotation of the system near the plate and temperature decreases far away from the plate .
4. Greater viscous dissipative heat causes rise in the axial velocity ,transverse velocity, temperature, transverse skin friction and fall in axial skin friction and rate of heat transfer for Pr=0.2.
5. Greater viscous dissipative heat causes rise in the axial velocity , temperature, transverse skin friction and fall in transverse velocity, axial skin friction and rate of heat transfer for air and water i.e. when Pr=0.71 and 7.0.
6. As Pr increase or the density of fluid increases there is fall in axial velocity, temperature, transverse skin friction and rise in transverse velocity ,axial skin friction and rate of heat transfer Rotation parameters E_k increases axial velocity, transverse velocity decreases for all Pr.
7. The rate of heat transfer decreases as E_k increases and axial skin friction increases for large Pr and decreases for small Pr,

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