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On g-Semi-Closed Sets in Generalized Topological Spaces

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Abstract: *In this paper, we have studied g-semi-closed sets in generalized topological space. We have obtained some significant properties of g-semi-closed sets and constructed various examples.*

Keywords: Generalized topological spaces, g-interior, g-closure, g-semi open set,g-semi-closed set

I. INTRODUCTION

Csaszar [1] has introduced the concept of generalized topology in 2002. Then in 2005, Csaszar [2] has defined the notion of g-semi-open in generalized topological space. In this paper we have obtained significant results which characterizes g-closed-sets and constructed useful examples.

II. PRELIMINARIES

We begin with the definition of generalized topological space, g-open sets and g-closed sets.

Definition 2.1 [1] Let X be a non empty set and letτ_g be a family of subsets of X. Then $\tau_g$is said to be a **generalized topology** on X, if following two conditions are satisfied viz.:

i) $\phi \in \tau_g$;

ii) Arbitrary union of members of τ_g is a member of τ_g.

The generalized topology τ_g is said to be **strong**if $X \in \tau_g$, and the pair (X, τ_g) is called a **generalized topological space**. The members of family τ_g are called g**-open sets** and their complements are called g**-closed sets**.

From the above Definition 2.1, we note that arbitrary intersection of g-closed-sets is a g-closed-set.

Proposition 2.2 [4]: Let (X, τ_g) be a generalized topological space and let $A \subseteq X$.Then A is g-open set in X iff for each point x$\in A$ there exists a g-open set U in X such that x $\in U \subseteq A$.

Corollary 2.3 [4]: Let (X, τ_g) be a generalized topological space and let $A \subseteq X$. Then A is g-closed set in X iff for each point x $\notin A$ there exists a g-open set U in X such that $x \in U$ and U$\cap$A = ϕ.

Definition 2.4 [1]: Let$X$be a generalized topological space and let$A \subseteq X$. Then **g-interior** of A is denoted by i_g(A) and is defined to be the union of all g-open sets in X contained in A. The **g-closure** of A is denoted by $c_g(A)$ and is defined to be the Intersection of all g-closed sets in X containing A.

Remark: Since arbitrary union of g-open sets is a g-open set and arbitrary intersection of g-closed sets is a g-closed set, it follows that $i_g(A)$is a g-open set and $c_g(A)$ is a g-closed set. Thus $i_g(A)$isthelargestg-opensetin X contained in A and $c_g(A)$ is the smallest g-closed set in X containing A.

Proposition 2.5 [4]: Let (X, τ_g)be a generalized topological space and let A $\subseteq X$. Then

i) Aisg-opensetiff$i_g(A) = A$.

ii) Aisg-closedsetiff$c_g(A) = A$.

Theorem 2.6 [4]: Let(X, τ_g)be a generalized topological space and let A, B be subsets of X. Then following properties holds:

1. $i_g(\phi) = \phi, i_g(X) = X$.
2. If $A \subseteq B$ then$i_g(A) \subseteq i_g(B)$.
3. $i_g(A) \cup i_g(B) \subseteq i_g(A \cup B)$.
4. $i_g(A \cap B) \subseteq i_g(A) \cap i_g(B)$.

5. $i_g\left(i_g(A)\right) = i_g(A)$.

Theorem 2.7 [4]: Let (X, τ_g) be a generalized topological space and let A, B be subsets of *X*. Then following properties holds:

1. $c_g(\phi) = \phi$, $c_g(X) = X$.
2. If $A \subseteq B$ then $c_g(A) \subseteq c_g(B)$.
3. $c_g(A) \cup c_g(B) \subseteq c_g(A \cup B)$.
4. $c_g(A \cap B) \subseteq c_g(A) \cap c_g(B)$.
5. $c_g\left(c_g(A)\right) = c_g(A)$.

Theorem 2.8 [4]: Let (X, τ_g) be a generalized topological space and $\{A_\propto\}_{\propto \in \Lambda}$ be a family of subsets of X. Then

1. $\bigcup_{\propto \in \Lambda} i_g(A_\propto) \subseteq i_g(\bigcup_{\propto \in \Lambda} A_\propto)$.
2. $i_g(\bigcap_{\propto \in \Lambda} A_\propto) \subseteq \bigcap_{\propto \in \Lambda} i_g(A_\propto)$.

Theorem 2.9 [4]: Let (X, τ_g) be a generalized topological space and $\{A_\propto\}_{\propto \in \Lambda}$be a family of subsets of X. Then

(i) $\bigcup_{\propto \in \Lambda} c_g(A_\propto) \subseteq c_g(\bigcup_{\propto \in \Lambda} A_\propto)$.
(ii) $c_g(\bigcap_{\propto \in \Lambda} A_\propto) \subseteq \bigcap_{\propto \in \Lambda} c_g(A_\propto)$.

Theorem 2.10 [4]:Let (X, τ_g) be a generalized topological space and $A \subseteq X$. Then

(i) $i_g(X - A) = X - c_g(A)$.
(ii) $c_g(X - A) = X - i_g(A)$.

III. g-SEMI-CLOSED SETS

In this section we have obtained significant results which characterizes g-semi-closed-sets and constructed useful examples.

Definition 3.1 [2]: Let X be a generalized topological space and let A $\subseteq X$. Then the set A is said to be g-**semi-open** set, if $A \subseteq c_g\left(i_g(A)\right)$. Further the set A is said to be **g-semi-closed** set if (X - A) is g-semi-open set in X.

Remark : The empty set ϕ and whole set X are always g-semi-closed set in any generalized topological space X.

Proposition 3.2 : Let X be a generalized topological space. If A is a g-closedset in X then A isg-semi-closed set.

Proof: Let X be a generalized topological space and let A be a g-closed set in X. Then $c_g(A) = A$. Now we have, $c_g(i_g(X - A)) = c_g(X - c_g(A)) = X - i_g(c_g(A)) = X - i_g(A) \supseteq (X - A)$(as $i_g(A) \subseteq A$). Thus $(X - A) \subseteq c_g\left(i_g(X - A)\right)$. Hence $(X - A)$ is a g-semi-open set in X and so A is a g-semi-closed set in X.

In the following Example we see that converse of above result is not necessarily true.

Example 3.3: Let $X = \{a, b, c, d\}$ and let consider generalized topology $\tau_g = \{\phi, X, \{a, b\}, \{b, d\}, \{a, b, d\}\}$ on X. Suppose $A = \{d\}$. Then we see that A is g-semi-closed set in X but not g-closed set in X.

Proposition 3.4 Let X be a generalized topological space and A $\subseteq$ X. Then A is g-semi-closed set iff $i_g\left(c_g(A)\right) \subseteq A$.

Proof: Let A be a g-semi-closed set in X. Then (X - A) is g-semi-open set in X. This means $(X - A) \subseteq c_g\left(i_g(X - A)\right)$. We have, $i_g(X - A) = X - c_g(A)$. Therefore $c_g(i_g(X - A) = c_g(X - c_g(A)) = X - i_g\left(c_g(A)\right)$. Thus we find that $(X - A) \subseteq X - i_g\left(c_g(A)\right)$. This implies $i_g\left(c_g(A)\right) \subseteq A$.

Conversely suppose that $A \subseteq X$ and $i_g\left(c_g(A)\right) \subseteq A$. Then we have $(X - A) \subseteq X - i_g\left(c_g(A)\right)$. As $X - i_g\left(c_g(A)\right) = c_g\left(i_g(X - A)\right)$, we find that $(X - A) \subseteq c_g\left(i_g(X - A)\right)$. Hence $(X - A)$ is g-semi open set and so A is g-semi-closed set in X.

Remark : In a generalized topological space X if A is g-semi-closed set and A $\neq$ X then $c_g(A) \neq$ X .

Proposition 3.5 : Let X be a generalized topological space and let A $\subseteq$ X. Then A is g-semi-closed set iff $i_g(A) = i_g\left(c_g(A)\right)$.

Proof : Let A be a g-semi-closed set in X. Then from Proposition 3.4, we have A $\supseteq i_g\left(c_g(A)\right)$. This implies $i_g(A) \supseteq i_g(\, i_g\left(c_g(A)\right)) = i_g\left(c_g(A)\right)$, i.e.,$i_g(A) \supseteq \left(c_g(A)\right)$. Since $c_g(A) \supseteq$ A, we have $i_g\left(c_g(A)\right) \supseteq i_g(A)$. Hence we have find that $i_g(A) = i_g\left(c_g(A)\right)$.

Conversely suppose that$i_g(A) = i_g\left(c_g(A)\right)$. Since $i_g(A) \subseteq A$, we have $i_g\left(c_g(A)\right) \subseteq A$. Hence from Proposition 3.4 it follows that A is g-semi-closed set in X .

Proposition 3.6 : Let Xbe a generalized topological space and let A $\subseteq$ X . Then A is g-semi-closed set iff there exits a g-closed set F in X such that i_g(F) $\subseteq A \subseteq$F.

Proof : Let A be a g-semi-closed set in X. Then from Proposition 3.4 , we have $i_g\left(c_g(A)\right) \subseteq A$. Suppose F=$c_g(A)$. Then F is a g-closed set in X and i_g(F) $\subseteq A$. Since$A \subseteq c_g(A)$, we have $A \subseteq$F. Hence we deduce that i_g(F) $\subseteq A \subseteq$F.
Conversely suppose there exit a g-closed set F in X such that i_g(F) $\subseteq A \subseteq$F. This implies F= c_g(F)$\supseteq c_g(A)$and therefore i_g(F)$\supseteq i_g\left(c_g(A)\right)$. Then by i_g(F) $\subseteq A$ and $i_g\left(c_g(A)\right) \subseteq i_g$(F)we find that $i_g\left(c_g(A)\right) \subseteq A$. Hence by Proposition 3.4, we find that A is g- semi-closed set in X.

Teorem 3.7 : Let X be a generalized topological space and let $\{A_\alpha\}_{\alpha\in\wedge}$ be a collection of g-semi-closed sets in X. Then A = $\cap_{\alpha\in\wedge} A_\alpha$ is a g-semi-closed set in X.

Proof: Let X be a generalized topological space and let $\{A_\alpha\}_{\alpha\in\wedge}$ be a collection of g-semi-closed sets in X. Then, $i_g\left(c_g(A_\alpha)\right) \subseteq A_\alpha$, for all $\alpha \in\wedge$. Put$A = \cap_{\alpha\in\wedge} A_\alpha$.Then we have,$i_g\left(c_g(A)\right) = i_g\left(c_g\left(\cap_{\alpha\in\wedge} A_\alpha\right)\right) \subseteq i_g\left(\cap_{\alpha\in\wedge} c_g(A_\alpha)\right) \subseteq \cap_{\alpha\in\wedge} i_g\left(c_g(A_\alpha)\right)$.Since$A_\alpha$is a g-semi-closed set in X for each $\alpha \in\wedge$,from Proposition 3.4 we have $i_g\left(c_g(A_\alpha)\right) \subseteq A_\alpha$, for all $\alpha \in\wedge$. Therefore$i_g\left(c_g(A)\right) \subseteq\cap_{\alpha\in\wedge} A_\alpha$= A.Thus we conclude that $i_g\left(c_g(A)\right) \subseteq A$. Hence from Proposition 3.4 it follows that A is a g-semi-closed set in X.
In the following Example we see that union of two g- semi-closed sets may not a g-semi-closed set.
Example 3.8 : Let X ={ a,b,c,d } and let us consider generalized topology τ_g= $\{\phi, X, \{b,c\}$,{b,d},{b,c,d}} on X. Suppose A= { a, c } and B= { a, d } . Then A and B are g-semi- closed sets in X but their union is {a, c, d} which is not a g-semi-closed set in X.

REFERENCES

[1].Csaszar, A., Generalized topology, generalized continuity, Acta Math. Hungar., 96 (2002), 351-357.
[2]. Csaszar, A., Generalized open sets in generalized topologies, Acta Math. Hungar, 106 (2005), 53-66.
[3]. Levine, N., Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36-41.
[4]. HrishikeshTripathi , g- semi-open sets in generalized topological spaces, ISSN NO 2456-5474,Vol.VII ,Issue-4th may 2022.
[5]. Njasted.O., On some classes of nearly open sets, pacific J.Math.15(1965)961-970.