

Kinematic Analysis of Robot Manipulator

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Abstract: Manipulator kinematics refers the analytical study of the motion of manipulator, such as positions, velocities, and accelerations of the links of a manipulator. As formulating the suitable kinematics models for a manipulator is very crucial for analyzing the behavior of manipulators. However, researches always focused on rigid manipulator, while the manipulator is always a rigid flexible coupling multibody system, which can affect the accuracy of kinematic analysis and numerical simulation. This paper proposed kinematic analysis of manipulator based on rigid-flexible coupling system, so that automation accuracies can be improved in mechanical industries.

Keywords: Kinematics analysis, Numerical simulation, Manipulator, Rigid-flexible coupling

I. INTRODUCTION

The increasing demands for high product quality, reduced product cost and shorter product development cycle result in a continuing need to achieve improvements in speed, versatility, and accuracy in machining operations [1]. The manipulator kinematics [2] is involved in the position and orientation relationship between the end of manipulator actuator and manipulate or joint variables, including forward kinematics [3] and inverse kinematics [4,5], which relates to the position [6] and gesture of moving objects [7-9]. It has two basic issues of forward kinematics and inverse kinematics. In general, the inverse kinematics of manipulator is much more complex than forward kinematics. The kinematics of manipulator is a solution to describe the position and orientation of the end of manipulator actuator in the space, while the solution of manipulator inverse kinematics has an important significance on the trajectory planning and motion control [11,12], etc.

Current researches on manipulator kinematics have some relationships with the space mechanisms, which has two basic types: one is descriptive geometry-based graphical method, and the other is analytical method, such as vector analysis, matrix analysis, binary number analysis, artificial intelligence approach, grid method, or other mathematical tools. Obviously, the graphical methods have strong limitations, so there is no more development in this way. Denavit and Hartenberg proposed homogeneous matrix, named D-H matrix. As the D-H matrix method can be used to determine the transformation matrix between adjacent links by a single variable with mature matrix theory, it has become a method for the analysis of manipulator kinematics.

This paper proposes a model of kinematics analysis and numerical simulation of manipulator. Section 2 proposes a model of manipulator kinematics based on the D-H method. Section 4 concludes this paper.

II. MANIPULATOR KINEMATICS

In order to study the manipulator kinematics, each connecting link was built in its separated coordinate system to describe the position and orientation of connecting links by showing the relationship between the coordinate system. Using the D-H method, a 4×4 homogeneous transformation matrix was proposed to establish the kinematic model of the manipulator.

A) Description of Position and Orientation

The description method of position and orientation of an object in the space include homogeneous transformation, vector method, spin or method, and four-parameter law. In this paper, the homogeneous transformation method was used, which can integrate motion, transformation, mapping, and matrix operations. Each object representation in the space can be achieved by its own coordinate system. The located objects in the coordinate system A with the base coordinate

system O that can be noted as; thus, the location of the origin point P of the coordinate system A in the base coordinate system O can be expressed as following:

$${}^A_P O = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \quad (1)$$

in which $p_x, p_y,$ and p_z are the X-, Y-, and Z-axis coordinate values of point P.

For object in the space, not only its location but also its orientation needs to be known. The orientation of an object can be represented with the three axes of the coordinate system in the reference coordinate system in the direction cosine matrix, thus a 3×3 matrix R can represent the orientation of the object coordinate system A in the base coordinate system O:

$${}^O_A R = \begin{bmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{bmatrix} \quad (2)$$

${}^O_A R$ Is often referred as the rotation matrix, with it's with three-column unit vector orthogonal to each other. Then n, o, and a stand for orientation vectors of the end of the manipulator in the coordinate system. The suffix stands for its corresponding component in the X-, Y-, or Z-axis. The position and orientation of an object can be represented in the Coordinate system A as the following:

$$\{A\} = \left\{ {}^O_A R {}^A_P O \right\} \quad (3)$$

B) Homogeneous Transformation

The position and orientation of an object can be represented by Eq. 3, a 3×4 matrix. On behalf of the same object position and orientation, multiply Eq. 3 by a matrix scale factor to form a 4×4 matrix, such as Eq. 4. The matrix of this form is known as a homogeneous matrix.

$$A = \begin{bmatrix} n_x & o_x a_x & p_x \\ n_y & o_y a_y & p_y \\ n_z & o_z a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

The translation and rotation can be transformed by above homogeneous matrix, with the coordinate system relative to the fixed reference coordinate system.

The translation homogeneous transformation is:

$$T = \text{Trans} (dx, dy, dz) = \quad (5)$$

The rotation homogeneous transformation is:

$$\text{Rot} (x, \theta) = \quad (6)$$

$$\text{Rot} (y, \theta) = \quad (7)$$

$$\text{Rot} (z, \theta) = \quad (8)$$

in which Trans stands for translation, Rot stands for rotation, cθ stands for cosθ, and sθ stands for sinθ.

C) Kinematics Solution

a) Kinematics Model Based on D-H Notation

The manipulator consisted of a series of joints and connecting links, whose joints may be a linear or rotated unit in any order and plane. The length of link might be any number (including zero), which might be distorted or bent, or be located in any plane. To carry out these transformation steps, it is needed to specify and then determine in a reference coordinate system for each joint.

The transformation can be obtained through the following standard steps,

1. Rotate the angle θ_{n+1} around the z_n axis, with the x_n and x_{n+1} axis parallel to each other.
2. Move the distance d_{n+1} along the z_n axis, with the x_n and x_{n+1} axis co-linear to each other.
3. Move the distance a_{n+1} along the x_n axis, with the origin point of the x_n and x_{n+1} axis coincidence to each other.
4. Rotate α_{n+1} around the z_n axis, with the z_n and z_{n+1} axis alignment to each other.

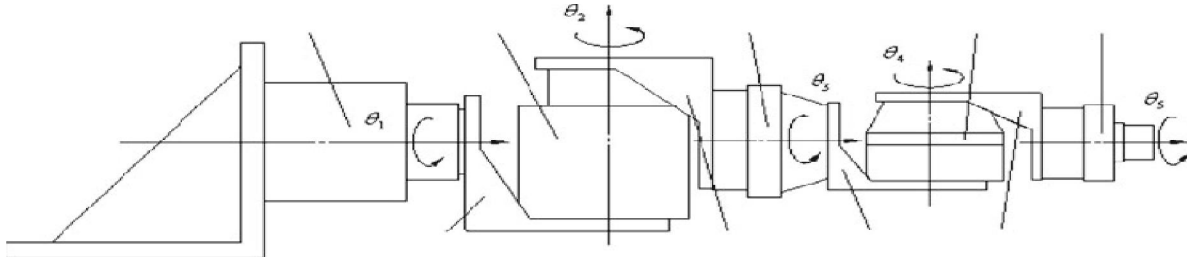


Figure 1: Basic structure of the manipulator

$$\begin{aligned}
 A_{n+1} &= \text{Rot}(z, \theta_{n+1}) \times \text{Trans}(0,0,d_{n+1}) \times \text{Trans}(a_{n+1},0,0) \times \text{Rot}(x, \alpha_{n+1}) \\
 &= \begin{bmatrix} a_1 & b_1 c_1 & d_1 \\ e_1 & f_1 g_1 & h_1 \\ 0 & i_1 j_1 & k_1 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}
 \end{aligned}$$

In which,

- a_1 $c\theta_{n+1}$
- b_1 $-s\theta_{n+1} \times c\alpha_{n+1}$
- c_1 $s\theta_{n+1} \times s\alpha_{n+1}$
- d_1 $a_{n+1} \times c\theta_{n+1}$
- e_1 $s\theta_{n+1}$
- f_1 $c\theta_{n+1} \times c\alpha_{n+1}$
- g_1 $-c\theta_{n+1} \times s\alpha_{n+1}$
- h_1 $a_{n+1} \times s\theta_{n+1}$
- i_1 $s\alpha_{n+1}$
- j_1 $c\alpha_{n+1}$
- k_1 d_{n+1}

b) Forward Kinematics Solution

According to the D-H method, a reference coordinate system of the manipulator was established, and the D-H parameter table was listed. With the establishment of the coordinate system shown in Fig. 2, the initial position of each joint angle is 0, where Y-axis direction is determined by the X- and Z -axes with the right hand rule. In order to make the coordinate system unified, a base coordinate system located at the base surface is used. According to the D-H method, the D-H parameters of the manipulator are listed in Table 1.

Link	Joint angle	d_n	α_n	a_n	Range of joint angle
1	θ_1	268	90	0	$-70^\circ \sim +70^\circ$
2	θ_2	0	-90	0	$-70^\circ \sim +70^\circ$
3	θ_3	256.6	90	0	$-76^\circ \sim +76^\circ$
4	θ_4	0	-90	0	$-76^\circ \sim +76^\circ$
5	θ_5	176.5	0	0	$-76^\circ \sim +76^\circ$

Table 1: D-H parameters of the manipulator

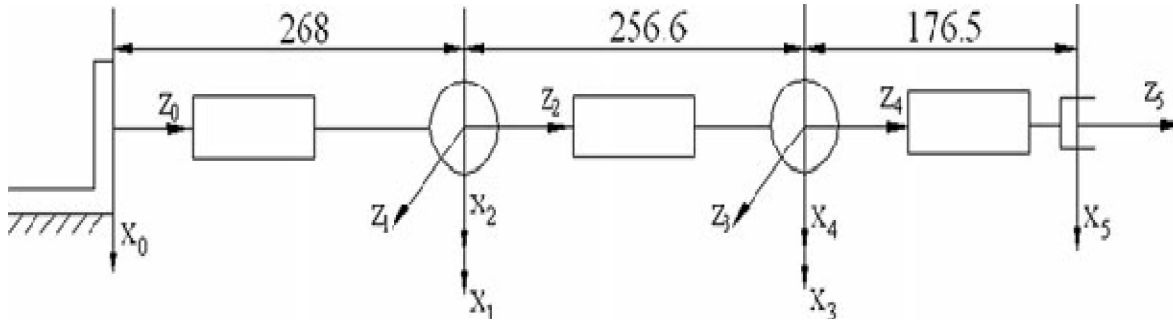


Figure 2: The coordinate system of the manipulator

From equation (9) And Table 1, each joint transformation matrix can be obtained.

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 10 & 268 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} C_2 & 0 & -S_2 & 0 \\ S_2 & 0 & C_2 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} C_3 & 0 & S_3 & 0 \\ S_3 & 0 & -C_3 & 0 \\ 0 & 10 & 256.6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_5 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & 268 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Its kinematics solution is:

$$T_5 = A_1 A_2 A_3 A_4 A_5 = \begin{bmatrix} n_x & o_x a_x & p_x \\ n_y & o_y a_y & p_y \\ n_z & o_z a_z & p_z \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

c) Inverse kinematics solution

According to the known end of the link relative to the position and orientation of the reference coordinate system, the inverse kinematics problem can be used to calculate each joint variable. In the manipulator control, it is often known to reach the target position and orientation, and the joint variables are used to drive the motor of each joint. Therefore, it is the basis of the manipulator motion planning and trajectory control. Kinematics solution is unique, and a given position and orientation of the manipulator is determined by each joint variable. However, for the inverse kinematics solution, it is often with multiple solutions, or no solutions. It seeks the kinematics using the inverse solution of the inverse transform method.

1. **To get θ_1** In the kinematic equations, multiply both sides of Eq. 10 by A_1^{-1}

$$A_1^{-1} T_5 = A_1 A_2 A_3 A_4 A_5 A_1^{-1} = A_2 A_3 A_4 A_5 \tag{11}$$

$$\begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 10 & 268 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x a_x & p_x \\ n_y & o_y a_y & p_y \\ n_z & o_z a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_2 & 0 & -S_2 & 0 \\ S_2 & 0 & C_2 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & 0 & S_3 & 0 \\ S_3 & 0 & -C_3 & 0 \\ 0 & 10 & 256.6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -10 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & 268 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, by simplifying equation we get,

$$\theta_1 = \arctan [(176.5 a_y - p_y) / (176.5 a_x - p_x)]$$

2. **To get θ_2** In the kinematic equations, multiply both sides of Eq. 11 by A_2^{-1}

$$A_2^{-1} A_1^{-1} T_5 = A_2 A_3 A_4 A_5 A_2^{-1} = A_3 A_4 A_5 \tag{12}$$

Then, by simplifying equation we get,

$$\theta_2 = \arctan [(176.5 a_x c_1 + 176.5 a_y s_1 - p_x c_1 - p_y s_1) / (-176.5 a_z + p_z - 268)]$$

3. **To get θ_3** In the kinematic equations, multiply both sides of Eq. 12 by A_3^{-1}

$$\begin{aligned} A_3^{-1} A_2^{-1} A_1^{-1} T_5 &= A_3 A_4 A_5 A_3^{-1} \\ &= A_4 A_5 \end{aligned} \quad (13)$$

Then, by simplifying equation we get,

$$\theta_3 = \arctan \left[\frac{-p_x s_1 + p_y c_1}{(p_x c_1 c_2 + p_y s_1 c_2 - s_2 p_2 + 268 s_2)} \right]$$

4. **To get θ_4** In the kinematic equations, multiply both sides of Eq. 13 by A_4^{-1}

$$\begin{aligned} A_4^{-1} A_3^{-1} A_2^{-1} A_1^{-1} T_5 &= A_4 A_5 A_4^{-1} \\ &= A_5 \end{aligned} \quad (14)$$

Then, by simplifying equation we get,

$$\theta_4 = -\arctan \frac{a}{b}$$

where, $a = (C_1 C_2 C_3 - S_1 S_3) P_x + (S_1 C_2 C_3 + C_1 S_3) P_y + S_2 C_3 P_z - 268 S_2 C_3$

$$b = -C_1 S_2 P_x - S_1 S_2 P_y + C_2 P_z - 268 C_2 - 256.6$$

5. **To get θ_5** In the kinematic equations, multiply both sides of Eq. 14 by A_5^{-1}

$$\begin{aligned} A_5^{-1} A_4^{-1} A_3^{-1} A_2^{-1} A_1^{-1} T_5 &= A_5 A_5^{-1} \\ &= I \end{aligned} \quad (15)$$

Then, by simplifying equation we get,

$$\theta_5 = -\arctan \left[\frac{-C_1 S_2 O_x - S_1 S_2 O_y + C_2 O_z}{(-C_1 S_2 n_x - S_1 S_2 n_y + C_2 n_z)} \right]$$

III. Kinematics ANALYSIS and NUMERICAL SIMULATION of MANIPULATOR

A) Kinematic Model

A three-dimensional model can be established in three-dimensional modeling software UG NX. Then this model can be imported into dynamic analysis software ADAMS thus, the rigid body modeling is complete. To achieve predetermined movement, the driving modes of each joint are then specified. By the joint model in UG NX, a variety of measure rotation angle of each joint, the angular velocity, and angular acceleration curve obtained from the measurement are created; in this way, the manipulator can be analyzed and optimized.

B) Simulation Model

On MATLAB platform, the coordinate system of the manipulator is defined, and the D-H parameters can be determined, then connecting rods are created with link parameters. Based on the simulation model of manipulator, the MATLAB simulation program can be used to achieve kinematic simulation. In Robotics Toolbox for MATLAB, the function “ ikine ” can be used to solve the manipulator inverse kinematics problem.

IV. CONCLUSION

This paper proposed a model of kinematics analysis of manipulator. The joint angles are obtained by the method of inverse kinematic analysis, and the predetermined trajectory is achieved by driving the joint motors through the joint angle control system. The kinematic analysis of five-degrees of freedom manipulator is given as an example, which demonstrates that the methodology is obviously helpful to manipulator design and can also profit from this way. Mathematical results can be validated using software, so that automation accuracies can be improved in mechanical industries.

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