

# Bending Stress Analysis of Spur Gear by Analytical Methods

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**Abstract:** *The main factors that cause the failure of gears are the bending stress and contact stress of the gear tooth. Stress analysis has been an important area in engineering to minimize failure and optimize the design. This project work gives details of bending stress and contact stress analysis of a spur gear tooth by analytical and numerical method. The investigation involves the involute profile of a spur gear. The geometrical parameters, such as the face width and module, are considered important for the variation of stresses in the design of gears. Specifically, the face width is important for spur gears. Using modeling software, 3-D models for different modules in spur gears are generated, and the simulation was performed using ANSYS to estimate the bending and contact stresses. The Lewis formula and Hertzian equation were used to calculate the bending stress and contact stress, respectively. The results of the theoretical stress values are compared with the stress values from the finite element analysis.*

**Keywords:** Bending Stress, Contact Stress, Module, etc.

## I. INTRODUCTION

Gears are known to be the essential and most efficient mechanical components in many machinery applications for power and motion transmission. Gears are used in machines and vehicles for the transmission of power. The design of gears is highly complicated involving many constraints such as strength, pitting resistance, bending stress, scoring wear, and interference in involutes gears etc. The concentration is focused on spur gear sets which are used to transmit motion between parallel shafts. The toothed gear transmission stands unique due to its high efficiency, reliable operation. The spur gear is the first-choice option for gears except when high speeds, loads and ratios direct towards other by using the vibration analysis and parameters such as natural frequency and vibration mode can be calculated. Gear transmissions are widely used in the mechanical engineering, and they efficiently transmit power. While the bending fatigue break of gear tooth is one of the most important failure modes. The degeneration of bending fatigue strength of gears is due to the initiation and extension of fatigue cracks on the root. In addition, the mild wear of gear flanks of interacting gear teeth causes unfavorable changes of the surface topography, giving non-uniform gear rate, increasing dynamic effects and perhaps more severe forms of tooth failures

Gearing is one of the most effective methods for transmitting power from one shaft to another with or without changing the speed. Spur gears are the most common type of gears. Spur gears have straight teeth, are mounted on parallel shafts and are mainly used to create very large gear reductions. Modern mechanical design involves complicated shapes, which are sometimes made of different materials that as a whole cannot be modeled by existing mathematical tools. Engineers need the FEA to evaluate their design. A complex problem is divided into smaller and simpler problems that can be solved using the existing knowledge of the mechanics of materials and mathematical tools.

The finite element method can proficiently supply this information, but the generation of a proper model is time consuming. Therefore, a pre-processor method that builds up the geometry required for finite element analysis may be used to reduce the modeling time, such as Pro/Engineer. Pro/Engineer can generate three-dimensional models of gears. In Pro/Engineer, the generated model geometry is opened in ANSYS for analysis. The application of finite element analysis allows the formation of bearing contacts during the cycle of meshing to be investigated and a stress analysis to be performed. The design of finite element models and the settings of boundary conditions are automated [1]. The theory of gearing and the modifications of the gear geometry are necessary to improve the conditions of meshing [2].

The calculation of the tooth bending strength and surface durability of normal and high contact ratios may be sufficient for preliminary designs or standardized purposes, but the stresses calculated using these simple equations derived from the linear theory of elasticity and the Hertzian contact model are not in good agreement with experimental results [2, 3]. The contact stress between two gear teeth was analyzed for different contact positions, which represented a pair of mating gears during rotation. Each case was represented by a sequence position of contacts between these two teeth. Finite element models were generated for these cases, and the stress was analyzed. The results were presented and finite element analysis results were compared with theoretical calculations [1]. The contact stress and bending stress of a helical gear set with localized bearing contact were predicted using finite element analysis (FEA).

The gear stress distribution was investigated using the commercial FEA package, ABAQUS/Standard. Furthermore, several examples have been presented to demonstrate the influences of the gear's design parameters and the contact positions on the stress distribution [4]. Researchers have analyzed the contact stresses between spur gear teeth using a plane model and validated the Hertz stress and AGMA contact stress with the finite element contact stress [5, 6]. The comparative reasoning of AGMA standards with FEA is main driver of the recent design and manufacturing of gears [7]. The quasi-static characteristic of finite element analysis allows the model to accurately simulate the distribution of equivalent stress and displacement change in the process of teeth meshing. The results agree well with the actual meshing [8]. The stress calculated for a pair of gears using the Lewis formula, Hertz equation and AGMA standards is comparable with FEA, and the PRO-E software and finite element software are good tools to define a safe design [9, 10].

Gearing is one of the most effective methods for transmitting power from one shaft to another with or without changing the speed. Spur gears are the most common type of gears. Spur gears have straight teeth, are mounted on parallel shafts and are mainly used to create very large gear reductions. The pressure angle is an important factor for spur gears to prevent undercutting when the number of teeth is small and to adjust the center distance. Modern mechanical design involves complicated shapes, which are sometimes made of different materials that as a whole cannot be modeled by existing mathematical tools. Engineers need the FEA to evaluate their design.

A complex problem is divided into smaller and simpler problems that can be solved using the existing knowledge of the mechanics of materials and mathematical tools. The finite element method can proficiently supply this information, but the generation of a proper model is time consuming. Therefore, a pre-processor method that builds up the geometry required for finite element analysis may be used to reduce the modeling time, such as Pro/Engineer. Pro/Engineer can generate three-dimensional models of gears. In Pro/Engineer, the generated model geometry is opened in ANSYS for analysis. The application of finite element analysis allows the formation of bearing contacts during the cycle of meshing to be investigated and a stress analysis to be performed. The design of finite element models and the settings of boundary conditions are automated [11]. The theory of gearing and the modifications of the gear geometry are necessary to improve the conditions of meshing [12]

## II. BENDING STRESS (LEWIS EQUATION)

In 1893, Wilfred Lewis provided a formula to estimate the bending stress in a tooth. He modeled a gear tooth that takes the full load at its tip as a simple cantilever beam. If we substitute a gear tooth for the rectangular beam, we can find the critical point in the root fillet of the gear by inscribing a parabola, as illustrated in Figure-1

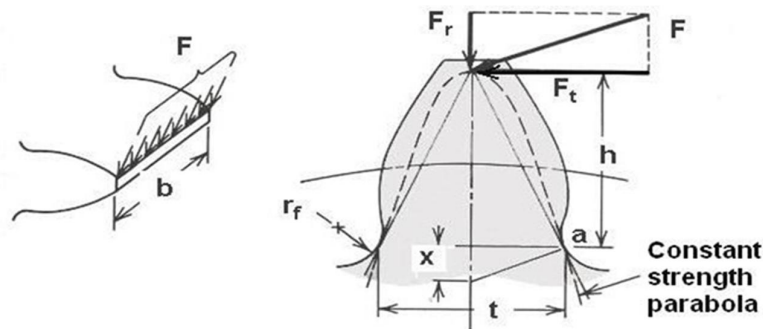


Figure 1: Forces Acting on Gear Tooth

Lewis considered gear teeth as a cantilever beam with a static normal force,  $F_t$ , applied at the tip. The assumptions are made in the derivation: The full load is applied to the tip of a single tooth at the static condition. The radial component is negligible. The load is distributed uniformly across the full-face width. Forces due to tooth sliding friction are negligible. The stress concentration in the tooth fillet is negligible.

$$\sigma_b = \frac{F_t}{b * m * Y}$$

$$F_t = (P/v) * K_v$$

The Lewis equation indicates that the tooth bending stress varies with, directly with the load,  $F_t$ , inversely with the tooth width,  $b$ , inversely with the tooth module,  $m$ , inversely with the tooth form factor,  $y$ . When teeth mesh, the load is delivered to the teeth with a certain impact. If we simply calculate the bending stress, the velocity factor should be used in the calculation [16]. Thus, the Lewis equation takes the following form:

$$\sigma_b = \frac{F_t}{b m n Y} (K_v * K_o * K_s (0.93 K_m))$$

### III. EFFECT OF MODULES IN SPUR GEAR

In spur gears, the bending stresses are calculated by varying the modules (2, 3, 4, 5, 6, and 7 mm) for a constant load as well as by keeping the other parameters, such as the number of teeth, pressure angle and face width, constant. The comparison of the theoretical bending stress values obtained from Lewis's equation and AGMA for different modules is shown in Table-3. The table clearly shows that the bending stress values obtained from the Lewis equation and AGMA are very close to each other and that the bending stress decreases with an increase in the module. Bending stress calculation using the Lewis equation from equation

Tangential load,  $F_t = 7462.68 \text{ N}$

Face width,  $b = 50 \text{ mm}$

Module,  $m = 5 \text{ mm}$

Lewis form factor,  $Y = 0.3$

$$\sigma_b = 5617 / (51 * 5 * 0.3) = 73.42 \text{ N/mm}^2$$

Bending stress calculation using AGMA from equation-3

Geometric factor,  $Y_j = 0.5$

Dynamic Velocity factor,  $K_v = 1.89$

Overload factor,  $K_o = 1$

Size factor,  $K_s = 0.75$

Load distribution factor,  $K_m = 1.4$

$$\sigma_b = 7462.68 / (50 * 5 * 0.5) = 71.42 \text{ N/mm}^2$$

In spur gears, the bending stresses are calculated by varying the modules (3, 4, 5, 6, 7 and 8 mm) for a constant load as well as by keeping the other parameters, such as the number of teeth, pressure angle and face width, constant.

**Table 1:** Geometric Input Parameters for Spur Gear

Description	Gear	Pinion
Material	Steel 15Ni2Cr1Mo28	Steel 15Ni2Cr1Mo28
Number of teeth (Z)	90	18
Young's Modulus (E)	$2.08 * 10^5 \text{ N/mm}^2$	$2.08 * 10^5 \text{ N/mm}^2$
Speed (N)	144 rpm	720 rpm
Power (P), kW	5	5
Poisson Ratio	0.3	0.3
Normal Module (m), mm	2,3,4,5,6,7	2,3,4,5,6,7
Normal Pressure Angle	20°	20°

**IV. RESULTS**

**Table 2:** Results Obtained for Different Modules of Spur Gear

Module (m)							
Description	Formula	2	3	4	5	6	7
Pitch Diameter (d) mm	$m \cdot Z_1$	36	54	72	90	108	126
Circular Pitch ( $P_c$ ) mm	$\pi d_1 / Z_1$	6.28	9.42	12.56	15.7	18.84	21.98
Diameter Pitch ( $P_d$ )	$Z_1 / d_1$	36	54	72	90	108	126
Centre Distance (a) mm	$m(Z_1 + Z_2) / 2$	6.28	9.42	12.56	15.7	18.84	21.98
Velocity	$(3.14 \cdot d_1 \cdot N) / 60$	0.50	0.33	0.25	0.20	0.17	0.14
Velocity factor $K_v$	$(6 + v) / 6$	81	121.5	162	202.5	243	283.5

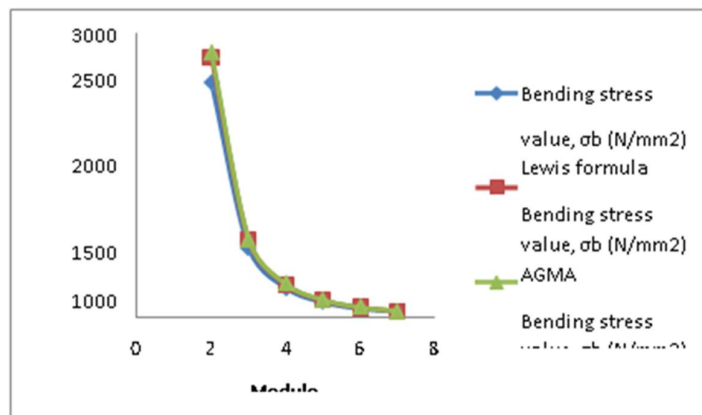
**Table 3:** Comparison of Maximum Bending Stress Values by Lewis Equation and AGMA for Different Module in Spur Gear

S. No	Module (mm)	Bending stress by Lewis's Formula (N/mm <sup>2</sup> )	Bending Stress by AGMA (N/mm <sup>2</sup> )
1	2	2488	2746
2	3	737.18	813.85
3	4	311	343.32
4	5	159.22	175.78
5	6	92.14	101.73
6	7	58.02	64.06

**Table 4:** Comparison of Maximum Bending Stress Values for Different Modules in Spur Gear

S. No.	Module, m (mm)	Bending stress value, $\sigma_b$ (N/mm <sup>2</sup> )			Difference (%)
		Lewis Formula	AGMA	ANSYS	
1	2	2488	2746	2796.14	1.79
2	3	737.18	813.85	823.25	1.14
3	4	311	343.32	352.12	2.50
4	5	159.22	175.78	181.66	3.24
5	6	92.14	101.73	103.36	1.58
6	7	58.02	64.06	65.14	1.66

**V. GRAPHICAL REPRESENTATION**



## **VI. CONCLUSION**

In spur gears, the design of the teeth is purely based on bending and contact stresses. The bending and contact stresses using agma for different modules in spur gears are calculated. The stresses are also calculated for spur gears using the lewis and hertz equations. The results obtained for the bending and contact stresses by agma, lewis and hertz equations are validated using the fea approach. The variation of the bending stress with various angular misalignments parallel to the plane of action is studied for spur gear and load concentration factor is calculated.

Finite element analysis for maximum bending stress is performed for parallel shafts to the plane of action and compared with the maximum bending stress for angle 10 and 2°. The maximum bending stress increases with the increase in angular misalignment and the load concentration is more on the edge of the gear tooth as the angle increases. To achieve accurate stress values, mesh sensitivity analysis is conducted under five categories for each model ranging from 1340 nodes to 4756 nodes. The results have shown that the stress does not appear to be converged even when mesh density has reached 4756.

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