IJARSCT
International Journal of Advanced Research in Science, Communication and Technology
International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal
ISSN: 2581-9429
Volume 6, Issue 1, November 2026
Impact Factor: 8.2

The Time Shifting Theorem and the Convolution for Iman Transform

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Abstract : *Integral transform methods have been modified to solve the several dynamic equations with initial or boundary conditions in many ways. Laplace, Sumudu and Iman transforms are such typical things. Among these, Iman transform is an efficient and novel tool. In this article, we have proposed the solution of differential equation with variable coefficients using Iman transform, and proved the time shifting theorem and the convolution one for Iman transform.*

Keywords: time shifting, variable coefficients, Laplace transform, Iman transform, Sumudu transform, convolution

1. INTRODUCTION

The Laplace transform method is performing a worthy role for solving linear ordinary differential equations and corresponding initial value problems, having an idea to replace operations of calculus on functions with operations of algebra on transforms, but it is not appropriate for differential equations with variable coefficients.
In order to overcome this limitation, many researches[1, 8-12, 16, 20] for integral transform have been pursued such as Sumudu transform and Iman one, and they applied to the transform for solving differential equations with variable coefficients. Sumudu transform/Iman transform, a modified Laplace transform, was introduced by Watugala in 1993/Iman on 2023 to solve initial value problems in control engineering problems, and it is defined by

$$L(v) = \frac{1}{v^2}\int_0^\infty e^{-v^2 t} f(t)dt$$

respectively. This slight difference makes many change. Iman insists that the Iman transform can be easily applied to the initial value problems with less computational work, and can solve various examples which is not solved by the Laplace or Sumudu transform. By the reason, we would like to propose the solution of differential equation with variable coefficients using Iman transform, and investigate some examples and theorems(time shifting/convolution) for Iman transform.
We have cognizance that the Iman transform is well applied for initial value problems with variable coefficients and solving integral equations of convolution type, despite of this may have the maladapted disadvantage for finding a general solution.
In this article, we have showed Iman transform of Euler-Cauchy equation, the Volterra integral equation of the second kind, its convolution theorem, and time shifting one.

II. THE SOLUTION OF EULER-CAUCHY EQUATION USING IMAN TRANSFORM

The topic for differential equation with variable coefficients has aroused deep interest of many researchers. In relation with this, let us check the definition of Iman transform.

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Definition 1. The Iman transform of the functions belonging to a class

A, where $A = \{f(t): \exists\, M, k_1, k_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}}, if\ t \in (-1)^j \times [0,\infty)\}$

Where $f(t)$ is denoted by $I[f(t)] = L(v)$ and defined as

$$I[f(t)] = L(v) = \frac{1}{v^2}\int_0^\infty e^{-v^2 t} f(t)dt\,, v \in k_1, k_2$$

We can naturally obtain the following results from the definition and simple calculations.

$$1)\ I\big(tf'(t)\big) = v^2\, L(v) - \frac{1}{v^2}f(0)$$

$$2)\ I(f''(t)) = v^4 L(v) - f(0) - \frac{1}{v^2}f'(0)$$

$$3)\ I(tf'(t)) = \frac{1}{v^4}\frac{d}{dv}\left[v^2 L(v) - \frac{1}{v^2}f(0)\right] - \frac{1}{v^2}\left[v^2 L(v) - \frac{1}{v^2}f(0)\right]$$

$$4)\ I(t^2 f'(t)) = \frac{1}{v^8}\frac{d^2}{dv^2}\left[v^2 L(v) - \frac{1}{v^2}f(0)\right]$$

$$5) I(tf''(t)) = \frac{1}{v^4}\frac{d}{dv}\left[v^4 L(v) - f(0) - \frac{1}{v^2}f'(0)\right] - \frac{1}{v^2}\left[v^2 L(v) - f(0) - \frac{1}{v^2}f'(0)\right]$$

$$6)\ I(t^2 f''(t)) = \frac{1}{v^8}\frac{d^2}{dv^2}\left[v^4 L(v) - f(0) - \frac{1}{v^2}f'(0)\right]$$

$$for\ I\big(f(t)\big) = L(v)$$

On the other hand, Euler-Cauchy equations are ordinary differential equations of the form $t^2y'+aty+by = 0$ with given constants a and b and unknown y(t). And the equation appears in a numbers of physics and engineering applications, such as when solving Laplace's equation in a polar coordinates, describing time-harmonic vibrations of a thin elastics rod, boundary value problem in spherical coordinates. Since the equation consist of variable coefficients, the Laplace transform method is not appropriate for this.

Theorem 1. The solution of Euler-Cauchy equation $t^2y'' + at\,y' + by = 0$
can be expressed by

$$y = \frac{(a-3)f(0) + tf'(0)}{b - a + 3}$$

using Iman transform.

Proof. Let us take Iman transform on both sides. Then

$$I(t^2 y'') = \frac{1}{v^8}\frac{d^2}{dv^2}\left[v^4 L(v) - f(0) - \frac{1}{v^2}f'(0)\right]$$

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$$= \frac{1}{v^4}L''(v) - \frac{4}{v^2}L'(v) + 6L(v)$$

$$I(aty') = \frac{a}{v^4}\frac{d}{dv}\left[v^2L(v) - \frac{1}{v^2}f(0)\right] - \frac{a}{v^2}\left[v^2L(v) - \frac{1}{v^2}f(0)\right]$$
$$= \frac{a}{v^4}L'(v) - 2aL(v)$$

And I[by] = b L(v). Arranging the equality, we get

$$\frac{1}{v^4}L''(v) - \frac{(a-4)}{v^2}L'(v) + (b - 2a + 6)L(v) = 0$$

where $I[f(t)] = L(v)$ is Iman transform of $f(t)$. Thus, we have

$$\frac{1}{v^4}\left[v^4L(v) - f(0) - \frac{1}{v^2}f'(0)\right] + \frac{(a-4)}{v^2}\left[v^2L(v) - \frac{1}{v^2}f(0)\right] + (b - 2a + 6)L(v) = 0$$

Collecting the $L(v)$-terms, we get

$$(b - a + 3)L(v) - \frac{(a-3)}{v^4}f(0) - \frac{1}{v^2}f'(0) - \frac{1}{v^6}f'(0) = 0$$

Since I(1) = $\frac{1}{v^4}$ and I(t) = $\frac{1}{v^6}$, we obtain

$$y = L^{-1}(v) = \frac{(a-3)f(0) + tf'(0)}{b - a + 3}$$

In [17], we proposed that a solution of Euler-Cauchy equation $t^2y'' + at\,y' + by = 0$
using Laplace transform can be expressed by

$$y = my(0)e^{\frac{a}{2}t}\ cos\ \sqrt{\frac{b}{m} - \frac{a^2}{4}}t + \left[ay(0)\left(\frac{m}{2} - 1\right) + my'(0)\right]\left[1\Big/\sqrt{\frac{b}{m} - \frac{a^2}{4}}\right]e^{\frac{a}{2}t}\ sin\ \sqrt{\frac{b}{m} - \frac{a^2}{4}}t$$

where d/ds = m. The result of theorem 1 is simpler than that of Laplace transform.

III. THE TIME SHIFTING THEOREM AND THE CONVOLUTION FOR IMAN TRANSFORM

Let us begin the lemma 1.

Lemma 1. Let L(v) be Iman transform of the function f(t) in

$A = \{f(t): \exists\, M, k_1, k_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}}, if\ t \in (-1)^j \times [0,\infty)\}$ with

Laplace transform F(s). Then Iman transform L(v) of $f\,f(t)$ is given by

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$$L(v) = \frac{1}{v^2} F(v^2)$$

Next, we consider convolution theorem for Iman transform. Iman [9] showed convolution theorem for Iman transform using lemma 1, and Ali [2] showed the theorem using infinite convergent series, but we would like to propose some more general manners as follows;

Theorem 2. (Convolution theorem for Iman transform)

$$I(f * g) = v^2 I(f) I(g)$$

for $I(f)$ is the Iman transform of f.

proof:

$$I(f)I(g) = \frac{1}{v^2}\int_0^\infty f(\tau)\, e^{-v^2\tau} d\tau \;.\; \int_0^\infty e^{-v^2 u} g(v) dv \;.$$

$$I(g) = \int_\tau^\infty g(t-\tau)\, e^{-v^2(t-\tau)} dt \;.$$

$$= e^{-v^2\tau} \int_\tau^\infty g(t-\tau)\, e^{-v^2 t} dt \;.$$

Thus

$$= \frac{1}{v^2}\int_0^\infty f(\tau)\, e^{-v^2\tau}\; e^{-v^2\tau} \int_\tau^\infty g(t-\tau)\, e^{-v^2 t} dt\, d\tau \,.$$

$$= \frac{1}{v^2}\int_0^\infty f(\tau) \int_\tau^\infty g(t-\tau)\, e^{-v^2 t} dt\, d\tau \,.$$

$$= \frac{1}{v^2}\int_0^\infty e^{-v^2 t} \int_\tau^\infty f(\tau) g(t-\tau)\, dt\, d\tau \,.$$

$$= \frac{1}{v^2}\int_0^\infty e^{-v^2 t} \quad (f * g)(t) dt \;.$$

$$= \frac{1}{v^2} I\ (f * g)$$

Theorem 3. (Time shifting theorem for Iman transform)

If $I[f(t)] = L(v)$, then

$$I\{f(t-a)v * (t-a)\} = e^{-av^2} L(v)$$

for v*(t) is the unit step function.

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Proof. The shifted function $I\{f(t-a)v*(t-a)\}$ is defined by

$$\frac{1}{v^2}\int_a^\infty e^{-v^2t}\, f(t-a)dt$$

and putting τ = t − a, we obtain

$$I\{f(t-a)v*(t-a)\} = \frac{1}{v^2}\int_0^\infty e^{-v^2(\tau+a)}\, f(\tau)d\tau$$

$$= \frac{1}{v^2}e^{-av^2}\int_0^\infty e^{-v^2\tau}\, f(\tau)d\tau$$

$$= \frac{1}{v^2}e^{-av^2}\int_0^\infty e^{-v^2\tau}\, f(\tau)d\tau = e^{-av^2}L(v)$$

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Volume 6, Issue 1, November 2026
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Table of Functions and their Iman Transform

$f(t)$	$I[f(t)] = F(v)$
1	$\frac{1}{v^4}$
t	$\frac{1}{v^6}$
t^2	$\frac{2!}{v^8}$
$t^n \quad n \in N$	$\frac{n!}{v^{2n+4}}$
e^{at}	$\frac{1}{v^2(v^2-a)}$
$\sin(at)$	$\frac{a}{v^2(v^4+a^2)}$
$\cos(at)$	$\frac{1}{v^4+a^2}$
$H(t-a)$	$\frac{1}{v^4}e^{-av^2}$
$\delta(t-a)$	$\frac{1}{v^2}e^{-av^2}$
$\sinh(at)$	$\frac{a}{v^2(v^4-a^2)}$
$\cosh(at)$	$\frac{a}{v^4-a^2}$
$t^{a-1}/\Gamma(a)$, a>0	$\left(\frac{1}{v^2}\right)^{a+1}$

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ISSN: 2581-9429
Volume 6, Issue 1, November 2026
Impact Factor: 8.2

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DOI: 10.48175/IJARSCT-40562