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Common Fixed- Point Theorems for Four Mappings Via FR-Contraction and FR-Expansive Maps Using CLR Property in Bipolar S_b-Metric Space

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Abstract: *In this paper, we introduce bipolar S_b-metric space with binary relation $\mathcal{R}$ via $F_{\mathcal{R}}$-contraction and $F_{\mathcal{R}}$-expansive maps. Two common fixed- point results are proved for four mappings under $CLR_{S,T}$ property and weak compatibility with graphical verification. The first result ensures common fixed point without continuity and completeness, while the second result for expansive maps does not require continuity, completeness and surjectivity. Illustrative examples with graphs and an application to integral equations are given.*

Keywords: bipolar S_b-metric space, $F_{\mathcal{R}}$-contraction, $F_{\mathcal{R}}$-expansive, common fixed point, $CLR_{S,T}$ property, binary relation

I. INTRODUCTION

The notion of metric space lays the foundation for the study of convergence, continuity and fixed -point theory. Motivated by the limitations of the classical triangle inequality in handling complex situations arising in integral equations and nonlinear analysis, a great deal of effort has been devoted to generalizing this structure.

A major breakthrough in this direction was made by Czerwik [4], who relaxed the triangle inequality by a constant $b \geq 1$ and introduced the concept of b-metric space. This idea opened a new pathway for obtaining fixed point results under weaker contraction conditions. In a different direction, Sedghi et al. [6] introduced S-metric spaces, where the distance is a function of three variables, thereby unifying and extending several known metric-type spaces. The natural combination of these two generalizations led to the notion of S_b-metric space [13], which inherits both the coefficient b and the three-variable structure and is strictly more general than both b-metric and S-metric spaces.

Parallel to this line, Mutlu and Gürdal [10] proposed a completely novel framework called bipolar metric space. Unlike classical metrics, the underlying set is decomposed into two poles X and Y, and the distance $d: X \times Y \to [0, \infty)$ is defined only between a left element and a right element. This bipolar viewpoint is particularly natural when the domain and range of the mappings lie in different sets. Recently, this concept was significantly enriched by Malhotra and Kumar [1], who introduced a binary relation R on $X \times Y$ and defined bipolar $\mathcal{R}$-metric spaces. The advantage of this approach is that the contractive condition is required to hold only for R-comparable pairs, rather than for all pairs, which is a genuine improvement.

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On the other hand, Wardowski [11] introduced a powerful new contraction, the F-contraction, via an auxiliary map $F:(0,\infty)\to\mathbb{R}$. Due to its generality over the Banach principle, this concept has attracted considerable attention and has been successfully extended to bipolar and bipolar b-metric spaces by many authors [15,2,18,25,20,22]. Simultaneously, the relation-theoretic fixed- point theory initiated by Alam and Imdad [8] has emerged as an effective tool to avoid the universality of the contraction condition by restricting it to a relation.

Although extensive research has been carried out on bipolar metrics, bipolar b-metrics and F-contractions, to the best of our knowledge, no result is available in the literature concerning common fixed points for four mappings satisfying the $CLR_{S,T}$ property in bipolar S_b-metric spaces endowed with a binary relation R via F_R-contractive and F_R-expansive conditions. The interaction of the third variable of S_b-metric with R-comparability creates new challenges that are not present in the bipolar b-metric case.

Motivated by these observations, we introduce the notion of bipolar S_b-metric space equipped with a binary relation R and define F_R-contractive and F_R-expansive mappings in this setting. We establish common fixed- point theorems for four self-mappings $A,B,S,T:\mathcal{X}\cup\mathcal{Y}\to\mathcal{X}\cup\mathcal{Y}$ satisfying $CLR_{S,T}$ property and weak compatibility. Our main results generalize and improve the results of Mutlu and Gürdal [10], Malhotra and Kumar [1], Wardowski [11], Alamri [15], Sedghi et al. [18] and several others [3,12,14]. Furthermore, we provide non-trivial examples to justify our results and an application to the existence of a common solution for a system of nonlinear integral equations is presented.

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II. PRELIMINARIES

Definition 1 (Bipolar metric space [10]). *Let $\mathcal{X},\mathcal{Y}$ be two non-empty sets and $d:\mathcal{X}\times\mathcal{Y}\to[0,\infty)$ be a mapping satisfying:*

$d(x,y)=0$ *iff* $x=y$, *for* $x=y\in\mathcal{X}\cap\mathcal{Y}$,

$d(x,y)=d(y,x)$ *if* $x,y\in\mathcal{X}\cap\mathcal{Y}$,

$d(x_1,y_2)\le d(x_1,y_1)+d(x_2,y_1)+d(x_2,y_2)$, *for all* $x_1,x_2\in\mathcal{X}$ *and* $y_1,y_2\in\mathcal{Y}$.

Then $(\mathcal{X},\mathcal{Y},d)$ is called a bipolar metric space.

Definition 2 (Bipolar S_b-metric space [6,13]). *Let $\mathcal{X},\mathcal{Y}$ be non-empty sets and $S_b:(\mathcal{X}\times\mathcal{Y})^3\to[0,\infty)$ be a mapping where $S_b:\mathcal{X}\times\mathcal{Y}\times\mathcal{X}\to[0,\infty)$ and $S_b:\mathcal{Y}\times\mathcal{X}\times\mathcal{Y}\to[0,\infty)$. The triple $(\mathcal{X},\mathcal{Y},S_b)$ is called a bipolar S_b-metric space with parameter $b\ge1$ if for all $x,y,z,a\in\mathcal{X}\cup\mathcal{Y}$:*

$S_b(x,y,z)=0$ *iff* $x=y=z$,

$S_b(x,y,z)\le b[S_b(x,x,a)+S_b(y,y,a)+S_b(z,z,a)]$.

Note: When $b=1$, it is called bipolar S-metric space. When we write S_b in this paper, we mean bipolar S_b-metric.

Definition 3 (Bipolar S_b-metric with binary relation $\mathcal{R}$[1,8]). *Let $(\mathcal{X},\mathcal{Y},S_b)$ be a bipolar S_b-metric space and $\mathcal{R}\subseteq\mathcal{X}\times\mathcal{Y}$ be a binary relation. Then $(\mathcal{X},\mathcal{Y},S_b,\mathcal{R})$ is called a bipolar S_b-metric space endowed with $\mathcal{R}$. We say $(x,y)\in\mathcal{R}$ is $\mathcal{R}$-comparable. $\mathcal{R}$ is called $\mathcal{R}$-complete if for all $(x,y)\in\mathcal{X}\times\mathcal{Y}$, either $(x,y)\in\mathcal{R}$ or $(y,x)\in\mathcal{R}$.*

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Example 1 (Example of S_b-metric). *Let* $\mathcal{X} = [0,1]$, $\mathcal{Y} = [0,1]$ *and define* $S_b(x,x,y) = |x-y|^2, \quad x \in \mathcal{X}, y \in \mathcal{Y}$. *Then with* $b = 2$, $(\mathcal{X}, \mathcal{Y}, S_b)$ *is a bipolar* S_b*-metric space. Indeed* $S_b(x,y,z) = 0$ *iff* $x = y = z$ *and triangle inequality holds with* $b = 2$. *Define* $\mathcal{R} = \{(x,y): x \geq y\}$, *then* $(\mathcal{X}, \mathcal{Y}, S_b, \mathcal{R})$ *is a bipolar* S_b*-metric with relation.*

Definition 4 (Bisequence and convergence [10,1]). *Let* $(\mathcal{X}, \mathcal{Y}, S_b, \mathcal{R})$ *be a bipolar* S_b*-metric space.*

A bisequence $\{x_n, y_n\}$ *is a sequence with* $x_n \in \mathcal{X}$ *and* $y_n \in \mathcal{Y}$.

$\{x_n, y_n\}$ *is* $\mathcal{R}$*-bisequence if* $(x_n, y_n) \in \mathcal{R}$ *and* $(x_n, y_{n+1}) \in \mathcal{R}$.

$\{x_n, y_n\}$ *is convergent if* $x_n \to p \in \mathcal{Y}$ *and* $y_n \to q \in \mathcal{X}$.

It is biconvergent if both converge to same point $z \in \mathcal{X} \cap \mathcal{Y}$, *i.e.,* $\lim_{n\to\infty} S_b(x_n, y_n, z) = 0$.

It is $\mathcal{R}$*-Cauchy if* $\lim_{n,m\to\infty} S_b(x_n, y_m, x_m) = 0$ *with* $(x_n, y_m) \in \mathcal{R}$. *The space is* $\mathcal{R}$*-complete if every* $\mathcal{R}$*-Cauchy bisequence is biconvergent.*

Definition 5 ($\mathcal{F}$-function [11]). *Let* $\mathcal{F}$ *be the family of functions* $F: (0,\infty) \to \mathbb{R}$ *such that:*

F *is strictly increasing,*

$\lim_{n\to\infty} \alpha_n = 0$ *iff* $\lim_{n\to\infty} F(\alpha_n) = -\infty$,

There exists $k \in (0,1)$ *such that* $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

Example: $F(\alpha) = \ln\alpha$, $F(\alpha) = \ln\alpha + \alpha$.

Definition 6 ($CLR_{S,T}$ property [1]). *Let* $A, B, S, T: \mathcal{X} \cup \mathcal{Y} \to \mathcal{X} \cup \mathcal{Y}$. *The pairs* (A,S) *and* (B,T) *are said to satisfy* $CLR_{S,T}$ *property if there exist bisequences* $\{x_n\} \subset \mathcal{X}$, $\{y_n\} \subset \mathcal{Y}$ *such that* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z \in \mathcal{X} \cap \mathcal{Y}$, *with* $(Ax_n, Sx_n) \in \mathcal{R}$ *and* $(By_n, Ty_n) \in \mathcal{R}$.

Definition 7 (Weak compatibility). *Pairs* (A,S) *and* (B,T) *are weakly compatible if they commute at coincidence points, i.e.,* $Ax = Sx \Longrightarrow ASx = SAx, \quad By = Ty \Longrightarrow BTy = TBy$.

Definition 8 (Cauchy-Schwarz inequality). *For integrable functions* f, g *on* $[0,T]$,

$$\left(\int_0^T f(s)g(s)ds\right)^2 \leq \left(\int_0^T f^2(s)ds\right)\left(\int_0^T g^2(s)ds\right).$$

In particular, $|\int_0^T f(s)ds|^2 \leq T\int_0^T |f(s)|^2 ds$. *This inequality will be used in our application to integral equations.*

Definition 9 ($F_{\mathcal{R}}$-contraction with four mappings). *Let* $(\mathcal{X}, \mathcal{Y}, S_b, \mathcal{R})$ *be a bipolar* S_b*-metric space. Four mappings* A, B, S, T *are called* $F_{\mathcal{R}}$*-contractive if there exist* $F \in \mathcal{F}$ *and* $\tau > 0$ *such that for all* $(x,y) \in \mathcal{R}$ *with* $S_b(Ax, Ax, By) > 0$,

$$\tau + F(S_b(Ax, Ax, By)) \leq F(M(x,y)),$$

where $M(x,y) = \max\left\{\begin{matrix} S_b(Sx, Sx, Ty), \\ S_b(Ax, Ax, Sx), \\ S_b(By, By, Ty), \\ \frac{S_b(Ax,Ax,Ty)+S_b(By,By,Sx)}{2} \end{matrix}\right\}$

Definition 10 ($F_{\mathcal{R}}$-expansive with four mappings). A, B, S, T *are called* $F_{\mathcal{R}}$*-expansive if there exist* $F \in \mathcal{F}$ *and* $\tau > 0$ *such that for all* $(x,y) \in \mathcal{R}$ *with* $S_b(Ax, Ax, By) > 0$,

$$F(S_b(Ax, Ax, By)) \geq F(M(x,y)) + \tau,$$

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with same $M(x,y)$ as in Definition 9.

Remark 1.

If we take $S = T =$ identity mapping, Definitions 9 and 10 reduce to $F_{\mathcal{R}}$-contraction for two mappings A, B.

If $\mathcal{R} = \mathcal{X} \times \mathcal{Y}$, then $(\mathcal{X}, \mathcal{Y}, \mathcal{S}_b, \mathcal{R})$ becomes usual bipolar $\mathcal{S}_b$-metric space and our definitions coincide with F-contraction in bipolar $\mathcal{S}_b$-metric space, generalizing [10,1,11,15,2].

The $CLR_{S,T}$ property does not require completeness of $\mathcal{X}$ or $\mathcal{Y}$ or continuity of mappings, which is an advantage over earlier results.

III. MAIN THEOREMS

Theorem 1. *Let $(\mathcal{X}, \mathcal{Y}, \mathcal{S}_b, \mathcal{R})$ be a bipolar $\mathcal{S}_b$-metric space with binary relation $\mathcal{R}$ on $\mathcal{X} \times \mathcal{Y}$. Let $A, B, S, T: \mathcal{X} \cup \mathcal{Y} \to \mathcal{X} \cup \mathcal{Y}$ be four mappings satisfying:*

The pair (A,S) and (B,T) satisfies $CLR_{S,T}$ property, i.e., there exist bisequences $\{(x_n, y_n)\} \subset \mathcal{X} \times \mathcal{Y}$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = Su = z \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = Tv = z$ for some $u, v \in \mathcal{X} \cup \mathcal{Y}$ and $z \in \mathcal{X} \cap \mathcal{Y}$ with $(u,v) \in \mathcal{R}$.

For some $F \in \mathcal{F}$ and $\tau > 0$, A, B, S, T is an $F_{\mathcal{R}}$-contraction mapping i.e., for all $(x,y) \in \mathcal{R}$ with $\mathcal{S}_b(Ax, Ax, By) > 0$,

$$\tau + F(\mathcal{S}_b(Ax, Ax, By)) \leq F(M(x,y))$$

where $$M(x,y) = \max \begin{Bmatrix} \mathcal{S}_b(Sx, Sx, Ty), \\ \mathcal{S}_b(Ax, Ax, Sx), \\ \mathcal{S}_b(By, By, Ty), \\ \frac{\mathcal{S}_b(Ax, Ax, Ty) + \mathcal{S}_b(By, By, Sx)}{2} \end{Bmatrix}$$

If $(x,y) \in \mathcal{R}$ then $(Ax, By) \in \mathcal{R}$ and $(Sx, Ty) \in \mathcal{R}$;

The pairs (A,S) and (B,T) are weakly compatible i.e., $Ax = Sx \Longrightarrow ASx = SAx$ and $By = Ty \Longrightarrow BTy = TBy$.

Then A, B, S, T have a common fixed point in $\mathcal{X} \cap \mathcal{Y}$. Moreover if $(z_1, z_2) \in \mathcal{R}$ for any two common fixed points z_1, z_2, then the common fixed point is unique.

*Proof.***Step 1: -** By the $CLR_{S,T}$ Property, there exists a bisequence $\{(x_n, y_n)\} \subset \mathcal{X} \times \mathcal{Y}$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = Su = z$$

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = Tv = z$$

Now, we first show that $Au = z$. Suppose, let $Au \neq z$, which means $\mathcal{S}_b(Au, Au, z) > 0$. As $n \to \infty$, it follows that $\mathcal{S}_b(Au, Au, By_n) > 0$. Since $(u, y_n) \in \mathcal{R}$, we can substitute $x = u$ and $y = y_n$ into the contraction condition (2):

$$\tau + F(\mathcal{S}_b(Au, Au, By_n)) \leq F(M(u, y_n))$$

Where

$$M(u, y_n) = \max \begin{Bmatrix} \mathcal{S}_b(Su, Su, Ty_n), \\ \mathcal{S}_b(Au, Au, Su), \\ \mathcal{S}_b(By_n, By_n, Ty_n), \\ \frac{\mathcal{S}_b(Au, Au, Ty_n) + \mathcal{S}_b(By_n, By_n, Su)}{2} \end{Bmatrix}$$

Taking limit as $n \to \infty$, then

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$$\mathcal{S}_b(Su, Su, Ty_n) \to \mathcal{S}_b(z, z, z) = 0$$
$$\mathcal{S}_b(Au, Au, Su) \to \mathcal{S}_b(Au, Au, z)$$
$$\mathcal{S}_b(By_n, By_n, Ty_n) \to \mathcal{S}_b(z, z, z) = 0$$
$$\frac{\mathcal{S}_b(Au, Au, Ty_n) + \mathcal{S}_b(By_n, By_n, Su)}{2} \to \frac{\mathcal{S}_b(Au, Au, z) + \mathcal{S}_b(z, z, z)}{2} = \frac{\mathcal{S}_b(Au, Au, z)}{2}$$

Thus $\lim_{n\to\infty} M(u, y_n) = \mathcal{S}_b(Au, Au, z)$.

Taking limit as $n \to \infty$ in the contraction yields:

$$\tau + F(\mathcal{S}_b(Au, Au, z)) \leq F(\mathcal{S}_b(Au, Au, z))$$
$$\Longrightarrow \tau \leq 0$$

This contradicts that $\tau > 0$. Therefore, we have $Au = z$. Since $Su = z$, so we get $Su = Au = z$.

Similarly taking $x = x_n$ and $y = v$ we get $Bv = Tv = z$, where $(x_n, v) \in \mathcal{R}$.

Step-2: - From condition (iv), the pairs (A, S) and (B, T) are weakly compatible. Since $Au = Su = z$

$$\Longrightarrow ASu = SAu \Longrightarrow AZ = SZ$$

Since $Bv = Tv = z$ it implies that $BTv = TBv$

$$\Longrightarrow Bz = Tz.$$

Step-3: - Next, we show that $Az = z$. Suppose that $Az \neq z$, then

$$\mathcal{S}_b(Az, Az, z) > 0.$$

Since $Bv = z$, then $\mathcal{S}_b(Az, Az, Bv) > 0$. Apply condition (ii) with $x = z$ and $y = v$, such that $(z, v) \in \mathcal{R}$

$$\tau + F(\mathcal{S}_b(Az, Az, Bv)) \leq F(M(z, v))$$

Where

$$M(z, v) = \max\left\{\begin{matrix}\mathcal{S}_b(Sz, Sz, Tv), \\ \mathcal{S}_b(Az, Az, Sz), \\ \mathcal{S}_b(Bv, Bv, Tv), \\ \dfrac{\mathcal{S}_b(Az, Az, Tv) + \mathcal{S}_b(Bv, Bv, Sz)}{2}\end{matrix}\right\}$$

$$= \max\left\{\begin{matrix}\mathcal{S}_b(Az, Az, z) \\ \mathcal{S}_b(Az, Az, Az) \\ \mathcal{S}_b(z, z, z) \\ \dfrac{\mathcal{S}_b(Az, Az, z) + \mathcal{S}_b(z, z, Az)}{2}\end{matrix}\right\}$$

$$= \mathcal{S}_b(Az, Az, z)$$

As $\mathcal{S}_b(Az, Az, z) = \mathcal{S}_b(z, z, Az)$ then by contraction inequality:

$$\tau + F(\mathcal{S}_b(Az, Az, z)) \leq F(\mathcal{S}_b(Az, Az, z))$$
$$\Longrightarrow \tau \leq 0.$$

This is a contradiction since $\tau > 0$. Hence $Az = z$. Since $Az = Sz$, so $Sz = z$. i.e. $Az = Sz = z$.

Similarly, using condition (ii) with general limits, it can be proved that $Bz = z$ and $Tz = z$. Combining these results:

$$Az = Bz = Sz = Tz = z.$$

Thus z is a common fixed point of the four mappings A, B, S and T.

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Step 4: - Uniqueness of the Common Fixed Point:

Let z_1 and z_2 be two distinct common fixed points such that $(z_1, z_2) \in \mathcal{R}$, and

$$Az_1 = Sz_1 = Bz_1 = Tz_1 = z_1 \text{ and } Az_2 = Sz_2 = Bz_2 = Tz_2 = z_2$$

Since $z_1 \neq z_2$ we have $\mathcal{S}_b(Az_1, Az_1, Bz_2) = \mathcal{S}_b(z_1, z_1, z_2) > 0$. Applying condition (2) for $x = z_1$ and $y = z_2$

$$\tau + F(\mathcal{S}_b(Az_1, Az_1, Bz_2)) \leq F(M(z_1, z_2))$$

where

$$M(z_1, z_2) = \max\left\{\begin{matrix}\mathcal{S}_b(Sz_1, Sz_1, Tz_2), \\ \mathcal{S}_b(Az_1, Az_1, Sz_1), \\ \mathcal{S}_b(Bz_2, Bz_2, Tz_2), \\ \frac{\mathcal{S}_b(Az_1, Az_1, Tz_2) + \mathcal{S}_b(Bz_2, Bz_2, Sz_1)}{2}\end{matrix}\right\}$$

$$= \max\left\{\begin{matrix}\mathcal{S}_b(z_1, z_1, z_2) \\ 0, \\ 0, \\ \frac{\mathcal{S}_b(z_1, z_1, z_2) + \mathcal{S}_b(z_2, z_2, z_1)}{2}\end{matrix}\right\}$$

$$= \mathcal{S}_b(z_1, z_1, z_2)$$

Then by contraction relation,

$$\tau + F(\mathcal{S}_b(z_1, z_1, z_2)) \leq F(\mathcal{S}_b(z_1, z_1, z_2))$$

$$\Longrightarrow \tau \leq 0$$

Which is impossible as $\tau > 0$. Thus $z_1 = z_2$.

Hence, the common fixed point is unique for four self- mappings A, B, S and T when $(z_1, z_2) \in \mathcal{R}$. □

Theorem 2 . *Let $(\mathcal{X}, \mathcal{Y}, \mathcal{S}_b, \mathcal{R})$ be a bipolar S_b-metric space and $A, B, S, T: \mathcal{X} \cup \mathcal{Y} \to \mathcal{X} \cup \mathcal{Y}$ be four mappings satisfying:*

The pairs (A, S) and (B, T) satisfy $CLR_{S,T}$ Property, as in theorem 1,

For some $F \in \mathcal{F}$ and $\tau > 0$, A, B, S, T is an $F_{\mathcal{R}}$-expansive mapping i.e., for all $(x, y) \in \mathcal{R}$, with $\mathcal{S}_b(Ax, Ax, By) > 0$, $F(\mathcal{S}_b(Ax, Ax, By)) \geq F(M(x, y)) + \tau$

Where $M(x, y) = \max\left\{\begin{matrix}\mathcal{S}_b(Sx, Sx, Ty), \\ \mathcal{S}_b(Ax, Ax, Sx), \\ \mathcal{S}_b(By, By, Ty), \\ \frac{\mathcal{S}_b(Ax, Ax, Ty) + \mathcal{S}_b(By, By, Sx)}{2}\end{matrix}\right\}$

Pairs (A, S) and (B, T) are Weakly Compatible.

If $(x, y) \in \mathcal{R}$ then $(Ax, By) \in \mathcal{R}$ and $(Sx, Ty) \in \mathcal{R}$.

Then A, B, S, T have a common fixed point in $\mathcal{X} \cap \mathcal{Y}$. Moreover if $(z_1, z_2) \in \mathcal{R}$ for any two common fixed points z_1 and z_2 then the common fixed point is unique.

Proof.

Step 1: Following the exact same line of argument as in theorem 1 (Step-1) via the $CLR_{S,T}$ Property, we assume $Au \neq z$ (i.e., $\mathcal{S}_b(Au, Au, z) > 0$). Taking the limit as $n \to \infty$ in the $F_{\mathcal{R}}$-expansive mapping condition (2) gives:

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$$F(\mathcal{S}_b(Au, Au, z)) \geq F(\mathcal{S}_b(Au, Au, z)) + \tau$$
$$\Rightarrow 0 \geq \tau$$

Which contradicts $\tau > 0$. Thus $Au = Su = z$. Similarly, it follows that $Bv = Tv = z$.

Step 2: As proven in theorem 1 (Step-2) the weak compatibility of the pairs implies $Az = Sz$ and $Bz = Tz$.

Step 3: To show $Az = z$, Assume that $Az \neq z$. Applying the $F_{\mathcal{R}}$-expansive condition for the pair $(z, v) \in \mathcal{R}$ yields:

$$F(\mathcal{S}_b(Az, Az, Bv)) \geq F(M(z, v)) + \tau$$

Since $M(z, v) = \mathcal{S}_b(Az, Az, z)$ and $Bv = z$, this implies to:

$$F(\mathcal{S}_b(Az, Az, z)) \geq F(\mathcal{S}_b(Az, Az, z)) + \tau$$
$$\Rightarrow \tau \leq 0$$

Which is a Contradiction. Hence $Az = Sz = z$. By the similar procedure for the other pair, we obtain $Bz = Tz = z$. Thus z is a Common fixed Point.

Step 4: If z_1, z_2 are two distinct common fixed points with $(z_1, z_2) \in \mathcal{R}$, substituting them into condition (ii) gives:

$$F(\mathcal{S}_b(z_1, z_1, z_2)) \geq F(\mathcal{S}_b(z_1, z_1, z_2)) + \tau$$
$$\Rightarrow \tau \leq 0$$

Which is a contradiction on $\tau > 0$. So $z_1 = z_2$. Hence the Common fixed Point is unique. □

Corollary 1.*Let $(\mathcal{X}, \mathcal{Y}, \mathcal{S}_b, \mathcal{R})$ be a bipolar S_b-metric space and let $\mathcal{R} \subseteq \mathcal{X} \times \mathcal{Y}$ be a closed binary relation. Let $A, B: \mathcal{X} \cup \mathcal{Y} \to \mathcal{X} \cup \mathcal{Y}$ be two self mappings satisfying the conditions:*

There exists a bisequence $\{(x_n, y_n)\} \subset \mathcal{X} \times \mathcal{Y}$ such that: $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n = z$ for some $z \in \mathcal{X} \cap \mathcal{Y}$ satisfying $(z, z) \in \mathcal{R}$.

For some function $F \in \mathcal{F}$ and a constant $\tau > 0$, the mappings satisfy one of the structural requirements $\forall (x, y) \in \mathcal{R}$, with $\mathcal{S}_b(Ax, Ax, By) > 0$:

[Contraction]: $\tau + F(\mathcal{S}_b(Ax, Ax, By)) \leq F(M(x, y))$

[Expansive]: $F(\mathcal{S}_b(Ax, Ax, By)) \geq F(M(x, y)) + \tau$

Where $M(x, y) = \max \begin{Bmatrix} \mathcal{S}_b(x, x, y), \\ \mathcal{S}_b(Ax, Ax, x), \\ \mathcal{S}_b(By, By, y), \\ \frac{\mathcal{S}_b(Ax, Ax, y) + \mathcal{S}_b(By, By, x)}{2} \end{Bmatrix}$ *Then A and B have a unique common fixed point $z \in \mathcal{X} \cap \mathcal{Y}$.*

Remark 2. *We are taking S and T as Identity mappings in theorem 1 and theorem 2.*

Corollary 2 (Corollary 2). *Let $(\mathcal{X}, \mathcal{Y}, \mathcal{S}_b, \mathcal{R})$ be a bipolar S_b-metric space and $A, S: \mathcal{X} \cup \mathcal{Y} \to \mathcal{X} \cup \mathcal{Y}$ be weakly compatible mappings satisfying the CLR_S property, and for some function $F \in \mathcal{F}$ and a constant $\tau > 0$, the mappings satisfy one of the structural requirements for all $(x, y) \in \mathcal{R}$ with $\mathcal{S}_b(Ax, Ax, Ay) > 0$:*

Contraction Case: $\tau + F(\mathcal{S}_b(Ax, Ax, Ay)) \leq F(M(x, y))$

Expansive Case: $F(\mathcal{S}_b(Ax, Ax, Ay)) \geq F(M(x, y)) + \tau$

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Where $M(x,y) = \max\left\{\begin{matrix}\mathcal{S}_b(Sx,Sx,Sy),\\ \mathcal{S}_b(Ax,Ax,Sx),\\ \mathcal{S}_b(Ay,Ay,Sy),\\ \frac{\mathcal{S}_b(Ax,Ax,Sy)+\mathcal{S}_b(Ay,Ay,Sx)}{2}\end{matrix}\right\}$ *Then A and S have a unique common fixed point* $z \in \mathcal{X} \cap \mathcal{Y}$.

Remark 3. *We taking* $A = B$ *and* $S = T$ *in theorem 1 and theorem 2.*

Remark 4. *If* $\mathcal{R} = \mathcal{X} \times \mathcal{Y}$, *then theorem 3.1 reduces to common fixed point for four maps without binary relation in bipolar* S_b*-metric via* $CLR_{S,T}$. *This generalizes Malhotra et al. [theorem 1].*

IV. EXAMPLE

Example 2. *Let* $\mathcal{X} = [0,1]$ *and* $\mathcal{Y} = [0,1]$. *We define the bipolar* S_b*-metric space* $(\mathcal{X}, \mathcal{Y}, \mathcal{S}_b, \mathcal{R})$ *with* $\mathcal{R} \subseteq \mathcal{X} \times \mathcal{Y}$ *a binary relation by* $\mathcal{S}_b(x,x,y) = |x-y|^2, \quad x \in \mathcal{X}, y \in \mathcal{Y}$. *The binary Relation* $\mathcal{R}$ *on* $\mathcal{X} \times \mathcal{Y}$ *is defined as* $\mathcal{R} = \{(x,y) \in \mathcal{X} \times \mathcal{Y} : x \geq y\}$ *Let the function* $F \in \mathcal{F}$ *be defined as* $F(\alpha) = \ln\alpha$ *for* $\alpha > 0$ *and* $\tau = \ln 2 > 0$.

Define the four mappings $A, B, S, T : \mathcal{X} \cup \mathcal{Y} \to \mathcal{X} \cup \mathcal{Y}$ *as follows,* $Ax = \frac{x}{4}, \quad By = \frac{y}{4} Sx = x, \quad Ty = y$.

(A) Verifying Conditions: -

$CLR_{S,T}$*Property: - Consider bisequence* $\{(x_n, y_n)\} = \langle(\frac{1}{n}, \frac{1}{n})\rangle$ *in* $\mathcal{X} \times \mathcal{Y}$ *such that,* $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = 0$ *Here* $u = 0, v = 0$ *and common limit is* $z = 0 \in \mathcal{X} \cap \mathcal{Y}$ *with* $(0,0) \in \mathcal{R}$.

Weak Compatibility: - Since $Sx = x$ *and* $Ty = y$ *are identity mappings, then any coincidence point commutes, always.*

Relation Preservation: - If $x \geq y \Longrightarrow \frac{x}{4} \geq \frac{y}{4} \Longrightarrow Ax \geq By$. *So* $(Ax, By) \in \mathcal{R}$.

(B) Checking the Contraction Inequality

$$\mathcal{S}_b(Ax,Ax,By) = \left|\frac{x}{4} - \frac{y}{4}\right|^2 = \frac{1}{16}|x-y|^2$$

$$M(x,y) = \max\left\{\begin{matrix}\mathcal{S}_b(Sx,Sx,Ty)\\ \mathcal{S}_b(Ax,Ax,Sx)\\ \mathcal{S}_b(By,By,Ty)\\ \frac{\mathcal{S}_b(Ax,Ax,Ty)+\mathcal{S}_b(By,By,Sx)}{2}\end{matrix}\right.$$

$$= \max\left\{\begin{matrix}\mathcal{S}_b(x,x,y),\\ \mathcal{S}_b(\frac{x}{4},\frac{x}{4},x),\\ \mathcal{S}_b(\frac{y}{4},\frac{y}{4},y),\\ \frac{\mathcal{S}_b(\frac{x}{4},\frac{x}{4},y)+\mathcal{S}_b(\frac{y}{4},\frac{y}{4},x)}{2}\end{matrix}\right.$$

$$\geq \mathcal{S}_b(x,x,y) = |x-y|^2$$

Now $L.H.S = \tau + F(\mathcal{S}_b(Ax,Ax,By)) = \ln 2 + \ln(|Ax-By|^2) = \ln 2 + \ln\left(\left|\frac{x}{4} - \frac{y}{4}\right|^2\right) = \ln 2 + \ln\left(\frac{1}{16}|x-y|^2\right) = \ln\left(2 \cdot \frac{1}{16}|x-y|^2\right) = \ln\left(\frac{1}{8}|x-y|^2\right)$

$R.H.S = F(M(x,y)) \geq \ln(|x-y|^2)$

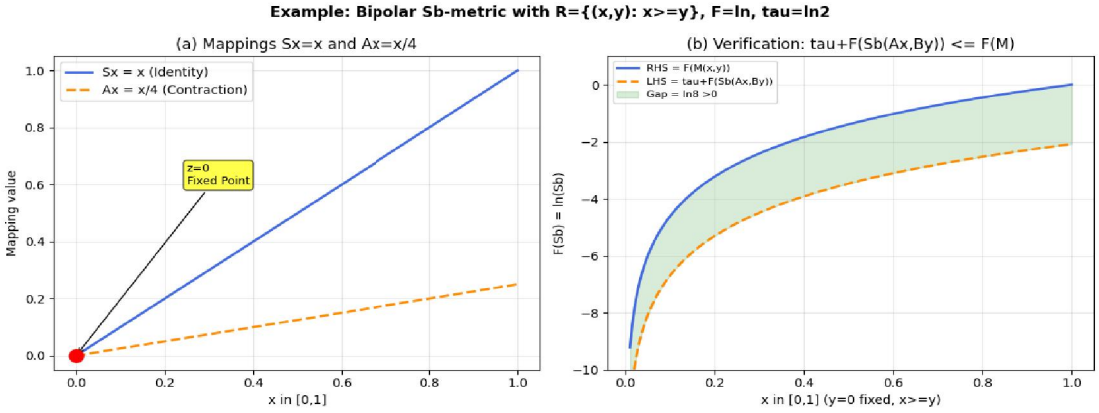
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Therefore

$\ln\left(\frac{1}{8}|x-y|^2\right) \le \ln(|x-y|^2) \Rightarrow \ln\left(\frac{1}{8}|x-y|^2\right) \le \ln(|x-y|^2) \le F(M(x,y)) \Rightarrow \tau + F(\mathcal{S}_b(Ax,Ax,By)) \le F(M(x,y))$ *for all* $(x,y) \in \mathcal{R}$ *where* $\mathcal{S}_b(Ax,Ax,By) > 0$.

Therefore, all conditions are satisfied. Then by theorem 1, we say that $z = 0$ *is the unique common fixed point such that* $A(0) = B(0) = S(0) = T(0) = 0$.



(a) Graphs of $Sx = x$ and $Ax = x/4$ with common fixed point $z = 0$. (b) Verification of F-contraction: $\tau + F(S_b(Ax,Ax,By)) \le F(M(x,y))$ with $F = ln, \tau = ln2$.

Example 3 (Validating the F_R-Expansive Mapping (Theorem 2)). *Let* $X = [0,1]$ *and* $Y = \{0\}$. *We define bipolar* S_b*-metric space* (X,Y,S_b,R) *with binary relation* $R \subseteq X \times Y$ *by* $S_b(x,x,y) = |x-y|^2, \quad \forall x \in X,\ y \in Y,$ *and* $R = \{(x,y) \in X \times Y : x \ge y\}$. *Let* $F \in \mathcal{F}$ *be defined as* $F(\alpha) = \ln\alpha$ *and* $\tau = \ln(1.5) > 0$.

Define four mappings $A,B,S,T: X \cup Y \to X \cup Y$ *as* $Ax = 8x, \quad By = 0, \quad Sx = 4x, \quad Ty = 0$. *In similar process as in Example 4.1,* $CLR_{S,T}$ *property and weak-compatibility property are also satisfied.*

Now, checking the Expansive Inequality:

Now, $S_b(Ax,Ax,By) = |8x-0|^2 = 64x^2$.

$$\text{Where } M(x,y) = \max\begin{cases} S_b(Sx,Sx,Ty), \\ S_b(Ax,Ax,Sx), \\ S_b(By,By,Ty), \\ \frac{S_b(Ax,Ax,Ty)+S_b(By,By,Sx)}{2} \end{cases}$$

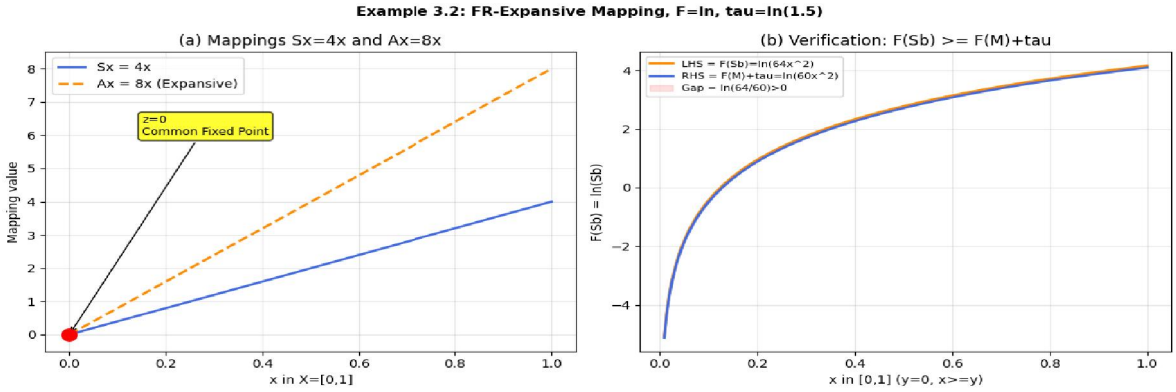
$$= \max\begin{cases} S_b(4x,4x,0), \\ S_b(8x,8x,4x), \\ S_b(0,0,0), \\ \frac{S_b(8x,8x,0)+S_b(0,0,4x)}{2} \end{cases} = \max\left\{16x^2,\ 0,\ 0,\ \frac{64x^2+16x^2}{2}\right\} = 40x^2.$$

Now, for $(x,y) \in R$, *with* $S_b(Ax,Ax,By) > 0$, $L.H.S = F(S_b(Ax,Ax,By)) = \ln(64x^2)$
$R.H.S = F(M(x,y)) + \tau = \ln(40x^2) + \ln 1.5 = \ln(60x^2)$.

Since $\ln(64x^2) \ge \ln(60x^2)$, *we have* $F(S_b(Ax,Ax,By)) \ge F(M(x,y)) + \tau, \quad \forall (x,y) \in R$.

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So, all properties of Theorem 3.2 are satisfied and $z = 0 \in X \cap Y$ *is confirmed as the unique common fixed point since* $A(0) = B(0) = S(0) = T(0) = 0$.



$Sx = 4x$, $Ax = 8x$ (b) $F(S_b) \geq F(M) + \tau$ verification

V. APPLICATION

Let $I = [0, T]$ and $E = C[0, T]$ the set of all real valued continuous functions defined on I, where $T > 1$.

We consider the system of non- linear integral equations:

$$x(t) = q(t) + \int_0^T k_1\,(t, s, x(s))\,ds$$

$$y(t) = q(t) + \int_0^T k_2\,(t, s, y(s))\,ds,$$

Where $q \in E$ and $k_1, k_2 : I \times I \times \mathbb{R} \to \mathbb{R}$ are continuous kernel.

Let $X = C[0, T]$ and $Y = C[0, T]$. Define bipolar S_b-metric by

$$S_b(x, x, y) = \sup_{t \in I} \left(|x(t) - y(t)|^2 e^{-\lambda t} \right)$$

Where $\lambda > 0$ is a scaling parameter.

Define the binary relation R on $X \times Y$ as:

$$R = \{(x, y) \in X \times Y : x(t) \leq y(t)\} \quad \text{for } t \in I.$$

We define four mappings $A, B, S, T : X \cup Y \to X \cup Y$ as

$$Sx = x$$
$$Ty = y$$

$$Ax(t) = q(t) + \int_0^T k_1\,(t, s, x(s))\,ds$$

$$By(t) = q(t) + \int_0^T k_2\,(t, s, y(s))\,ds.$$

Suppose that, for all $t, s \in I$ and $(x, y) \in R$

There exists a constant $\delta > 0$ such that

$$|k_1(t, s, x(s)) - k_2(t, s, y(s))| \leq \delta |x(s) - y(s)|$$

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Let $\frac{\delta^2 T e^{\lambda T}}{\lambda} \leq k$ for k be an arbitrary constant such that $0 < k < 1$.

Now,

$$\begin{aligned}|Ax(t) - By(t)| &= \left|\int_0^T [k_1(t,s,x(s)) - k_2(t,s,y(s))]\, ds\right| \\ &\leq \int_0^T |\, k_1(t,s,x(s)) - k_2(t,s,y(s))|\, ds \\ &\leq \delta \int_0^T |\, x(s) - y(s)|\, ds \\ &= \delta \int_0^T \left(|x(s) - y(s)| \cdot e^{-\frac{\lambda s}{2}}\right) e^{\frac{\lambda s}{2}}\, ds.\end{aligned}$$

Squaring both sides, and applying Cauchy-Schwarz inequality,

$$\begin{aligned}|Ax(t) - By(t)|^2 &\leq \delta^2 \left(\int_0^T |\, x(s) - y(s)|^2 e^{-\lambda s}\, ds\right) \cdot \left(\int_0^T e^{\lambda s}\, ds\right) \\ &\leq \delta^2 S_b(x,x,y)\, T \left[\frac{e^{\lambda T} - 1}{\lambda}\right].\end{aligned}$$

Multiply both sides by $e^{-\lambda t}$ and evaluate the supremum over $t \in I$,

$$\begin{aligned}\sup_{t \in I} e^{-\lambda t} |Ax(t) - By(t)|^2 &\leq \delta^2 S_b(x,x,y) \cdot T \cdot \left[\frac{e^{\lambda T} - 1}{\lambda}\right] \sup_{t \in I} e^{-\lambda t} \\ \Rightarrow S_b(Ax, Ax, By) &\leq \left(\frac{\delta^2 T e^{\lambda T}}{\lambda}\right) S_b(x,x,y) \quad \text{as } e^{-\lambda t} \leq 1 \text{ for } t \in I.\end{aligned}$$

Now,

$$M(x,y) = \max \begin{cases} S_b(x,x,y), \\ S_b(Ax,Ax,x), \\ S_b(By,By,y), \\ \dfrac{S_b(Ax,Ax,y) + S_b(By,By,x)}{2} \end{cases} \geq S_b(x,x,y).$$

Then using the assumption (ii) we have

$$\begin{aligned} S_b(Ax, Ax, By) &\leq k S_b(x,x,y) \\ \Rightarrow S_b(Ax, Ax, By) &\leq k M(x,y), \end{aligned}$$

since F is strictly increasing function, so

$$F(S_b(Ax, Ax, By)) \leq F(k \cdot M(x,y)).$$

Taking $F(x) = \ln x$,

$$\begin{aligned} \ln(S_b(Ax, Ax, By)) &\leq \ln k + \ln(M(x,y)) \\ \Rightarrow -\ln k + \ln(S_b(Ax, Ax, By)) &\leq \ln(M(x,y)). \end{aligned}$$

Let $\tau = -\ln k > 0$ as $0 < k < 1$. So,

$$\tau + F(S_b(Ax, Ax, By)) \leq F(M(x,y)).$$

So, by Theorem 3.1, A, B, S, T possess a unique common fixed point $z \in X \cap Y$.

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VI. CONCLUSION

In this paper, we have introduced the structure of bipolar S_b-metric space endowed with a binary relation $\mathcal{R}$ and established two common fixed- point theorems for four self-mappings A, B, S, T satisfying $F_{\mathcal{R}}$-contractive and $F_{\mathcal{R}}$-expansive conditions via $CLR_{S,T}$ property.

The main advantage of our approach is threefold. First, the bipolar S_b-metric with parameter $b \geq 1$ simultaneously generalizes bipolar metric [10], b-metric [4], S-metric [6] and S_b-metric [13]. Second, the restriction of contraction only on $\mathcal{R}$-comparable pairs is much weaker than the global contraction, following the idea of Alam and Imdad [8] and Malhotra and Kumar [1]. Third, the $CLR_{S,T}$ property allows us to obtain common fixed points without requiring completeness of the whole space or continuity of the mappings, which improves many earlier results [1,11,15,2,18].

We have validated our results through non-trivial examples with $S_b(x, x, y) = |x - y|^2$ and shown that the contractive factor $\tau = \ln 2$ and expansive factor $\tau = \ln 1.5$ satisfy the required inequalities. Moreover, the application to a system of nonlinear integral equations demonstrates that our abstract theorems can be used to ensure the existence of a common solution in $C[0, T]$, where the Cauchy-Schwarz inequality plays a crucial role to verify $F_{\mathcal{R}}$-contraction.

Our theorems generalize and extend the results of Mutlu and Gürdal [10], Wardowski [11], Malhotra and Kumar [1], Alamri [15], Mani et al. [2] and several others [20,22,5].

For future research, one may extend these results to bipolar fuzzy S_b-metric spaces, bipolar C^*-algebra valued S_b-metric spaces, or to multi-valued mappings with graph structure. The study of $F_{\mathcal{R}}$-contractions under w-compatibility with simulation functions in this new setting would also be an interesting direction.

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