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# Design of ARIMA-GARCH Control Chart for Volatile and Autocorrelated Processes

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**Abstract:** *This study proposes an ARIMA-GARCH control chart so as to improve shift detection in process mean when the assumption of independence of observation is violated. In this study, different hypothetical processes following based on different level of violation of the said assumption were simulated. The ARIMA-GARCH control chart was alongside Shewhart, CUSUM, EWMA and EWMA-CUSUM charts compared in terms of* $ARL_1$, $SDRL_1$ *and* $AEQL$. *The comparison results revealed that the proposed chart outperformed the traditional charts under both small and high level of autocorrelation and volatility clustering with special performance under high level of autocorrelation and volatility clustering, in terms of* $ARL_1$ *and* $AEQL$. *The implication of these findings is that the ARIMA-GARCH chart can be used effectively under volatile and autocorrelated processes.*

## I. INTRODUCTION

According to Bentes et al., (2008), one of the statistical process control (SPC) tools is the control chart. It was first created for industrial process monitoring (Shewhart, 1924). A number of these charts, each intended for a particular purpose, may be found in literature. For example, Shewhart control chart of 1924 is meant for monitoring and detection of large shift in mean of statistical process (Waqas et al., 2024). Sometimes, industrial and financial data are characterized by small changes whose monitoring requires methods different from Shewhart chart. As a result, Roberts, 1959 and Page, 1954 proposed EWMA and CUSUM charts, respectively (Riaz et al., 2024). As a set, Shewhart, EWMA and CUSUM charts are referred to as standard control charts and they are considered the basis of other hybrid/mixed charts in available literature. Like many other statistical methods, traditional control charts require some assumptions (Montgomery, 2019). The assumptions required are that the process being monitored comes from a population of independent data, follows normal distribution and that it is independent and identically distributed (iid) (Mingot et al., 2008, da Silva et al., 2007, Claro et al., 2007 and Ramos & Ho, 2003). However, many real-world processes exhibit complex behaviors, including autocorrelations, trends and volatility 2 clustering, which are not fully captured by these traditional control charts. Thus, in a situation where a process exhibits volatility clustering and autocorrelations, such as financial markets, industrial operations, the limitations of traditional control charts become apparent (Souza et al., 2015). Over decades, improving process monitoring ability of the standard control charts has always been a topic of interest to many researchers. Classical control charts struggle to detect subtle shifts, trend, or handle noisy and non-stationary or complex temporal dependency in the data (Montgomery, 2019) even as the control charts assume that the quality characteristics being monitored are independent and stationary both in mean and variance. Russo et al., (2012) added that under such circumstances that the process behavior is more complex than the fundamental supposition, there is no reason for using traditional charts to monitor quality, because they will induce erroneous conclusions and facilitate a wrong conclusion that the process is under statistical control with flaws in the identification of systematic variation of the process. To mitigate the flaws, Kovářík, (2014) emphasized the need to incorporate autoregressive integrated moving average (ARIMA) and Generalized autoregressive conditional heteroskedasticity (GARCH) models into statistical processes process control particularly when monitoring processes with heteroskedasticity and autocorrelation. According to Kovářík, (2014), ARIMA, GARCH and ARIMA-GARCH


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models handle the autocorrelation, heteroskedasticity and the combination of autocorrelation and heteroskedasticity, respectively. Nonetheless, studies by Loredo et al., (2002) and Žmuk, (2016) have it that the existing residual-based control charts, including ARIMA-based and ARIMA-GARCH-based control charts, were developed under the assumption that the residuals are independent identically distributed normal variable, with mean zero and homoscedastic variance. Wang (2016) used a residual-based EWMA control chart to improve air handling unit fault monitoring and detection. By removing the detrimental effects of dynamic characteristics, serial correlation, and typical transient system changes through time series modelling errors, the suggested fault detection method of the residual-based EWMA control chart demonstrated an improved fault detection accuracy. Harris & Ross, (1991), on the other hand, acknowledged that the EWMA and CUSUM control charts for the residuals from a first-order autoregressive AR(1) process were not be very good at detecting the process mean change. EWMA charts performed poorly when processes were positively autocorrelated (at the first lag), as demonstrated by Wardell et al., (1994). Zhang, (1998) also investigated the detection capability of residual charts for autocorrelated data, defining a measure of the residual $\bar{X}$-chart's detection capability for the general stationary process and demonstrating that the residual chart's detection capability for the AR(2) process was small in comparison to the $\bar{X}$-chart's detection capability. However, because it is a product of financial data (which is typically skewed and heavy-tailed), the residuals are rarely normal, and according to the standards for the validity of traditional control charts, it may be practically risky to apply Shewhart, EWMA, and CUSUM charts directly to such misbehaving processes. This work suggests an important method to overcome this difficulty viz the ARIMA-GARCH control chart, which dynamically monitors a process's mean and volatility clustering by combining GARCH and ARIMA. The control chart is more sensitive to changes in process behaviour when the GARCH model takes variable volatility into account, but the ARIMA model captures mean dynamics, including autocorrelations and trends. In particular, an ARIMA-GARCH control chart is proposed with the goal of effectively monitoring processes in the event of both autocorrelation and volatility clustering. More precisely, the proposed chart seeks to offer a more precise and flexible instrument for monitoring process mean when autocorrelation and volatility are present. The goal of this study is to design and evaluate this control chart's performance in order to assess its superiority over the conventional statistical process control charts in practical applications. The control limits of the proposed control chart will be based on the normal distribution, which is a commonly assumed distribution for process data under stable conditions.

## II. METHODOLOGY

### 2.1 Integration of $ARIMA$ and $GARCH$ models in Control Chart Design

The integration of ARIMA and GARCH models in the control chart design can be achieved by modeling conditional variance as a function of present and past shocks and the lagged conditional variances using the residuals from an ARIMA model to estimate conditional standard deviations for the process of interest. The procedure is complete by constructing an appropriate control chart on a standardized residual. The standardized residuals will be obtained by dividing the estimated residuals from the ARIMA model by the estimated conditional standard deviation from the GARCH model. Therefore, given the estimates of $\varepsilon_t$ from the mean model, $\sigma_t^2$ from the variance model and the standardized variate $Z_t = {}^{\varepsilon_t}/_{\sigma_t}$, where $Z_t$ is theoretically independent and identically distributed variable, the proposed control chart will be constructed on a Johnson distribution-based variate given by:

$$Z_t^* = \gamma + \delta f(.) \qquad (2.1)$$

### 2.2 Choice of Johnson Distribution

The standard control charts assume normality about the data being monitored. For large sample $(n > 30)$, it was established that the central limit theorem (CLT) allows many distributions to approximate normal distributions (Huberts *et al*., 2018). However, this may not always be assured in many practical situations, especially in stock exchange where data are skewed and heavy tailed. In this case, traditional control charts are less relevant and other statistics associated about the control charts are practically unreliable. To sum it up, when process exhibits non-normal



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distribution, it is erroneous to use traditional control charts for its monitoring. There arises the need to ensure normality using an established probability distribution such as Box-Cox, log-normal and Johnson's distributions. The ability to compute an optimal transformation function from its three flexible distribution families ($S_U, S_B$ and $S_L$) makes Johnson distribution most powerful of the three distributions. Therefore, with good estimates of its parameters, Johnson distribution can accurately transform skewed and kurtotic distributions to normal. $S_U$ denotes system of unbounded curve for the set of distributions whose support is the set of real numbers, $S_B$ denotes system of bounded curve whose support has a fixed boundary on either the upper or lower or both and $S_L$ denotes a system of log-normal curves for the set of distributions whose support is the border between bounded and unbounded systems. Worth noting is that $S_U$ covers the $t$ and normal distribution (D'Agostino, 1970), $S_B$ covers gamma, beta and many other distributions and $S_L$ covers log-normal distribution. Dealing with non-normality problem, this study will be using the most suitable curve to normalize both the simulated and the empirical data sets.

### 2.2.1 $S_L$ Johnson Distribution

The $S_L$ Johnson distribution will be used to transform the random variable $Z_t$ that has a log normal form to a new variable $Z_t^* \sim N(0,1)$. The transformed variable takes the form:

$$Z_t^* = \gamma + \delta log\left(\frac{Z_t - \xi}{\lambda}\right) \tag{2.2}$$

The probability density function of the variable $Z_t$ is:

$$f(Z_t) = \frac{\delta}{\lambda\sqrt{2\pi}} \cdot \frac{1}{(Z_t - \xi)} \cdot exp\left[-\frac{1}{2}\left(\gamma + \delta log\left(\frac{Z_t - \xi}{\lambda}\right)\right)^2\right] \tag{2.3}$$

### 2.2.2 $S_U$ Johnson Distribution

The $S_U$ Johnson distribution will be used to transform the random variable $Z_t$ that is unbounded. to a new variable $Z_t^* \sim N(0,1)$. The transformed variable takes the form:

$$Z_t^* = \gamma + \delta sinh^{-1}\left(\frac{Z_t - \xi}{\lambda}\right) \tag{2.3}$$

The probability density function of the variable $Z_t$ is:

$$f(Z_t) = \frac{\delta}{\lambda\sqrt{2\pi}} \cdot \frac{1}{\sqrt{1 + \left(\frac{Z_t - \xi}{\lambda}\right)^2}} \cdot exp\left[-\frac{1}{2}\left(\gamma + \delta sinh^{-1}\left(\frac{Z_t - \xi}{\lambda}\right)\right)^2\right] \tag{2.4}$$

### 2.2.3 $S_B$ Johnson Distribution

The $S_B$ Johnson distribution will be used to transform the random variable $Z_t$ that is bounded. to a new variable $Z^* \sim N(0,1)$. The transformed variable takes the form:

$$Z_t^* = \gamma + \delta log\left(\frac{\lambda - \xi}{\xi + \lambda - Z_t}\right) \tag{2.4}$$

The probability density function of the variable $Z_t$ is:

$$f(Z_t) = \frac{\delta}{\lambda\sqrt{2\pi}} \cdot \frac{1}{(Z_t - \xi)(\xi + \lambda - Z_t)} \cdot exp\left[-\frac{1}{2}\left(\gamma + \delta log\left(\frac{Z_t - \xi}{\xi + \lambda - Z_t}\right)\right)^2\right] \tag{2.5}$$

This is equivalently expressed as:

$$f(Z_t) = \frac{\delta}{\lambda\sqrt{2\pi}} \cdot \frac{1}{(Z_t - \xi)(\xi + \lambda - Z_t)} \cdot exp\left[-\frac{1}{2} Z_t^{*2}\right] \tag{2.6}$$

## 2.3 Johnson Systems Parameters Estimation

### 2.3.1 The $S_B$ system Parameter Estimation

The Johnson $S_B$ system transform $Z_t = \frac{\varepsilon_t}{\sigma_t}$ to $Z_t^* \sim N(0,1)$ using the expression:

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$$Z_t^* = \gamma + \delta log\left(\frac{\lambda-\xi}{\xi+\lambda-Z_t}\right) \qquad (2.7)$$

The log-likelihood function is calculated from the pdf of Johnson's distribution as:

$$L(\theta) = \sum_{i=1}^{n}\left[-\frac{1}{2}log(2\pi) - \frac{{Z_i^*}^2}{2} + log(\delta) - log(\lambda) - \frac{1}{2}log\left[1 + \left(\frac{Z_t-\xi}{\lambda}\right)^2\right]\right] \qquad (2.8)$$

The expression in equation (2.8) can equivalently expressed as:

$$L(\theta) = -\frac{n}{2}log(2\pi) + nlog(\delta) - nlog(\lambda) - \frac{1}{2}\sum_{i=1}^{n}\left[{Z_{ti}^*}^2 + log\left[1 + \left(\frac{Z_t-\xi}{\lambda}\right)^2\right]\right] \qquad (2.9)$$

where

$$\theta = (\gamma, \delta, \xi, \lambda) \text{ and } Z_{ti}^* = \gamma + \delta sinh^{-1}\left(\frac{Z_{ti}-\xi}{\lambda}\right) \qquad (2.10)$$

The MLE is obtained by choosing some arbitrary initial values for the parameters denoted and expressed as:

$$\theta^{(0)} = \left(\gamma^{(0)}, \delta^{(0)}, \xi^{(0)}, \lambda^{(0)}\right) \qquad (2.11)$$

Then, the data set $Z_{ti}$ is transformed to $Z_{ti}^*$ using the initialized parameters based on equation (2.11) and the likelihood function is calculated using equation (2.10). Therefore, maximizing equation (2.10) with respect to $\theta = (\gamma, \delta, \xi, \lambda)$ gives the MLE estimates $\hat{\theta} = \left(\hat{\gamma}, \hat{\delta}, \hat{\xi}, \hat{\lambda}\right)$ of $\theta = (\gamma, \delta, \xi, \lambda)$ expressed as:

$$\hat{\theta}_{MLE} = \arg\max_{\gamma,\delta,\xi,\lambda} L(\theta) \qquad (2.12)$$

### 2.3.2 The $S_U$ System Parameter Estimation

The Johnson $S_U$ system transform $Z_t = \frac{\varepsilon_t}{\sigma_t}$ to $Z_t^* \sim N(0,1)$ using the expression:

$$Z_t^* = \gamma + \delta sinh^{-1}\left(\frac{Z_t-\xi}{\lambda}\right) \qquad (2.13)$$

The log-likelihood function is calculated from the pdf of Johnson's distribution as:

$$L(\theta) = -\frac{n}{2}log(2\pi) + nlog(\delta) - nlog(\lambda) - \frac{1}{2}\sum_{i=1}^{n}\left[\left(\gamma + \delta sinh^{-1}\left(\frac{Z_t-\xi}{\lambda}\right)\right)^2 + log\left[1 + \left(\frac{Z_t-\xi}{\lambda}\right)^2\right]\right] \qquad (2.14)$$

The expression in equation (2.14) can be written as:

$$L(\theta) = -\frac{n}{2}log(2\pi) + nlog(\delta) - nlog(\lambda) - \frac{1}{2}\sum_{i=1}^{n}\left[{Z_{ti}^*}^2 + log\left[1 + \left(\frac{Z_t-\xi}{\lambda}\right)^2\right]\right] \qquad (2.15)$$

where $\theta = (\gamma, \delta, \xi, \lambda)$

The MLE is obtained by choosing some arbitrary initial values for the parameters denoted and expressed as:

$$\theta^{(0)} = \left(\gamma^{(0)}, \delta^{(0)}, \xi^{(0)}, \lambda^{(0)}\right) \qquad (2.16)$$

Then, the data set $Z_t$ is transformed to $Z^*$ using the initialized parameters based on equation (2.16) and the likelihood function is calculated using equation (2.15). Therefore, maximizing equation (2.15) with respect to $\theta = (\gamma, \delta, \xi, \lambda)$ gives the MLE estimates $\hat{\theta} = \left(\hat{\gamma}, \hat{\delta}, \hat{\xi}, \hat{\lambda}\right)$ of $\theta = (\gamma, \delta, \xi, \lambda)$ expressed as:

$$\hat{\theta}_{MLE} = \arg\max_{\gamma,\delta,\xi,\lambda} L(\theta) \qquad (2.17)$$

### 2.3.3 The $S_L$ System Parameter Estimation

The Johnson $S_L$ system transform $Z_t = \frac{\varepsilon_t}{\sigma_t}$ to $Z_t^* \sim N(0,1)$ using the expression:

$$Z_t^* = \gamma + \delta log\left(\frac{Z_t-\xi}{\lambda}\right) \qquad (2.18)$$

The log-likelihood function is calculated from the pdf of Johnson's distribution as:

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$$L(\theta) = -\frac{n}{2}log(2\pi) + nlog(\delta) - nlog(\lambda) - \frac{1}{2}\sum_{i=1}^{n}\left[\left(\gamma + \delta log\left(\frac{Z_t-\xi}{\lambda}\right)\right)^2 + \left[\left(\gamma + \delta log\left(\frac{Z_t-\xi}{\lambda}\right)\right)^2\right]\right] \quad (2.19)$$

The expression in equation (2.19) can be written as:

$$L(\theta) = -\frac{n}{2}log(2\pi) + nlog(\delta) - nlog(\lambda) - \frac{1}{2}\sum_{i=1}^{n}\left[{Z_{ti}^*}^2 + \left[\left(\gamma + \delta log\left(\frac{Z_t-\xi}{\lambda}\right)\right)^2\right]\right] \quad (2.20)$$

where $\theta = (\gamma, \delta, \xi, \lambda)$

The MLE is obtained by choosing some arbitrary initial values for the parameters denoted and expressed as:

$$\theta^{(0)} = \left(\gamma^{(0)}, \delta^{(0)}, \xi^{(0)}, \lambda^{(0)}\right) \quad (2.21)$$

Then the data set $Z_t$ is transformed to $Z_t^*$ using the initialized parameters based on equation (2.21) and the likelihood function is calculated using equation (2.20). Therefore, maximizing equation (2.20) with respect to $\theta = (\gamma, \delta, \xi, \lambda)$ gives the MLE estimates $\hat{\theta} = \left(\hat{\gamma}, \hat{\delta}, \hat{\xi}, \hat{\lambda}\right)$ of $\theta = (\gamma, \delta, \xi, \lambda)$ expressed as:

$$\hat{\theta}_{MLE} = \arg\max_{\gamma,\delta,\xi,\lambda} L(\theta) \quad (2.22)$$

## 2.4 Control Charts Design

### 2.4.1 Design Structure of ARMA-GARCH-EWMA Chart

The $ARMA - GARCH - EWMA$ chart, hence forth, AGE, integrates the properties of Exponential Weighted Moving Average (EWMA), $ARMA$ and $GARCH$ . The statistics for $AGE$ is given as:

$$AGE_t = \theta Z_t^* + (1-\theta)AGE_{t-1} \quad (2.23)$$

The exact control limits are derived and given as in equation (44). From Equation (33), we can obtain as follows:

$$AGE_1 = \theta Z_1^* + (1-\theta)AGE_0 \quad (2.24)$$

Substituting recursively, gives:

$$AGE_2 = \theta Z_2^* + \theta(1-\theta)Z_1^* + (1-\theta)^2 AGE_0 \quad (2.25)$$

$$AGE_3 = \theta Z_3^* + \theta(1-\theta)Z_2^* + \theta(1-\theta)^2 Z_1^* + (1-\theta)^3 AGE_0 \quad (2.26)$$

This recursive substitution gives equation (2.27) in the long run.

$$AGE_t = \theta\sum_{j=0}^{t-1}(1-\theta)^j Z_{t-j}^* + (1-\theta)^t AGE_0 \quad (2.27)$$

The mean of $AGE_t$ is given as:

$$E(AGE_t) = E\left(\theta\sum_{j=0}^{t-1}(1-\theta)^j Z_{t-j}^* + (1-\theta)^t AGE_0\right) \quad (2.28)$$

$$E(AGE_t) = \theta\sum_{j=0}^{t-1}(1-\theta)^j E\left(Z_{t-j}\right) + (1-\theta)^t E(AGE_0) \quad (2.29)$$

Let $E(AGE_t) = \mu_K$ and $E\left(Z_{t-j}^*\right) = \mu_{Z_t^*}$. Then equation (2.29) can be written as:

$$\mu_K = \theta\sum_{j=0}^{t-1}(1-\theta)^j \mu_{Z_t^*} + (1-\theta)^t \mu_K \quad (2.30)$$

Recall that,

$$\sum_{j=0}^{t-j} P^j = \frac{1-P^t}{1-P}\ for\ |P| < 1 \quad (2.31)$$

Now, for $P = 1 - \theta$ in equation (2.31), we have,

$$\sum_{j=0}^{t-j}(1-\theta\,)^j = \frac{1-(1-\theta\,)^t}{1-(1-\theta\,)} = \frac{1-(1-\theta\,)^t}{\theta} \quad (2.32)$$

Substituting equation (2.32) in equation (2.30), we have,

$$\mu_K = \theta\frac{1-(1-\theta\,)^t}{\theta}\mu_{Z_t^*} + (1-\theta)^t \mu_K \quad (2.33)$$

Therefore,

$$\mu_K = [1-(1-\theta\,)^t]\mu_{Z_t^*} + (1-\theta)^t \mu_K$$

$$\mu_K[1-(1-\theta)^t] = [1-(1-\theta\,)^t]\mu_{Z_t^*}$$

and this implies that the mean of the statistic $AGE_t$ is given by:

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$$\mu_K = \mu_{Z_t^*} \quad (2.34)$$

The variance of $AGE_t$ is given as:

$$Var(AGE_t) = Var\left[\theta \sum_{j=0}^{t-1}(1-\theta)^j Z_{t-j}^* + (1-\theta)^t AGE_0\right]$$
$$= \theta^2 \sum_{j=0}^{t-1}(1-\theta)^{2j} Var\left(Z_{t-j}^*\right) + (1-\theta)^t Var(AGE_0)$$
$$= \theta^2 \sum_{j=0}^{t-1}(1-\theta)^{2j}\sigma_{Z_t^*}^2 \quad (2.35)$$

Applying equation (2.32), we can express equation (2.35) as,

$$Var(AGE_t) = \theta^2\sigma_{Z_t^*}^2\left[\frac{1-(1-\theta)^{2t}}{1-(1-\theta)^2}\right] = \sigma_{Z_t^*}^2\frac{\theta}{2-\theta}\left[1-(1-\theta)^{2t}\right]$$

Thus, the standard deviation of the statistic $AGE_t$ is given as:

$$\sigma_{AGE_t} = \sigma_{Z_t^*}\sqrt{\frac{\theta}{2-\theta}\left[1-(1-\theta)^{2t}\right]} \quad (2.36)$$

Where, the estimate of $\mu_{Z_t^*}$ and $\sigma_{Z_t^*}$ respectively given by $\hat{\mu}_{Z_t^*}$and $\hat{\sigma}_{Z_t^*}$ will be used in practice. Hence, the exact control limits of $AGE$ based on the derived properties of the $AGE$ statistics in Equations (2.37) and (2.38) are given by:

$$UCL = \hat{\mu}_{Z_t^*} + L\hat{\sigma}_{Z_t^*}\sqrt{\frac{\theta}{2-\theta}\left[1-(1-\theta)^{2t}\right]}$$
$$LCL = \mu_{Z_t^*} - L\hat{\sigma}_{Z_t^*}\sqrt{\frac{\theta}{2-\theta}\left[1-(1-\lambda)^{2t}\right]} \quad (2.37)$$

where $\hat{\mu}_{Z_t^*}$is regarded as the target mean of the process, equally denoted by $\mu_0$.

For the steady state control limits, the expressions in Equation (2.37) reduce to:

$$UCL = \mu_0 + L\hat{\sigma}_{Z_t^*}\sqrt{\frac{\theta}{2-\theta}}$$
$$LCL = \mu_0 + L\hat{\sigma}_{Z_t^*}\sqrt{\frac{\theta}{2-\theta}} \quad (2.38)$$

The decision is that the process whose quality characteristic is $Z_t^*$ will be adjudged as being out of control if an EWMA statistic falls outside the control limits in equations (2.37) or (2.38).

### 2.4.2 Design Structure of ARMA-GARCH-EWMA-CUSUM Chart

The ARMA-GARCH-EWMA-CUSUM chart, hence forth, $AGEC$, integrates the properties of EWMA, CUSUM, ARMA model and GARCH model. The AGEC chart is designed by building a $CUSUM$ chart on AGE chart statistic. Like in the traditional $CUSUM$ chart, AGEC chart will be based on upper and lower AGEC statistics. These statistics are respectively given by:

$$AGEC_t^+ = Max\left[0, (AGE_t - \mu_0) - K_{AGE_t} + AGEC_{t-1}^+\right] \quad (2.39)$$
$$AGEC_t^- = Max\left[0, (\mu_0 - AGE_t) - K_{AGE_t} + AGEC_{t-1}^-\right] \quad (2.40)$$

where:

$$K_{AGE_t} = k_{AGE_t} \times \sigma_{AGE_t},$$
$$k_{AGE_t} = \frac{|\mu_1 - \mu_0|}{\sigma_{AGE_t}},$$

$\sigma_{AGE_t}$ is standard deviation of $AGE_t$,

$\mu_0$ and $\mu_1$ are the target mean and the out of control mean, respectively.

The decision about the state of the process $Z_t^*$ is based on the statistic given in equation (2.40):

$$H = h \quad (2.41)$$

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### 2.4.3 Design Structure of ARMA-GARCH-CUSUM Chart

The ARMA-GARCH-CUSUM will be designed by integrating CUSUM chart into ARMA-GARCH. The ARMA-GARCH-CUSUM one-sided upper and lower CUSUMs are given in equation (2.42):

$$C_t^+ = Max[0, Z_t^* - (\hat{\mu}_t + K) + C_{t-1}^+]$$
$$C_t^- = Max[0, (\hat{\mu}_t - K) - y_t + C_{t-1}^-] \quad (2.43)$$

where $K = k\hat{\sigma}_t$ is the reference value and $H_t = h\hat{\sigma}_t$. The starting values of the CUSUM statistics is are $C_0^+ = C_0^- = 0$. Montegomery, (2020) recommends that parameters $h$ and k be chosen as $h = 4 \ or\ 5$ and $k = \frac{1}{2}$, respectively, for a shift of about $1\sigma$.

### 2.4.4 Design Structure of ARIMA-GARCH-Shewhart-EWMA Chart

The ARIMA-GARCH-Shewhart-EWMA chart for individual observation, hence forth, $AGSE$, is built on the idea of Malela-Majika *et al.,* (2022) used to develop a single composite Shewhart-EWMA $\bar{X}$. This idea was based on a weighting parameter $\varpi$ $(0 \leq \varpi \leq 1)$ introduced to make the Shewhart-EWMA $\bar{X}$ more flexible. The parameter was used to form an additive weighted model. For variable $Z_t^*$, the additive model has a functional form defined as:

$$W_t = (1 - \varpi)Z_t^* + \varpi EWMA_{Z_t^*} \quad (2.44)$$

where $EWMA_{Z_t^*}$ is the EWMA statistic for the variable $Z_t^*$. It can be observed that setting the weighting parameter to $\varpi = 1$ reduces equation (2.44) to an EWMA statistic $EWMA_{Z_t^*}$ while setting it to $\varpi = 0$ reduces the equation to $Z_t^*$ which is the Shewhart statistic for individual observation. Therefore, the expression in equation (2.44) possesses the features of both Shewhart and EWMA charts when the weighting parameter $\varpi$ is within the range $0 < \varpi < 1$.

The $AGSE$ chart is then built based on the principles of traditional Shewhart. That is, Shewhart properties of $\mu_0$ and $\sigma_0$ have to be estimated on the additive model in equation (2.44) and substituted in the control limits for Shewhart chart.

The mean property of $W_t$.

$$E(W_t) = E\left[(1 - \varpi)Z_t^* + \varpi EWMA_{Z_t^*}\right] \quad (2.45)$$

Let $EWMA_{Z_t^*}$ be defined as $\gamma_t$, then (2.45) can be expressed as:

$$E(W_t) = E[(1 - \varpi)Z_t + \varpi\gamma_t]$$

where

$$\gamma_t = \theta Z_t^* + (1 - \theta)\gamma_{t-1}$$

then,

$$W_t = (1 - \varpi)Z_t^* + \varpi[\theta Z_t^* + (1 - \theta)\gamma_{t-1}]$$
$$= [1 - \varpi + \varpi\theta]Z_t^* + \varpi(1 - \theta)\gamma_{t-1} \quad (2.46)$$

Since the statistic $EWMA_{Z_t^*}$ is defined as

$$EWMA_{Z_t^*} = \gamma_t = \theta\sum_{j=0}^{t-1}(1 - \theta)^j Z_{t-j-1}^* + (1 - \theta)^t\, EWMA_{Z_0}$$

Then,

$$EWMA_{Z_{t-1}^*} = \gamma_{t-1} = \theta\sum_{j=0}^{t-2}(1 - \theta)^j Z_{t-j-1}^* + (1 - \theta)^{t-1}\, EWMA_{Z_0}$$
$$= \theta\sum_{j=0}^{t-2}(1 - \theta)^j Z_{t-j-1}^* + (1 - \theta)^{t-1}\gamma_0$$
$$= \theta\sum_{j=0}^{t-2}(1 - \theta)^j Z_{t-j-1}^* + (1 - \theta)^{t-1}\gamma_0 \quad (2.47)$$

Substituting (2.47) in (2.46) and taking the expectation yields

$$E[W_t] = E\left[[1 - \varpi + \varpi\theta]Z_t^* + \theta\varpi(1 - \theta)\left[\sum_{j=0}^{t-2}(1 - \theta)^j Z_{t-j-1}^* + (1 - \theta)^{t-1}\gamma_0\right]\right]$$
$$= [1 - \varpi + \varpi\theta]\mu_{Z_t^*} + \theta\varpi(1 - \theta)\left[\frac{1 - (1 - \theta)^{t-1}}{1 - (1 - \theta)}\mu_{Z_t^*} + (1 - \theta)^{t-1}\mu_{Z_t^*}\right]$$

$$= \left[[1-\varpi+\varpi\theta] + \varpi(1-\theta)(1-(1-\theta)^{t-1}) + \varpi(1-\theta)(1-\theta)^{t-1}\right]\mu_t$$

Thus,

$$E[W_t] = \mu_{Z_t^*} \quad (2.48)$$

The variance property of $W_t$.

$$Var(W_t) = Var\left[(1-\varpi)Z_t^* + \varpi EWMA_{Z_t^*}\right] \quad (2.49)$$

Letting $EWMA_{Z_t^*} = \gamma_t$ as before, then (2.49) can be expressed as:

$$Var(W_t) = Var[(1-\varpi)Z_t^* + \varpi\gamma_t]$$

$$= Var\left[[1-\varpi+\varpi\theta]Z_t^* + \theta\varpi(1-\theta)\left[\sum_{j=0}^{t-2}(1-\theta)^j Z_{t-j-1}^* + (1-\theta)^{t-1}\gamma_0\right]\right]$$

$$= (1-\varpi+\varpi\theta)^2\sigma_{Z_t^*}^2 + \theta^2\varpi^2(1-\theta)^2\left[\frac{1-(1-\theta)^{2(t-1)}}{1-(1-\theta)^2}\sigma_{Z_t^*}^2 + (1-\theta)^{t-1}\times 0\right]$$

$$= (1-\varpi+\varpi\theta)^2\sigma_{Z_t^*}^2 + \theta^2\varpi^2(1-\theta)^2\frac{1-(1-\theta)^{2(t-1)}}{1-(1-\theta)^2}\sigma_{Z_t^*}^2$$

$$= \left[(1-\varpi+\varpi\theta)^2 + \frac{\theta}{2-\theta}\varpi^2(1-\theta)^2(1-(1-\theta)^{2t-2})\right]\sigma_{Z_t^*}^2$$

Therefore,

$$Var(W_t) = \left[(1-\varpi+\varpi\theta)^2 + \frac{\theta}{2-\theta}\varpi^2(1-\theta)^2(1-(1-\theta)^{2t-2})\right]\sigma_{Z_t^*}^2 \quad (2.50)$$

Equation (2.50) can equivalently written as

$$Var(W_t) = \left[(1-\varpi+\varpi\theta)^2 - \theta\varpi^2(1-\theta)^2\left(\theta - \frac{1-(1-\theta)^{2t-2}}{2-\theta}\right)\right]\sigma_{Z_t^*}^2 \quad (2.51)$$

Hence, if $\varpi = 0$, the variance of the additive model reduces to the variance of a Shewhart chart defined by:

$$Var(W_t) = \sigma_{Z_t^*}^2 \quad (2.52)$$

and also, if $\varpi = 1$, the variance of the additive model reduces to the variance of the EWMA chart defined by:

$$Var(W_t) = \frac{\theta}{2-\theta}[1-(1-\theta)^{2t}]\sigma_{Z_t^*}^2 \quad (2.53)$$

So, utilizing equations (2.49) and (2.52), the control limits of the proposed AGSE chart is defined as:

$$UCL_t = \mu_0 + L\left[\left[(1-\varpi+\varpi\theta)^2 - \theta\varpi^2(1-\theta)^2\left(\theta - \frac{1-(1-\theta)^{2t-2}}{2-\theta}\right)\right]\sigma_{Z_t^*}^2\right] \quad (2.54)$$

$$LCL_t = \mu_0 - L\left[\left[(1-\varpi+\varpi\theta)^2 - \theta\varpi^2(1-\theta)^2\left(\theta - \frac{1-(1-\theta)^{2t-2}}{2-\theta}\right)\right]\sigma_{Z_t^*}^2\right] \quad (2.55)$$

### 2.4.5 Design Structure of ARIMA-GARCH-Shewhart Chart

The design of ARMA-GARCH-Shewhart for normal distribution, hence forth, (AGS), is predicated on the properties of the traditional Shewhart chart which is also a normal distribution-based. In a more explicit manner, traditional Shewhart chart assumes that the process $Z_t^*$ is normal, independent and identically distributed, that is, $Z_t^* \sim N(\mu, \sigma^2)$. Therefore, to build $AGS$ for individual observation on $Z_t^*$, then $Z_t^*$ also has to be independent and identically normal with mean $\mu_t$ and variance $\sigma_t^2$. Hence, the Johnson's-based distribution-based $AGS$ for individual observation is defined as:

$$UCL = \mu_0 + L\sigma_{Z_t^*} \quad (2.56)$$

$$CL = \mu_0 \quad (2.57)$$

$$LCL = \mu_0 - L\sigma_{Z_t^*} \quad (2.58)$$

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where $\mu_0$ and $Z_t^*$ are the target mean and the process standard deviation, respectively, and $L$ is the distance of the control limits from the center. The process is said to be in statistical control at period $t$ if $LCL < Z_t^* < UCL$, otherwise it is out of control.

### 2.5 Traditional Control Charts

Traditional control charts are designed to monitor process parameters when the underlying form of the process distributions are known. Specifically, traditional control charts are those charts that assume normality, independence and constant variance on the monitored process (Montgomery, 2020). Thus, traditional control charts maintain constant process parameters, analogous to non-traditional adaptive control charts. These charts include Shewhart, EWMA and CUSUM and were discussed in the subsequent unit.

#### 2.5.1 Traditional CUSUM chart

Page, (1954) developed a cumulative sum (CUSUM) chart that enhances the small or moderate mean shift ability of Shewhart chart. Of the several CUSUM charts available in the literature (Montgomery, 2013; Khoo & Teh, 2009), tabular CUSUM is mostly used in practice and concept. CUSUM charts, like Shewhart and EWMA, is valid when the process quality characteristic follows independent and identically distributed normal distribution assumption and is covariance stationary. Tabular CUSUM accumulates the deviation of each observation from target value $\mu_0$ that are above the reference value $K$ with the statistic $C_i^+$ and the deviation from target value $\mu_0$ that are below the reference value $K$ with the statistic $C_i^-$. $C_i^+$ and $C_i^-$ are called the one-sided upper and lower CUSUM and are defined initially as $C_i^+ = C_i^- = 0$. As per Hawkins & Olwell, (1998) and Woodall & Adams, (1993), the statistics $C_i^+$ and $C_i^-$ are respectively defined as:

$$C_i^+ = Max[0, y_i - (\mu_0 + K) + C_{i-1}^+] \tag{2.59}$$

$$C_i^- = Max[0, (\mu_0 - K) - y_i + C_{i-1}^-] \tag{2.60}$$

where $y_i$ is the data set (quality characteristic) assumed to follow normal distribution and $\mu_0$ is the target mean.

The tabular CUSUM is based two parameters, namely the reference value K and the decision interval H. K and H are defined as $K = k\sigma$ and $H = h\sigma$.

Montegomery, (2020) suggests that the reasonable value for the decision interval H is $H = 5\sigma$ and that of the reference value $K$ is $K = \delta/2 \times \sigma = |\mu_1 - \mu_0|/2$ for a process shift expressed in standard deviation unit as $\mu_1 = \mu_0 + \delta\sigma$ or $\delta = |\mu_1 - \mu_0|/2$, where $\mu_0$ is the target mean, $\mu_1$ is the out of control mean and $\delta$ is the process shift.

The decision about the state of the process is based on the decision interval $H$. Thus, a process is said to be out of control if either $C_i^+$ or $C_i^-$ exceeds the decision interval $H$.

#### 2.5.2 Traditional EWMA Chart

Roberts, (1959) complemented the work of Page by developing a new chart alternative to Shewhart namely exponential weighted moving average (EWMA) chart. The purpose was to handle some of the limitations of the Shewhart chart. Therefore, EWMA is as well a chart for monitoring small or moderate shift in mean of statistical process (Abbas et al., 2011). According to Montgomery, (2020) and Lee *et al.,* (2013), EWMA and CUSUM are very similar in performance based on ARL metric. EWMA is based on a statistic$Z_t^*$, called the EWMA statistic and defined as:

$$Z_i = \theta y_i + (1 - \theta) y_{i-1} \tag{2.61}$$

The initial value of $Z_i$ denoted by $Z_0$ is conventionally set to zero as there is no preceding $Z_i$ value to compute it, that is, $Z_0 = 0$.

where $Z_t^*$ is the data set (quality characteristic) assumed to follow normal distribution and $\theta$ is the smoothing parameter which is said to be in the interval $0 < \theta \leq 1$.

The control limits for non-steady state traditional EWMA is given as:

$$UCL_i = \mu_0 + L\sigma\sqrt{\frac{\theta}{2-\theta}[1 - (1 - \theta)^{2i}]} \tag{2.62}$$


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$$LCL_i = \mu_0 - L\sigma\sqrt{\frac{\theta}{2-\theta}[1-(1-\theta)^{2i}]} \quad (2.63)$$

While for a steady state is given as:

The control limits for non-steady state traditional EWMA is given as:

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\theta}{2-\theta}} \quad (2.64)$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\theta}{2-\theta}} \quad (2.65)$$

If the $LCL_i < Z_i < UCL_i$ and $LCL < Z_i < UCL$ the process is said to be out of control at period $i$ under non-steady state and steady state respectively. In Equations (2.61), (2.62), (2.64) and (2.65), $\mu_0$ is the target mean estimated from $y_i$ if unknown, $\sigma$ is the standard deviation of $y_i$ and $L$ is the width of the control limits. Note that $y_i$ is replaced by $\bar{y}_i$ in constructing all the traditional control charts because the study is dealing with subgroup data sets.

**2.5.3 Traditional Shewhart Chart**

A Shewhart chart is a graphical representation of a sequential statistical significance test for an out-of-control condition (Montgomery, 2020). It follows from this that the chart's control limits represent the confidence interval for the true process mean. In order for the chart to be applied effectively, the monitored process must be normal, normal, independent and has constant variance. The chart is designed to monitor process mean when there occurs a large shift in the process (Malindzakova *et al*., 2023). The lower control limit, center line and upper control limit for Shewhart control chart is given by:

$$LCL = \mu_0 - L\frac{\sigma}{\sqrt{n}} \quad (2.66)$$

$$LCL = \mu_0 \quad (2.67)$$

$$LCL = \mu_0 + L\frac{\sigma}{\sqrt{n}} \quad (2.68)$$

where $L$ is the multiplicative constant, $\sigma$ is the process standard deviation and $\mu_0$ is the target mean of the process, estimated as $\mu_0 = \bar{\bar{y}}$.

**2.6 Model Performance Evaluation**

The performance of the proposed ARIMA-GARCH control chart is evaluated in comparison to the current traditional and non-traditional control charts as part of the design process. The average run length (ARL), a control chart performance metric, serves as the foundation for this evaluation. For this aim, the study specifically uses in-control average run length ($ARL_0$) and out-of-control average run length ($ARL_1$). It is anticipated that the proposed control chart's $ARL_0$ will be greater than that of the conventional control charts.

Literature has shown that Monte Carlo simulation (MC), the Markov chain approach, explicit formulas, and integral equations are the widely used techniques for estimating average run length (ARL). For example, Sales et al., (2020) estimated ARL in Poisson mixed integer autoregressive processes using the MC approach. The Tukey mixed extended exponentially weighted moving average control chart was evaluated by Talordphop et al., (2025) using MC. In a similar vein, Malela-Majika et al., (2022) estimated ARL for their suggested single composite Shewhart-EWMA control chart for process mean monitoring using the MC approach. Aslam, (2016) and Aslam et al., (2018) employed the MC to evaluate the ARL on mixed HEWMA and CUSUM control chart for the Weibull distribution. Brook & Evans, (1972) used the Markov chain approach to investigate the run length properties of a CUSUM control chart under the assumption of independent and identically distributed (i.i.d.) observations. Hawkins, (1992) improved the Markov chain approach by using Richardson extrapolation for an entire family of distributions including the Chi-squared distribution. Champ & Rigdon, (1992) used integral equations to determine the ARL. Fredholm integral equations of the second kind have been employed as part of the NIE approach used to calculate the ARL (Wieringa, 1999). Acosta-Mejia et al., (1999) used an integral equation approach for the ARL numerically approximated using the Gauss-

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Legendre quadrature. Knoth, (2006) proposed utilizing a piecewise collocation method rather than the Gauss-Legendre quadrature to approximate the ARL.

Monte Carlo simulation will be used to estimate the performance metrics in this study because the technique has high accuracy with large samples, it is very flexible and can be applied to both complex models and unknown distributions. Thus, MC estimates ARL for all distributions, including non-linear, non-normal, and autocorrelated data. Areepong & Sukparungsee (2025) state that the MC approach begins by simulating an out-of-control process $(\mu = \mu_0 + \delta\sigma)$ $N$ times, where $N$ is the number of simulations, $\sigma$ is the process standard deviation and $\delta$ is the magnitude of the process shift. Secondly, establishing the chart's control limits. Third, run length (RL), which is the number of observations until the control chart triggers the first warning. Lastly, the expression in equation (2.69) provides the average RL for all simulations:

$$ARL_1 = \frac{1}{N}\sum_{i=1}^{n} RL_i \qquad (2.69)$$

The term "out-of-control average run length" refers to the value in equation (2.69). Equation (2.70) presents the in-control average run length ($ARL_0$), which is obtained by repeating the four phases of the MC approach with the modification of simulating an in-control process (($\mu = \mu_0$)) to the first step.

$$ARL_0 = \frac{1}{N}\sum_{i=1}^{N} RL_i \qquad (2.70)$$

Other measures used are the Standard Deviation of Run Length (SDRL) when the process is out-of-control and the Average Extra Quadratic Loss (AEQL).

The out-of-control $SDRL_1$ is expressed as:

$$SDRL_1 = \sqrt{\frac{\left(\sum RL_1\right)^2}{N} - ARL_1^2} \qquad (2.71)$$

This metric measures the charts' stability or consistency in observing $ARL_1$. Finally, $AEQL$ is according to Sunthornwat *et al*., (2024) expressed as:

$$AEQL = \frac{1}{k}\sum_{i=1}^{k} \delta^2 ARL\left(\delta_i\right) \qquad (2.72)$$

The metric gives a measure of expected loss caused by delayed detection of process shift. In all the formulae discussed in this subsection, $k$, $\delta$ and $ARL\left(\delta_i\right)$ are the number of shifts, shift magnitude and shift-specific $ARL_1$ respectively.

## III. RESULTS AND DISCUSSIIN

### 3.1 RESULTS

This section presents the performance of the proposed ARIMA-GARCH control charts in comparison with the traditional charts under different level of autocorrelation and volatility clustering. For each distribution, the ARL, SDRL and AEQL values are summarized in separate tables.

#### 3.1.1 Performance of the charts under low level of autocorrelation and volatility clustering

Table 3.1 represents the performance of the proposed ARIMA-GARCH and the traditional control charts under low level of autocorrelation and volatility clustering.


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Table 3.1. Comparative Performance of the Optimal Values of the ARIMA-GARCH and the Traditional Control Charts Under Low Level of Autocorrelation and Volatility Clustering.

| Control Charts | Metrics | Shifts ( $\delta$ ) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| ARIMA-GARCH | $ARL_1$ | 255.21 | 74.04 | 32.61 | 15.91 | 10.46 | 7.64 | 5.34 | 4.46 | 3.82 | 3.19 |
| | $SDRL_1$ | 233.04 | 74.38 | 29.83 | 12.38 | 7.45 | 4.84 | 3.07 | 2.20 | 1.57 | 1.11 |
| | $AEQL$ | 2.55 | 2.96 | 2.93 | 2.55 | 2.62 | 2.75 | 2.62 | 2.85 | 3.09 | 3.19 |
| Shewhart | $ARL_1$ | 83.10 | 27.90 | 15.80 | 13.30 | 10.20 | 8.00 | 6.80 | 5.50 | 5.30 | 5.00 |
| | $SDRL_1$ | 51.59 | 9.91 | 4.66 | 3.27 | 3.29 | 2.40 | 1.23 | 0.85 | 1.05 | 1.35 |
| | $AEQL$ | 0.83 | 1.12 | 1.42 | 2.13 | 2.55 | 2.88 | 3.33 | 3.52 | 4.29 | 5.00 |
| CUSUM | $ARL_1$ | 108.18 | 45.85 | 24.57 | 15.62 | 10.67 | 8.24 | 6.62 | 5.60 | 4.96 | 4.44 |
| | $SDRL_1$ | 100.75 | 40.05 | 19.05 | 10.82 | 6.50 | 4.34 | 2.94 | 2.24 | 1.70 | 1.40 |
| | $AEQL$ | 1.08 | 1.83 | 2.21 | 2.50 | 2.67 | 2.97 | 3.24 | 3.58 | 4.02 | 4.44 |
| EWMA | $ARL_1$ | 123.77 | 59.40 | 35.16 | 22.61 | 15.97 | 12.43 | 10.58 | 8.82 | 7.85 | 7.23 |
| | $SDRL_1$ | 118.03 | 51.89 | 29.11 | 16.50 | 10.35 | 6.81 | 5.33 | 3.84 | 3.00 | 2.52 |
| | $AEQL$ | 1.24 | 2.38 | 3.16 | 3.62 | 3.99 | 4.47 | 5.18 | 5.64 | 6.36 | 7.23 |
| EWMA-CUSUM | $ARL_1$ | 118.62 | 49.73 | 29.12 | 19.82 | 15.19 | 12.84 | 11.16 | 10.29 | 9.56 | 9.07 |
| | $SDRL_1$ | 104.70 | 37.83 | 18.28 | 9.97 | 5.99 | 4.27 | 2.94 | 2.29 | 1.70 | 1.31 |
| | $AEQL$ | 1.19 | 1.99 | 2.62 | 3.17 | 3.80 | 4.62 | 5.47 | 6.59 | 7.74 | 9.07 |

From Tables 3.1 above, it is observed that all charts under Normal Distribution exhibit improved shift detection as the shift magnitude increases, as reflected by decreasing $ARL_1$ values. In terms of $AEQL$, the proposed chart was equally seen to have outperformed the traditional charts for the same shifts as in terms of $ARL_1$. No clear superiority exhibited by the proposed chart over the traditional charts in terms of $SDRL_1$. The traditional Shewhart chart outperformed the proposed chart for small shifts, reflecting the efficiency of the Shewhart chart when its assumptions are satisfied. The proposed ARIMA-GARCH chart, however, maintained competitive detection performance for moderate shifts.

On comparing the proposed chart with the traditional Shewhart chart in a pair-wise manner under low level of autocorrelation and volatility clustering, it is observed that the proposed ARIMA-GARCH control chart is more efficient in detecting moderate shifts while the traditional Shewhart chart performs better for small shifts. The proposed ARIMA-GARCH control chart also outperforms the CUSUM chart for 60% of the shifts considered in the simulation. The only situation when the CUSUM chart beats the proposed chart under the normal distribution is for the first four smallest shifts. Comparing the proposed chart with the EWMA control chart, it was seen that the proposed chart dominates the traditional EWMA chart in terms of shift detection performance for all the shifts except for the smallest shift. In a similar performance, the proposed chart outperformed the EWMA-CUSUM chart for all the shifts except for

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the two smallest of them. This indicates that even under normality, the ARIMA-GARCH control chart can be chosen over EWMA-CUSUM control chart.

**3.1.2 Performance of the charts under Moderate Level of Autocorrelation and Volatility Clustering**

Table 3.2 presents the comparative performance of the proposed ARIMA-GARCH, traditional Shewhart, CUSUM, EWMA and EWMA-CUSUM control charts under moderate level of autocorrelation and volatility clustering.

Table 3.2. Comparative Performance of the Optimal Values of the Bootstrap-EWMA-MW and the Traditional Control Charts Under $t_4$ Distribution.

| Control Charts | Metrics | Shifts ( $\delta$ ) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| ARIMA-GARCH | $ARL_1$ | 191.44 | 62.21 | 26.50 | 15.49 | 9.92 | 7.38 | 6.06 | 5.25 | 4.39 | 4.11 |
| | $SDRL_1$ | 202.11 | 55.07 | 20.43 | 10.24 | 5.15 | 3.38 | 2.50 | 1.78 | 1.67 | 1.33 |
| | $AEQL$ | 1.91 | 2.49 | 2.39 | 2.48 | 2.48 | 2.66 | 2.97 | 3.36 | 3.56 | 4.11 |
| Shewhart | ARL | 166.04 | 74.37 | 34.61 | 18.56 | 13.33 | 10.11 | 8.14 | 6.97 | 5.80 | 5.31 |
| | SDRL | 158.85 | 69.39 | 29.20 | 11.71 | 7.85 | 5.19 | 4.37 | 3.05 | 2.28 | 2.00 |
| | AEQL | 1.66 | 2.97 | 3.11 | 2.97 | 3.33 | 3.64 | 3.99 | 4.46 | 4.70 | 5.31 |
| CUSUM | $ARL_1$ | 138.37 | 53.31 | 30.92 | 22.62 | 17.58 | 14.59 | 12.51 | 11.05 | 9.77 | 8.87 |
| | $SDRL_1$ | 99.94 | 26.47 | 11.44 | 7.22 | 5.00 | 3.62 | 2.84 | 2.33 | 1.88 | 1.62 |
| | $AEQL$ | 1.38 | 2.13 | 2.78 | 3.62 | 4.40 | 5.25 | 6.13 | 7.07 | 7.91 | 8.87 |
| EWMA | $ARL_1$ | 129.00 | 48.00 | 26.06 | 17.90 | 13.53 | 10.92 | 9.35 | 8.09 | 7.36 | 6.64 |
| | $SDRL_1$ | 116.56 | 34.86 | 15.76 | 8.86 | 5.92 | 4.28 | 3.20 | 2.51 | 2.14 | 1.78 |
| | $AEQL$ | 1.29 | 1.92 | 2.35 | 2.86 | 3.38 | 3.93 | 4.58 | 5.18 | 5.96 | 6.64 |
| EWMA-CUSUM | $ARL_1$ | 204.74 | 77.20 | 35.45 | 19.58 | 13.49 | 9.93 | 7.83 | 6.70 | 5.77 | 5.13 |
| | $SDRL_1$ | 203.18 | 72.99 | 28.79 | 14.46 | 9.14 | 5.60 | 4.12 | 2.92 | 2.28 | 1.80 |
| | $AEQL$ | 2.05 | 3.09 | 3.19 | 3.13 | 3.37 | 3.57 | 3.84 | 4.29 | 4.67 | 5.13 |

Table 3.2 showed that for the moderate level of autocorrelation and volatility clustering, the proposed ARIMA-GARCH control chart outperformed the traditional charts for most of the shifts as represented by small $ARL_1$ and $AEQL$ values despite random performance of $SDRL_1$ values.

In a pair-wise comparison, the proposed ARIMA-GARCH control chart was according to the Table 3.2 seen to be a better model than the traditional Shewhart chart for 90% of the shifts considered in the study. The proposed chart was inferior to the traditional Shewhart chart for only the first shift of $0.1\sigma$. Similarly, the proposed ARIMA-GARCH control chart was equally better than the traditional CUSUM control chart under the moderate level of autocorrelation and volatility clustering**.** The performance is in favour of the proposed chart for 80% of the ten shifts considered, with only $0.1\sigma$ and $0.2\sigma$ shift sizes being the cases for which the proposed chart was outperformed by the CUSUM chart. The EWMA control chart improves on the performance of CUSUM chart against the proposed ARIMA-GARCH control chart by beating the proposed chart in three occasions viz $0.1\sigma$, $0.2\sigma$ and $0.3\sigma$ which is 30% of the whole

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shift sizes. In a clear dominance performance, the proposed ARIMA-GARCH control chart outperformed the traditional EWMA-CUSUM control chart for all the shifts under the moderate level of autocorrelation and volatility clustering.

### 3.1.3 Performance of the charts under the high level of autocorrelation and volatility clustering.

Table 3.3 represents the comparative performance of the ARIMA-GARCH, traditional Shewhart, CUSUM, EWMA and EWMA-CUSUM control charts under high level of autocorrelation and volatility clustering.

Table 3.3. Comparative Performance of the Optimal Values of the ARIMA-GARCH and the Traditional Control Charts Under High Level of Autocorrelation and Volatility Clustering.

| Control Charts | Metrics | Shifts ( $\delta$ ) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| ARIMA-GARCH | $ARL_1$ | 113.91 | 35.26 | 19.69 | 12.52 | 9.26 | 7.59 | 6.26 | 5.29 | 4.85 | 4.14 |
| | $SDRL_1$ | 107.18 | 27.37 | 12.02 | 6.87 | 4.47 | 3.02 | 2.52 | 1.70 | 1.53 | 1.03 |
| | $AEQL$ | 1.14 | 1.41 | 1.77 | 2.00 | 2.32 | 2.73 | 3.07 | 3.39 | 3.93 | 4.14 |
| Shewhart | $ARL_1$ | 103.02 | 36.87 | 21.83 | 13.84 | 9.79 | 8.19 | 6.57 | 5.93 | 4.87 | 4.44 |
| | $SDRL_1$ | 86.56 | 27.11 | 13.66 | 6.86 | 4.10 | 3.30 | 2.65 | 1.77 | 1.47 | 1.32 |
| | $AEQL$ | 1.03 | 1.47 | 1.96 | 2.21 | 2.45 | 2.95 | 3.22 | 3.80 | 3.94 | 4.44 |
| CUSUM | $ARL_1$ | 332.97 | 97.65 | 39.00 | 21.00 | 13.83 | 10.00 | 8.01 | 6.55 | 5.60 | 5.01 |
| | $SDRL_1$ | 326.46 | 92.31 | 31.17 | 14.37 | 7.82 | 5.06 | 3.57 | 2.56 | 1.90 | 1.63 |
| | $AEQL$ | 3.33 | 3.91 | 3.51 | 3.36 | 3.46 | 3.60 | 3.92 | 4.19 | 4.54 | 5.01 |
| EWMA | $ARL_1$ | 123.01 | 54.53 | 31.93 | 21.55 | 16.35 | 13.27 | 11.42 | 10.04 | 9.03 | 8.45 |
| | $SDRL_1$ | 110.43 | 42.54 | 21.92 | 12.99 | 8.16 | 5.72 | 4.28 | 3.38 | 2.51 | 2.24 |
| | $AEQL$ | 1.23 | 2.18 | 2.87 | 3.45 | 4.09 | 4.78 | 5.60 | 6.43 | 7.31 | 8.45 |
| EWMA-CUSUM | $ARL_1$ | 131.56 | 54.77 | 30.31 | 20.55 | 15.84 | 12.89 | 11.18 | 10.09 | 9.24 | 8.70 |
| | $SDRL_1$ | 118.62 | 42.70 | 20.60 | 11.21 | 7.36 | 4.80 | 3.37 | 2.70 | 2.14 | 1.66 |
| | $AEQL$ | 1.32 | 2.19 | 2.73 | 3.29 | 3.96 | 4.64 | 5.48 | 6.46 | 7.48 | 8.70 |

From Table 3.3, it can be noted that the performance trends under high level of autocorrelation and volatility clustering were consistent in outperforming the traditional charts. The ARIMA-GARCH chart retained its overall consistency and economy advantage over the traditional charts for most of the shifts studied.

In one versus one comparison between the proposed ARIMA-GARCH control chart and the traditional Shewhart control chart under the under high level of autocorrelation and volatility clustering, it has been seen that the proposed chart has demonstrated superior performance over the traditional Shewhart chart for all the shift sizes except the smallest one. In a better performance and under the same autocorrelation and volatility levels, the proposed control chart has as well shown a complete dominance over the CUSUM control chart. The proposed chart outperformed the competing chart for all the shift sizes. Similarly, the results also revealed that the ARIMA-GARCH control chart dominates the EWMA control chart in terms of speed in detecting shifts ( $ARL_1$ ) of all sizes between $0.1\sigma$ to $1.0\sigma$ under the high level of autocorrelation and volatility clustering. On the performance of the proposed chart against the


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EWMA-CUSUM control chart under the same distribution, the results showed that the ARIMA-GARCH control chart maintains its superiority across all the shift sizes considered in the study.

### 3.2 DISCUSSION

The purpose of this study was to develop ARIMA-GARCH control chart and assess its ability to detect changes in process location across different level of autocorrelation and volatility clustering. Using Monte Carlo simulation, the study compared the proposed chart with the traditional charts in terms of out-of-control performance metrics. We interpret the findings of the study, explain the observed behavior of the proposed chart and connect the results to what is available in the literature domain.

The results of the Monte Carlo simulation show that under different settings considered, the proposed chart consistently performs better than the other charts. In particular, compared to the Shewhart chart and other parametric charts, the chart offers quicker detection of small and moderate shifts. The efficiency of integrating Johnson's distribution and time series with traditional control chart is demonstrated by the improvement in out-of-control detection, which is especially apparent under moderate and high level of autocorrelation and volatility clustering. These results imply that the proposed chart improves finite-sample performance, which is crucial in real-world Phase I and Phase II monitoring scenarios, while also maintaining the desired robustness of the proposed charts.

A number of well-known Shewhart-type control charts, such as the traditional Shewhart chart of Shewhart, (1924), the CUSUM chart of Page, (1954), the EWMA chart of Roberts, (1959), and the EWMA-CUSUM chart, were used to further contextualize the performance of the proposed chart. This comparison gives a clear picture of the relative efficiency of the developed chart by highlighting the advantages and disadvantages of each approach under the considered simulation settings. The ARIMA-GARCH control chart showed quicker shifts detection ability under very high level of autocorrelation and volatility clustering as compared to the traditional Shewhart chart and all the rest of the control charts.

In conclusion, by combining Johnson's distribution and time series with control chart idea, the proposed ARIMA-GARCH control chart successfully extends the traditional parametric charts techniques. It offers a versatile and useful structure for process monitoring under autocorrelation because of its increased robustness and enhanced sensitivity to shifts, especially under high autocorrelation and volatility clustering.

## IV. CONCLUSION

This study developed and evaluated an ARIMA-GARCH control chart for monitoring process location under various distributional conditions. The proposed chart showed stable in-control performance and faster detection of small and moderate shifts, particularly in skewed and heavy-tailed distributions. In comparison with traditional parametric charts, including the traditional Shewhart, CUSUM, EWMA, and EWMA-CUSUM charts, the proposed method revealed superior robustness and improved performance due to the integration of time series idea.

The results confirm that the ARIMA-GARCH chart provides a practical and flexible tool for quality monitoring in real-world industrial contexts, where process independence of observation assumption may not be met. The ability of the ARIMA-GARCH control chart to detect shifts efficiently while ensuring robustness under autocorrelated processes makes the proposed chart particularly suitable for industries such as food production, bakery, and other local manufacturing processes in Nigeria.

The proposed method, like any other, suffer from some drawbacks which include inability to address multivariate issues, non-adaptability of the proposed method, inability to jointly monitor both location and dispersion parameters as well as absence of non-parametric version of the method. In recognition and addressing the research gap these drawbacks might have created, future research works may extend the proposed methodology to joint monitoring of location and dispersion, multivariate processes, or adaptive procedures to further enhance efficiency. Overall, the study provides an important charting strategy for process monitoring and demonstrates the value of combining time series idea with control chart design.

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