

The Mathematical Evolution of Probability Theory

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Abstract: *Probability deals with issues that are uncertain or unpredictable. An explanation of the theory of permutations and combinations required to compute probabilities in games of chance, such as dice or cards, usually opens a first probability course. This article focuses on the evolution of the mathematical theory of probability from its inception in a correspondence between Pascal and Fermat in 1654 to its peak in Laplace's work in the early nineteenth century. An overview of early insurance mathematics' evolution is also discussed in this article.*

Keywords: *Probability*

I. INTRODUCTION

Gambling is one of the oldest pursuits of mankind. It was established in the laws of ancient China and Rome, and by Islam and Buddhism, and in the Jewish Talmud. The origins of gambling are thought to be divinatory: man tried to predict the future and the gods' intentions by tossing marked sticks and other items and analyzing the results. Additionally, anthropologists have noted that gambling is more common in communities where there is a general belief in gods and spirits whose goodwill may be sought. In various societies, including Sweden as late as 1803, the casting of lots and sometimes dice has been employed to administer justice and identify offenders during trials. Dike, the Greek word for justice, is derived from a verb that means to throw as in tossing dice.

Lotteries in the fifteenth century marked the beginning of organized gambling on a bigger scale that was approved by governments and other authorities in order to raise money. The area of probability theory emerged as a result of mathematicians' growing interest in games including randomizing elements, such as dice and cards, following the establishment of legal gambling establishments in the 17th century. In addition to its predecessors in ancient Greece and Rome, organized, legalized sports betting began in the late 1700s. Around that time, the official perspective on gambling started to gradually, if erratically, change from viewing it as a sin to viewing it as a vice and a human weakness to finally viewing it as a mostly harmless and even enjoyable activity. In the 1980s and 1990s, medical authorities in a number of nations identified pathological gambling as a cognitive illness affecting slightly more than 1% of the population. To address the issue, a number of treatment and rehabilitation programs were created. In Western countries, about four out of five persons gambled at least once by the start of the twenty-first century.

The Historical Foundations of Probability Theory

Probability originated in the covert realm of Arab cryptography rather than in a lab. In order to shield private messages from prying eyes, mathematicians worked to obfuscate and encrypt them between the eighth and thirteenth centuries. Al-Kindi was the first person known to apply statistical reasoning and considered to be a prominent character of this time. His finding that using probability, a seemingly distorted data can be made meaningful, was innovative. He put forth the idea that pattern identification using mathematical theory in the encrypted text can be used to segregate noises from signals.

The focus was on the games of chance in early Arab foundations during 16th Century and 17th Century. Combinatorics was established around this time. The solution to the logic of the die was found to be possible through probability. This emphasised the counting of finite possibilities and whole number outcomes. This approach grew into an advanced



mathematical framework that could address the problem of infinite during the 20th century. This significant advancement was led by the Austrian Richard von Mises and the Soviet mathematician Andrey Kolmogorov. Through their efforts, the discipline moved from discrete counts known as integers to continuous real values.

Before 1750, games of chance were the primary source of inspiration for probability theory. Both governmental and private lotteries, card games, and dice were significant social and commercial activities back then and now. Three periods will be distinguished; conversation on the problem of points by Pascal with Fermat, treatise on Reckoning at Games of Chance by Huygens and dissertation on the Arithmetical Triangle and its applications by Pascal mark the beginning of the period between 1654 and 1665 that laid the groundwork for probability theory. Pascal derives a rule of division based on the symmetric binomial tail probability and uses recursion to solve the points issue in his book. He and Fermat used combinatorial techniques to tackle the same problem in their correspondence. Huygens uses recursion to solve the problem numerically. Additionally, two linear equations connecting the conditional expectancies of the two players were also used by him to solve a case of games with a potentially endless number. Without disclosing their approach, all three of them were able to resolve the Gambler's Ruin dilemma. For more references one can read [1], [2], [4] and [5].

Combinations and permutations required to compute probabilities in games of chance like cards or dice. This idea of permutations and combinations originated in India, just as many other areas of mathematics. In actuality, the Indian conception of metre and music was fundamentally based on the science of permutations and combinations. Combinations of two syllables known as guru (deep, long) and laghu (short) were used by Vedic and post-Vedic musicians. Despite the fact that the Vedic metre incorporates this principle, the first known written explanation of permutations and combinations actually dates back to before Piṅgala and may be found in the Jain Bhagwatī Sūtra from the 4th century. Permutations were called vikalpa-ganitha and combinations bhanga [3].

Throughout the first half of the eighteenth century, gambling, whether on games or lives, remained the principal source of new concepts for probability theory, despite the philosophical and mathematical interest in Bernoulli's ideas. Laplace is famous for the principle of indifference, sometimes known as the principle of insufficient reason, which expands on the concept of equiprobability and states that all conceivable outcomes should be seen as equally likely in the absence of any information.

Probability theory offers the mathematical framework for our interactions with data from the real world. We use several methods to draw conclusions from data, whether we are training a large language model or forecasting the weather. Some of them include frequentist statistics, which focus on the frequency of events over time and are the primary framework for data inference employed in the majority of contemporary scientific research. As fresh data or evidence becomes available, we can update the likelihood of a hypothesis using Bayesian statistics. The cutting edge of artificial intelligence, machine learning, uses continuous probability to enable algorithms to learn from data and generate predictions on their own.

Development and evolution of Theory of Probability

Pascal and Fermat codified the rule that defines the probability as the ratio of the number of favourable events to the total number of equally probable cases. Later on, this idea came to be known as probability's classical definition. The Dutch mathematician Huygens (1657) published the first treatise on probability, which contributed to the spread of these new concepts. However, by establishing the initial framework for a mathematical theory of the probable, Pascal and Fermat are recognized for having started the mathematization of chance. They paved the way for a new understanding of reason, i.e. the incorporation of computation into uncertain decision-making.

This frequentist approach progressively gained traction in the nineteenth century. Mathematicians and philosophers like Borel (1909) and Venn (1866) emphasised that probability becomes relevant only as the long run limit of frequency. When the number of trials approaches infinity, the value to which the frequency of occurrence of an event tends toward, was characterised as probability subsequently. According to this perspective, time or at the very least, the number of repetitions is crucial; probability emerges when n increases.

Following a nearly 50-year period of stagnation, the basic and incomplete conclusions of these eminent scholars were



established into a cohesive probability theory during a decade of remarkable activity and advancement from 1708 to 1718. Basic probability rules, combinatorics, algorithms and recursion formulae, conditional probabilities and expectations, the inclusion and exclusion method and instances of limiting processes and infinite series were all covered. The hypergeometric distribution, multivariate version of the binomial and negative binomial distributions, the law of large numbers for the binomial, the Poisson approximation to the binomial and an approximation to the binomial's tail were all derived.

Application of probability theory to life insurance issues provided compelling evidence of its efficacy. States and localities raised money for public purposes throughout the 16th and 17th centuries by selling life annuities to their residents. Regardless of the nominee's age, the annual payout of annuity was set as a proportion of the invested capital, frequently two times the current rate of interest. In a 1671 report, De Witt demonstrated how to determine an annuity's value using a linear life table together with the nominee age and rate of interest. Despite including some studies on annuitant mortality, this life table was speculative. In 1693, Halley computed the table of annuity values as a function of the nominee age, created a life table based on the number of deaths in Breslau in an year and described how to compute joint-life annuities.

Probability became a technique for inductive inference in the eighteenth century, following a significant conceptual expansion following the combinatorial underpinnings of the seventeenth century. This intellectual revolution is exemplified by two key people. First, Bayes (1763) proposed the "problem of inverse probability," which is the process of determining the likelihood of a cause based on observed consequences. Second, Bayes's result was independently rediscovered by Laplace (1774), who used it as the basis for a general theory of scientific induction. Laplace created a cohesive theory of probability that combined a general inductive approach with Pascal's combinatorial calculus. He used probabilistic reasoning in a variety of domains, including demographics, insurance, astronomy with measurement errors, orbit determination, and even justice with testimony reliability assessment. Laplace popularized the modern version of Bayes's well-known formula and illustrated this in terms of likely causes-effects. The remarkable development with the contributions of Kolmogorov in 1933 was that probability could be treated as a complete mathematical field removing the dependency of philosophical interpretation. With this, probability theory gained more objectivity and a mathematical base.

II. CONCLUSION

Historically, probability theory has undergone multiple reformulations, each reflecting a shift in how reason addresses uncertainty. With the introduction of the Bayesian inversion in 1763 by Bayes and its generalization by Laplace 1812 and 1814, probability gained an obviously inductive and temporal component, enabling the interpretation of causes from effects and the updating of beliefs in response to new information. In 1837, Poisson brought in practical reality to probability which led to the rise of a true dynamics of events. This was the beginning of contemporary statistical thought. The 19th and early 20th centuries further solidified this path. Axiomatization by Kolmogorov led to the formal closure of probability as a theoretical measure, which gave it complete mathematical rigor. The emergence of probability represents a significant shift in thought.

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