

# **An Optimal Solution for Fuzzy Balanced Transportation Environment**

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**Abstract:** *In this paper, we are computed balanced transportation problem when Transportation problem contains fuzzy numbers, Intuitionistic fuzzy numbers and real numbers. The purpose of this paper is to find the optimal transportation cost in which cost, supply and demand are fuzzy numbers. This procedure is illustrated with numerical example.*

**Keywords:** Real numbers, hexagonal fuzzy numbers, trapezoidal fuzzy numbers, Intuitionistic fuzzy numbers, Pentagonal fuzzy numbers, ranking functions, optimal solution

## **I. INTRODUCTION**

In today's most competitive market, we need to find better way in transportation problem to minimize the transportation cost. Optimization methods and algorithms have lately become very important tools to optimize the problem. Due to incertitude in real life we have introduced fuzzy numbers in transportation problem. There may be such a situation in which we have to apply several of fuzzy numbers in transportation problem. The idea of fuzzy sets was introduced by Zadeh [8]. Klir [3] apply his logic in fuzzy sets and fuzzy logic. Ahmed et al. [5] develop a new algorithm for finding an initial basic feasible solution of transportation problem when transportation matrix contains fuzzy numbers and real numbers. Prabha et al. [7] worked on mixed Intuitionistic fuzzy transportation problem and solved by best candidate method. The transportation problem was first initiated by Hitcock [5] and it was developed with the systematic solution procedures from simplex algorithm, primarily by G.B. Dantzig [4] and then by Charnes and Cooper [3]. IBFS of Transportation problem is the first stage for an optimal solution. The well-known classical methods for finding an IBFS for the transportation problems are North West Corner Method (NWCN), Least Cost Method (LCM) and Vogel's Approximation Method (VAM). The researcher keeps going towards the development of the new algorithm of transportation problem to find excellent IBFS for transportation problem. In this work we have added a new algorithm [1] that provides a better IBFS, for both the balanced and unbalanced transportation problem. Fuzzy set theory used to many areas such as management sciences, mathematical modelling, control theory and industrial applications. In real world situations such as industry or corporate of distribution problems are indefinite in nature due to variations in the parameters. Zadeh [7] introduced the fuzzy set theory to deal with uncertainty due to imprecision and indefiniteness. Fuzzy linear programming was explained by Zimmermann [8]. Charnes et al [2] expressed as crisp values or defuzzified values into fuzzy linear programming problems.

In section 2 basic definitions are briefly explained. In section 3 we derived the algorithm. In section 4 we have computed some numerical example using different ranking techniques. In section 5 conclusions is discussed.

## **II. PRELIMINARIES**

### **2.1 Fuzzy set**

A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse  $X$  to the unit interval  $[0,1]$  i.e.,  $\tilde{A} = \{ X, \mu_{\tilde{A}}(X) : X \in X \}$ . Here  $\mu_{\tilde{A}}(X) : X \rightarrow [0, 1]$  is a mapping called the degree of



membership function of the fuzzy set  $\hat{A}$  and  $\mu_{\hat{A}}(X)$  is called the membership value of  $x \in X$  in the fuzzy set  $\hat{A}$ . These membership grades are often represented by real numbers ranging from  $[0, 1]$ .

### 2.2 Fuzzy number

A fuzzy number is a generalized of a regular real number and which does not refer to a single value but rather to a connected set of possible values, where each possible value has its weight between 0 and 1. This weight is called the membership function.

A fuzzy number  $\hat{A}$  is a convex normalized fuzzy set on the real line  $R$  such that:

There exist at least one  $x \in R$  with  $\mu_{\hat{A}}(X) = 1$

$\mu_{\hat{A}}(X)$  is piecewise continuous.

### 2.3 Triangular Fuzzy number

A triangular fuzzy number  $\hat{A}$  is denoted by 3- tuples  $(a_1, a_2, a_3)$ , where  $a_1, a_2$  and  $a_3$  are real numbers and  $a_1 \leq a_2 \leq a_3$  with membership function defined as

$$\mu_{\hat{A}}(X) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & , a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} & , a_2 \leq x \leq a_3 \\ 0 & , \text{otherwise} \end{cases}$$

### 2.4 Trapezoidal Fuzzy number

A trapezoidal fuzzy number is denoted by 4- tuples  $(a_1, a_2, a_3, a_4)$ , where  $a_1, a_2, a_3$  and  $a_4$  are real numbers and  $a_1 \leq a_2 \leq a_3 \leq a_4$  with membership function defined as

$$\mu_{\hat{A}}(X) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & , a_1 \leq x \leq a_2 \\ 1 & , a_2 \leq x \leq a_3 \\ \frac{a_4 - x}{a_4 - a_3} & , a_3 \leq x \leq a_4 \\ 0 & , \text{otherwise} \end{cases}$$

### 2.5: (Pentagonal Fuzzy Numbers):

A fuzzy number  $\hat{A} = (a_1, a_2, a_3, a_4, a_5)$  is called a pentagonal fuzzy number when the membership function has the form where the middle point  $a_3$  has the grade of membership 1 and  $w_1, w_2$  are the respective grades of points  $a_2, a_4$ . Note that every PFN is associated with two weights  $w_1$  and  $w_2$ .



$$\mu_A(X) = \left\{ \begin{array}{ll} w_1 \left( \frac{x-a_1}{a_2-a_1} \right) & , a_1 \leq x \leq a_2 \\ 1 - (1 - w_1) \left( \frac{x-a_2}{a_3-a_2} \right) & , a_2 \leq x \leq a_3 \\ 1 & , x = a_3 \\ 1 - (1 - w_2) \left( \frac{x-a_3}{a_4-a_3} \right) & , a_3 \leq x \leq a_4 \\ w_2 \left( \frac{x-a_4}{a_5-a_4} \right) & , a_4 \leq x \leq a_5 \\ 0 & , x > a_5 \end{array} \right.$$

### 2.6: (Hexagonal Fuzzy number):

A fuzzy number is denoted by 6- tuples =  $(a_1, a_2, a_3, a_4, a_5, a_6)$  , where  $a_1, a_2, a_3, a_4, a_5$  and  $a_6$  are real numbers and  $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6$  with membership function defined as

$$\mu_A(X) = \left\{ \begin{array}{ll} \frac{1}{2} \left( \frac{x-a_1}{a_2-a_1} \right) & , a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2} \left( \frac{x-a_2}{a_3-a_2} \right) & , a_2 \leq x \leq a_3 \\ 1 & , a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2} \left( \frac{x-a_4}{a_5-a_4} \right) & , a_4 \leq x \leq a_5 \\ \frac{1}{2} \left( \frac{x-a_5}{a_6-a_5} \right) & , a_5 \leq x \leq a_6 \\ 0 & , \text{otherwise} \end{array} \right.$$

### 2.7 Intuitionistic Fuzzy Set

Let X be the universal set. In the following, I will describe those aspects of intuitionistic fuzzy sets which will be needed in our next discussion. An intuitionistic fuzzy set (IFS)  $\mathcal{A}$  in X is given by  $\mathcal{A} = \{(x, \mu_{\mathcal{A}}(x), \vartheta_{\mathcal{A}}(x)) / x \in X\}$  , where the functions  $\mu_{\mathcal{A}}(x), \vartheta_{\mathcal{A}}(x)$  define the degree of membership and degree of non-membership of the element  $x \in X$  to the set  $\mathcal{A}$  respectively , which is a subset of X , and for every  $x \in X, 0 \leq \mu_{\mathcal{A}}(x) + \vartheta_{\mathcal{A}}(x) \leq 1$ .



### 2.8 Intuitionistic fuzzy numbers

An IFS  $A$  in  $X$  is given by  $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ , where the functions  $\mu_A, \nu_A : X \rightarrow [0, 1]$  define respectively, the degree of membership and degree of non-membership of the element  $x \in X$  to the set  $A$ , which is a subset of  $X$ , and for every  $x \in X$ ,  $0 \leq \mu(x) + \nu(x) \leq 1$ . Obviously, every fuzzy set has the form  $\{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ . For each IFS  $A$  in  $X$ , we will call  $\Pi_A(x) = 1 - \mu(x) - \nu(x)$  the intuitionistic fuzzy index of  $x$  in  $A$ . It is obvious that  $0 \leq \Pi_A(x) \leq 1, \forall x \in X$ .

### 2.9 Intuitionistic Triangular Fuzzy Number

An Intuitionistic triangular Fuzzy Number of a Intuitionistic Fuzzy set is  $\mathcal{A}$  is defined as  $\mathcal{A} = \{(a_1, a_2, a_3), (b_1, b_2, b_3)\}$ , where  $\mathcal{A} \in \mathbb{R}$  and its membership and non-membership is given by

$$\mu_{\mathcal{A}}(X) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \text{ and}$$

$$\vartheta_{\mathcal{A}}(X) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

### 2.10 Intuitionistic Trapezoidal Fuzzy Number

$\mathcal{A}$  is a trapezoidal intuitionistic fuzzy number with parameter  $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$  and denoted by  $\mathcal{A} = \{(b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)\}$

$$\mu_{\mathcal{A}}(X) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\ 0, & \text{otherwise} \end{cases}$$

$$\vartheta_{\mathcal{A}}(X) = \begin{cases} \frac{x - b_1}{b_2 - b_1}, & b_1 \leq x \leq b_2 \\ 1, & b_2 \leq x \leq b_3 \\ \frac{b_4 - x}{b_4 - b_3}, & b_3 \leq x \leq b_4 \\ 0, & \text{otherwise} \end{cases}$$



### 2.11 Ranking of hexagonal fuzzy numbers

A number of approaches have been proposed for the ranking of fuzzy numbers. In this paper for a hexagonal fuzzy number,  $A(H) = (a_1, a_2, a_3, a_4, a_5, a_6)$  then the ranking will be the magnitude of a HFN,  $A(H) = (a_1, a_2, a_3, a_4, a_5, a_6)$  is defined as

$$\text{Mag} ( A(H) ) = \frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6}{18}$$

#### Algorithm:

**Step 1:** Construct the mixed type fuzzy transportation table for a given fuzzy transportation problem.

**Step 2:** Using ranking function convert all fuzzy numbers into the crisp numbers.

**Step 3:** Check whether the transportation table is balanced or not, if not, make it balanced.

**Step 4:** After defuzzify the quantities of the problem, if any of values are not integers, round off into integers.

**Step 5:** Select the minimum odd cost from all cost in the matrix. Suppose all costs are even, multiply each column by 1/2.

**Step 6:** Subtract selected least odd cost only from odd cost in the matrix. Now there will be at least one zero and remaining all cost become even.

**Step 7:** Allocate minimum of supply/ demand at the place of zero.

**Step 8:** After the allotment, multiply each column by 1/2.

**Step 9:** Again select minimum odd cost in the remaining column except zeros in that column.

**Step 10:** Go to step 6 and repeat step 7 and 8 till optimal solution are obtained.

**Step 11:** Finally total minimum cost is calculated as sum of the product of the cost and corresponding allocated value of supply /demand.

#### Numerical Example:

Consider a fuzzy transportation problem

	<b>D<sub>1</sub></b>	<b>D<sub>2</sub></b>	<b>D<sub>3</sub></b>	<b>Supply</b>
<b>S<sub>1</sub></b>	( 4 , 4 , 4.5 )	(3, 4, 5, 5, 6, 7)	50	(3,5,7,9,11)
<b>S<sub>2</sub></b>	(5,4,8,13,15)	(2, 3, 4.5, 6; 1.5, 3, 4.5, 7)	(9,11,12,14)	35
<b>S<sub>3</sub></b>	(180,190,200, 200,210,220)	[4,8,12,16]	(13,14,15; 12,14,16)	(2, 4, 7, 10, 11)
<b>Demand</b>	( 20,40,60 )	23	(10,12,13,14,15,17)	

It is a balanced mixed fuzzy transportation problem since total demand equal to total supply.

i.e)  $\sum a_i = \sum b_j = 76.5$ .

	<b>D<sub>1</sub></b>	<b>D<sub>2</sub></b>	<b>D<sub>3</sub></b>	<b>Supply</b>
<b>S<sub>1</sub></b>	4	20	50	21
<b>S<sub>2</sub></b>	27	2	12	35
<b>S<sub>3</sub></b>	200	10	14	21
<b>Demand</b>	40	23	14	



Since the minimum odd cost in the odd matrix is 27, subtract 27 from all the odd costs and allocate minimum of supply or demand to the cell where there is zero cost then delete the row or column.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Supply
S <sub>1</sub>	4	20	50	21
S <sub>2</sub>	<b>35</b> 0	2	12	35
S <sub>3</sub>	200	10	14	21
Demand	40	23	14	

Now all the cost is even, hence multiply all the cost by ½ and subtract the minimum of odd cost from all the odd cost.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Supply
S <sub>1</sub>	2	10	20	21
S <sub>3</sub>	100	<b>21</b> 0	2	21
Demand	5	25	14	

The optimum cost is given by,

$$\text{Cost } Z = (5 \times 4) + (2 \times 20) + (14 \times 50) + (35 \times 27) + (21 \times 10) = \text{Rs. } 1915.$$

### III. CONCLUSION

In this paper, cost, supply and demand of fuzzy transportation problem are considered as mixed fuzzy numbers, i.e hexagonal fuzzy numbers, trapezoidal fuzzy numbers, triangular number, real numbers and intuitionistic pentagonal fuzzy numbers. By using different ranking functions the quantities have modified to crisp values and optimal solution has got by the given methodology. This method is very easy to solve the mixed fuzzy transportation problem.

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