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An Application of the Double Elzaki Transform in Partial Differential Equations

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Abstract: *The partial differential equation plays an important role in Physics, Chemistry, Differential Equations and many more applied subjects. Also various methods of solving partial differential equations are found in literature. In this paper, we use double Elzaki Transform to solve some partial Differential equations.*

Keywords: Elzaki Transform, Double Elzaki Transform, Function of exponential order, Partial Differential Equation

I. Introduction

In Science and Engineering issues, we generally look for answer of the various differential equations with initial and boundary conditions. On account of conventional differential conditions, we may initially observe the overall arrangement and decide the constants from conditions. But similar method is not working for partial differential equations. So it is challenging to change these constant and capacities to fulfil the given limit and conditions.The Boundary value problems, Initial value problems, Heat equations, Wave equations are mathematically represented in terms of partial differential equation. The partial differential equations are solved by various ways: i) by using method of separation, ii) by using integral Transforms, iii) by using Numerical methods, iv) Decomposition methods, etc. In literature various articles are found with this reference, [14] explains solution of partial differential equation by using numerical methods, [2] explains convolution method, [3], [7], [8], [9], [12], [15] presents integral Transform method. In method of integral transform involves Fourier Transform, Laplace Transform, Double Sumudu transform and Double Laplace Transform. In this paper we use Double Elzaki Transform to solving some partial differential equations. We use the following definitions and properties for further calculations.

II. Definitions And Properties

2.1 Definitions:

Double Elzaki Transform

Let $f(x,y)$with $x,y>0$ be a function which can be expressed as a convergent infinite series then its Double Elzaki Transform is defined as,

$$E_2\{f(x,y);u,v\}=T(u,v)=uv\int_0^\infty\int_0^\infty f(x,y)e^{-\left(\frac{x}{u}+\frac{y}{v}\right)}dxdy,$$

whenever integral exist.

Inverse double Elzaki Transform is defined as,

$$E_2{}^{-1}\{T(u,v)\}=f(x,y),\ x,y>0.$$

Function of exponential order:

The function $f(x,y)$ is said to be of exponential order,

$a>0$, $b>0$ on $0\le x<\infty$,

$0\le y<\infty$

if there exists a positive constants k such that,

$$|f(x,y)|\le ke^{\frac{x}{a}+\frac{y}{b}}.$$

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Existence condition for the Double Elzaki Transform:
If $f(x,y)$ is continuous in every finite interval and of exponential order then Double Elzaki Transform of $f(x,y)$is exist.
Now we discussed the some standard properties of double Elzaki Transform:

2.2 Standard Properties of double Elzaki Transform

A. Linearity Property

If $f(x,y)$ and $g(x,y)$ be two functions of $x,y>0$ such that,

$$E_2\{f(x,y)\}=T_1(u,v),$$

and

$$E_2\{g(x,y)=T_2(u,v)\} \text{ then,}$$

$E_2\{\alpha f(x,y)+\beta g(x,y)\}=\alpha E_2\{f(x,y)\}+\beta E_2\{g(x,y)\}=\alpha T_1(u,v)+\beta T_2(u,v)$.

Proof:

By using definition we have,

$$\begin{aligned} E_2\{\alpha f(x,y)+\beta g(x,y)\} &= uv\int_0^\infty\int_0^\infty e^{-\frac{x}{u}-\frac{y}{v}}[\alpha f(x,y)+\beta g(x,y)]dxdy,\\ &= u\int_0^\infty e^{-\frac{x}{u}}\left[v\int_0^\infty e^{-\frac{y}{v}}(\alpha f(x,y)+\beta g(x,y))dy\right]dx,\\ &= uv\int_0^\infty e^{-\frac{x}{u}}\left[\alpha\int_0^\infty e^{-\frac{y}{v}}f(x,y)dy+\beta\int_0^\infty e^{-\frac{y}{v}}g(x,y)dy\right]dx\\ &= \alpha\left[uv\int_0^\infty\int_0^\infty e^{-\frac{x}{u}-\frac{y}{v}}f(x,y)dxdy\right]+\beta\left[uv\int_0^\infty\int_0^\infty e^{-\frac{x}{u}-\frac{y}{v}}g(x,y)dxdy)\right]\\ &= \alpha E_2\{f(x,y)\}+\beta E_2\{g(x,y)\}\ ,\forall\, x,y\in\mathbb{R} \end{aligned}$$

B. Change of Scale Property

If $E_2\{f(x,y)\}=T(u,v)$ then $E_2\{f(ax,by)\}=\frac{1}{ab}T(au,\ bv)$.

Proof:

$$\begin{aligned} E_2\{f(ax,by)\} &= uv\int_0^\infty\int_0^\infty e^{-\frac{x}{u}-\frac{y}{v}}f(ax,by)dxdy,\\ &= \frac{uv}{ab}\int_0^\infty\int_0^\infty e^{-\frac{ax}{au}-\frac{by}{bv}}f(ax,by)abdxdy,\\ &= \frac{1}{ab}\left[uv\int_0^\infty\int_0^\infty e^{-\frac{ax}{au}-\frac{by}{bv}}f(ax,by)abdxdy\right]\\ &= \frac{1}{ab}[T(au,bv)]. \end{aligned}$$

C. First Shifting Property

If $E_2\{f(x,y)\}=T(u,v)$ then $E_2\{e^{ax+by}f(x,y)\}=T\left[\frac{u}{1-au},\frac{v}{1-bv}\right]$.

Proof:

$$\begin{aligned} \text{Let } E_2\{e^{ax+by}f(x,y)\} &= uv\int_0^\infty\int_0^\infty e^{-\frac{x}{u}-\frac{y}{v}}e^{ax+by}\,f(x,y)dxdy\\ &= uv\int_0^\infty\int_0^\infty e^{-\left(\frac{1}{u}-a\right)x-\left(\frac{1}{v}-b\right)y}f(x,y)dxdy\\ &= T\left(\frac{u}{1-au},\frac{v}{1-bv}\right). \end{aligned}$$

Similarly, after replacing a by $-a$ and b by $-b$ in (III), we have,

D. If $E_2\{f(x,y)\}=T(u,v)$then $E_2\{e^{-ax-by}f(x,y)\}=T\left(\frac{u}{1+au},\frac{v}{1+bv}\right)$.

E. Elzaki Transform of Partial Derivatives

1. $E_2\left\{\frac{\partial}{\partial x}f(x,y)\right\}=\frac{1}{u}T(u,v)-uT(0,v)$.

Proof:

By using definition we have,

$$E_2\left\{\frac{\partial}{\partial x}f(x,y)\right\} = uv\int_0^\infty\int_0^\infty e^{-\frac{x}{u}-\frac{y}{v}}\frac{\partial}{\partial x}f(x,y)dxdy,$$

$$= v\int_0^\infty e^{-\frac{y}{v}}\left\{u\int_0^\infty e^{-\frac{x}{u}}\frac{\partial}{\partial x}f(x,y)dx\right\}dy \qquad (1)$$

Using definition of Elzaki Transform we have,

$$E\left\{\frac{\partial}{\partial x}f(x,y)\right\} = u\int_0^\infty e^{-\frac{x}{u}}\frac{\partial}{\partial x}f(x,y)dx,$$

$= \frac{1}{u}T(u,y) - uf(0,y)$, substitute in (1) we obtained,

$$E_2\left\{\frac{\partial}{\partial x}f(x,y)\right\} = v\int_0^\infty e^{-\frac{y}{v}}\left[\frac{1}{u}T(u,y) - uf(0,y)\right]dy,$$

$$= \frac{1}{u}\left\{v\int_0^\infty e^{-\frac{y}{v}}T(u,y)dy\right\} - u\left\{v\int_0^\infty e^{-\frac{y}{v}}f(0,y)dy\right\}$$

$$= \frac{1}{u}T(u,v) - uT(0,v).$$

2. $E_2\left\{\frac{\partial}{\partial y}f(x,y)\right\} = \frac{1}{v}T(u,v) - vT(u,0)$

3. $E_2\left\{\frac{\partial^2}{\partial x^2}f(x,y)\right\} = \frac{1}{u^2}T(u,v) - T(0,v) - u\frac{\partial}{\partial x}T(0,v)$

4. $E_2\left\{\frac{\partial^2}{\partial y^2}f(x,y)\right\} = \frac{1}{v^2}T(u,v) - T(u,0) - v\frac{\partial}{\partial y}T(u,0)$

5. $E_2\left\{\frac{\partial^2}{\partial xy}f(x,y)\right\} = \frac{1}{uv}T(u,v) - \frac{v}{u}T(u,0) - \frac{u}{v}T(0,0) + uvT(0,0).$

VI. $E_2\{f(x)\} = uv^2T(u)$ and $E_2\{g(y)\} = u^2T(v)$.

Proof: Using definition,

$$E_2\{f(x)\} = uv\int_0^\infty\int_0^\infty e^{-\frac{x}{u}-\frac{y}{v}}f(x)dxdy,$$

$$= uv\int_0^\infty e^{-\frac{y}{v}}\int_0^\infty e^{-\frac{x}{u}}f(x)dxdy,$$

$$= uv^2\int_0^\infty e^{-\frac{1}{u}x}f(x)dx$$

$$= uv^2\int_0^\infty e^{-\frac{1}{u}x}f(x)dx$$

$$= v^2\left[u\int_0^\infty e^{-\frac{x}{u}}f(x)dx\right]$$

$$= v^2E\{f(x)\} = v^2T(u).$$

On the same way we have, $E_2\{g(y)\} = u^2T(v)$.

2.3 Double Elzaki Transform of Some Standard Functions

We calculate double Elzaki Transform of some standard functions using definition summarized in the following table:

Function $f(x,y)$	Double Elzaki Transform $T(u,v)$
1	u^2v^2
e^{ax+by}	$\frac{u^2v^2}{(1-au)(1-bv)}$
$\cos(ax+by)$	$\frac{u^2v^2(1-abuv)}{(1+a^2u^2)(1+b^2v^2)}$
$\sin(ax+by)$	$\frac{u^2v^2(au+bv)}{(1+a^2u^2)(1+b^2v^2)}$
$\sinh(ax+by)$	$\frac{1}{2}\left[\frac{u^2v^2}{(1-au)(1-bv)} - \frac{u^2v^2}{(1+au)(1+bv)}\right]$
$cosh(ax+by)$	$\frac{1}{2}\left[\frac{u^2v^2}{(1-au)(1-bv)} + \frac{u^2v^2}{(1+au)(1+bv)}\right]$

Table 1.3.1: Double Elzaki Transform of some standard function

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III. Applications in Partial Differential Equations

3.1 Inhomogeneous Partial Differential Equation

Consider inhomogeneous partial differential equation of the form,

$$u_{xy} = -csin(cy),\ y > 0$$

with initial condition, $u(x,0) = x$,

and boundary condition, $u(0,y) = 0,\ u(0,0) = 0$, where c is constant.

Solution:

Now taking double Elzaki transform of given partial differential equation and taking Elzaki transform of initial and boundary condition we have,

$$E_2\{u_{xy}\} = -cE_2\{sin(cy)\}$$

with $E\{u(x,0)\} = T(u,0) = u^3,\ E\{u(0,y)\} = T(0,v) = 0$, and $T(0,0) = 0$.

Then using above partial derivative formula we have,

$$\frac{1}{uv}T(u,v) - \frac{v}{u}T(u,0) - \frac{u}{v}T(0,v) + uvT(0,0) = -cu^2\left[\frac{cv^3}{1+c^2v^2}\right]$$

after solving we have,

$$T(u,v) = \frac{u^3v^2}{1+c^2v^2}$$

Taking inverse double Elzaki transform we get solution,

$u(x,y) = xcos(cy)$.

3.2 Partial Differential Equation of Order One

$$u_x = u_y,$$

with initial and boundary conditions $u(x,0) = x, u(0,y) = y$.

Solution:

Now taking double Elzaki transform of given partial differential equation and taking Elzaki transform of initial and boundary condition we have,

$$E_2\{u_x\} = E_2\{u_y\}$$

with $E\{u(x,0)\} = T(u,0) = u^3,\ E\{u(0,y)\} = T(0,v) = v^3$.

Then using above partial derivative formula we have,

$$\frac{1}{u}T(u,v) - uT(0,v) = \frac{1}{v}T(u,v) - vT(u,0)$$

after solving we have,

$$T(u,v) = u^2v^3 + u^3v^2$$

Taking inverse double Elzaki transform we get solution,

$$u(x,y) = x + y.$$

3.3 Inhomogeneous Partial differential Equation of Order One

$$u_x + u_y = e^y(1+x),$$

with initial and boundary conditions $u(x,0) = x, u(0,y) = 0$.

Solution:

Now taking double Elzaki transform of given partial differential equation and taking Elzaki transform of initial and boundary condition we have,

$$E_2\{u_x\} + E_2\{u_y\} = E_2\{e^y(1+x),\}$$

with $E\{u(x,0)\} = T(u,0) = u^3,\ E\{u(0,y)\} = T(0,v) = 0$.

Then using above partial derivative formula we have,

$$\frac{1}{u}T(u,v) - uT(0,v) + \frac{1}{v}T(u,v) - vT(u,0) = \left\{\frac{v^2(u^2+u^3)}{(1-v)}\right\}$$

after solving we have,

$$T(u,v) = \frac{u^3 v^2}{(1-v)}$$

Taking inverse double Elzaki transform we get solution,

$$u(x,y) = xe^y.$$

3.4 Homogeneous Partial Differential Equation of Order Two

$$u_{xx} + u_{yy} - 4u = 0,$$

with initial conditions $u(x,0) = x,\ u_y(x,0) = e^{-2x}$ and

and boundary condition $u(0,y) = y, u_x(0,y) = -2y$.

Solution:

Now taking double Elzaki transform of given partial differential equation and taking Elzaki transform of initial and boundary condition we have,

$$E_2\{u_{xx}\} + E_2\{u_{yy}\} - 4E_2\{u(x,y)\} = 0$$

with $E\{u(x,0)\} = T(u,0) = 0, E\{u_y(x,0)\} = \frac{u^2}{(1+2u)}$

$$E\{u(0,y)\} = T(0,v) = v^3,\ E\{u_x(0,y)\} = -2v^3.$$

Then using above partial derivative formula we have,

$$T(u,v)\left\{\frac{v^2 + u^2 - 4u^2v^2}{u^2v^2}\right\} = \frac{vu^2}{1+2u} + v^3 - 2uv^3$$

after solving we have,

$$T(u,v) = \frac{u^3 v^3}{(1+2u)}$$

Taking inverse double Elzaki transform we get solution,

$$u(x,y) = ye^{-2x}.$$

IV. Conclusion

In this paper, we use double Elzaki Transform for solving various partial differential equations. Also the standard properties of Elzaki Transforms are discussed in this work. Using some standard results and few calculations different partial differential equations are solved by double Elzaki transform in few steps.

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