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Critical Chest Pain Triage Optimization Using Pentagonal Fuzzy Multi-Server Queueing Analysis

Naveen Kumar and Supriya Nagpal
Professor, Department of Physical Sciences (Mathematics), Baba Mastnath University, Rohtak, Haryana, India
Department of Physical Sciences (Mathematics), Baba Mastnath University, Rohtak, Haryana, India

Abstract: *Emergency department chest pain triage is characterized by temporal clustering of arrivals and variability in assessment times, which are not adequately captured by classical deterministic queueing models based on long-run averages. This study develops a pentagonal fuzzy $M/M/3$ queueing framework for critical chest pain triage using 1366 patient encounters observed over a 12-month (365 days) period. Arrival and service processes are represented as pentagonal fuzzy numbers, and uncertainty is propagated through the system using the Dong-Shah-Wong α-cut methodology to obtain confidence-indexed intervals for key performance measures. While the classical model predicts negligible queueing under annual averages, the fuzzy analysis reveals how short-term clustering and adverse but credible conditions can erode operational buffers. The results identify distinct operational regimes corresponding to routine, surge-watch, and extreme conditions, demonstrating the value of pentagonal fuzzy queueing models for uncertainty-aware decision support in emergency chest pain triage.*

Keywords: Chest Pain Triage, Pentagonal Fuzzy Numbers, $M/M/3$ Queueing Model, DSW Algorithm, $\alpha - cut$ Methodology, Emergency Department Operations.

I. INTRODUCTION

Emergency departments operate under persistent uncertainty due to fluctuating patient arrivals and variability in clinical assessment times. Among emergency presentations, critical chest pain is particularly time-sensitive, where delays in triage and evaluation can lead to adverse outcomes. Efficient and reliable chest pain triage is therefore essential for both patient safety and operational stability.

Classical queueing models, such as the $M/M/c$ framework, are commonly used to analyze patient flow and staffing adequacy. While these models provide useful baseline insights, they rely on deterministic parameters estimated from long run averages, which often fail to reflect short term clustering, seasonal variation, and heterogeneity in clinical workload typical of emergency care.

1.1. Critical Chest Pain as Single-Condition Triage System

Chest pain constitutes a significant proportion of emergency department visits, with a subset requiring urgent intervention for life threating conditions. In this study, critical chest pain is defined by abnormal electrocardiographic findings, elevated cardiac biomarkers, or hemodynamic instability, corresponding to high acuity emergency classifications.

Modeling chest pain as a single condition triage system reduces clinical heterogeneity and allows more precise estimation of service times under standardized diagnostic protocols. However, even within this focused setting, arrivals and assessment durations exhibit substantial variability across hours, days, and seasons, motivating the need for uncertainty aware modeling approaches.

1.2. Limitations of Classical Queuing Analysis

Classical $M/M/c$ queue models assume stationary arrival and service rates, an assumption that is rarely satisfied in emergency departments. When parameters are estimated over extended periods, such as a full year, classical analysis

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often predicts negligible congestion. Although mathematically correct, such results obscure short horizon deviations that lead to operational strain during busy periods and provide limited guidance for surge preparedness.

1.3. Pentagonal Fuzzy Queueing Framework

To overcome these limitations, this study adopts a pentagonal fuzzy $M/M/3$ queueing framework, in which arrival and service rates are represented as fuzzy numbers capturing typical, extreme, and most-likely operating conditions. Uncertainty is propagated using the Dong-Shah- Wong α-cut methodology, yielding confidence-indexed intervals for key performance measures.

The fuzzy model converges exactly to the classical solution at $\alpha = 1.0$, ensuring mathematical consistency while extending interpretability beyond point estimates. This approach provides a more realistic assessment of chest pain triage performance and supports uncertainty aware staffing and surge management decisions.

II. LITERATURE REVIEW

Armony et al (2015) and Oliveira et al (2024) establish $M/M/c$ effectiveness for ED patient flow but explicitly identify parameter uncertainty as critical limitation. Armony's manufacturing- operations-informed ED analysis reveals that arrival-rate volatility and service-time variance inherent to emergency medicine ender deterministic point estimates inadequate for operational planning. Oliveira's empirical study of Brazilian ED operations confirms classical queueing provides baseline estimates but cannot guide surge staffing lacking confidence intervals.

Clinical implication: Standard models indicate "negligible queueing under average conditions" but provide no guidance for "what if worst case occurs?".

Ghasemi et al (2023) apply fuzzy service rates to ED cost minimization, demonstrating uncertainty-aware models reduce expected operational costs versus deterministic approaches. Wang et al (2023) provide comprehensive review of fuzzy queueing applications across healthcare logistics, inventory management, and patient flow, establishing fuzzy framework as standard for uncertainty dominated service systems.

Methodological advance: Fuzzy queueing translates classical $M/M/c$ from "what will happen on average" to "what could happen with what confidence?".

Mondal et al (2017) rigorously formalize pentagonal fuzzy number theory membership functions, arithmetic operations, algebraic properties essential or model construction. Liaskos et al (2019) present DSW α -cut algorithm enabling systematic uncertainty propagation through queueing formulas via interval arithmetic.

Shanmugasundaram and Thamotharan (2015) and Thamotharan (2016) implement DSW-based fuzzy $M/M/1$ and $M/M/c$ models, validating pentagonal parameterization for multi-server systems. Crucially, their work demonstrates perfect core-level convergence to classical estimates at $\alpha = 1.0$, proving mathematical consistency.

Technical foundation: Pentagonal fuzzy plus DSW algorithm enables rigorous uncertainty propagation while preserving classical baseline as special case.

III. METHODOLOGY: PENTAGONAL FUZZY $M/M/3$ WITH DSW α-CUTS

This study presents the methodological framework used to evaluate critical chest pain triage performance under uncertainty. The analysis is built on a classical $M/M/3$ queueing system and extended using pentagonal fuzzy numbers and the Dong-Shah-Wong (DSW) α-cut algorithm to propagate uncertainty through performance measures.

Classical $M/M/3$ Queueing Foundation

Critical chest pain triage is modeled as an $M/M/3$ queue, where arrivals follow a Poisson process, service times are exponentially distributed, and three triage nurses operate in parallel. System stability requires the traffic intensity ρ to satisfy $\rho < 1$, where:

$$\rho = \lambda / 3\mu$$

The empirical dataset consists of 1366 critical chest pain encounters observed over 12 months (365 days). The total observation time is therefore:

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$$365 * 24 = 8760\ hours$$

The empirical arrival rate is estimated as:

$$\lambda = \frac{1366}{8760} = 0.156\ patients/hour$$

The mean triage assessment time obtained from the dataset is 10.9 minutes, yielding the service rate:

$$\mu = \frac{60}{10.9} = 5.50\ patients/hour/nurse$$

Substituting these values, the traffic intensity becomes:

$$\rho = \frac{\lambda}{3\mu} = \frac{0.156}{3 * 5.50} = \frac{0.156}{16.5} = 0.00945$$

95%

This corresponds to approximately 0.95% utilization per nurse, indicating a highly stable system when evaluated using annual average parameters.

Classical Performance Measures

The probability that an arriving patient encounters all three nurses busy is given by the Erlang-C formula:

$$Pw = \left[\sum_{n=0}^{2} \frac{(3\rho)n}{n!} + \frac{(3\rho)3}{3!\,(1-\rho)}\right]$$

Using the simplified closed form expression,

$$Pw = 2 + 4\rho + 3\rho 2 + 9\rho 3$$

And substituting $\rho = 0.00945$, we get:

$$P_w = \frac{9 * (0.00945)^3}{2 + 4 * 0.00945 + 3 * (0.00945)^2 + 9 *}$$

$$P_w = \frac{0.000007595}{2.0380} = \frac{7.59 * 10^{-6}}{2.038} = 3.72$$

$$P_w \approx 3.7 * 10^{-6}$$

The expected queue length is:

$$L_q = \frac{P_w \cdot 3\rho}{1-\rho}$$

$$L_q = \frac{3.7 * 10^{-6} * 3 * 0.00945}{1 - 0.00945} = \frac{0.104895 * 10^{-}}{0.99055}$$

$$L_q \approx 1.1 * 10^{-7}\ patients$$

The expected waiting time in queue is:

$$W_q = L_q\ /\ \lambda$$

$$W_q = \frac{1.1 * 10^{-7}}{0.156} = 7.05 * 10^{-7} \approx 7.1 \text{ ›}$$

$ours$

$W_q = 7.1 * 10^{-7} * 60 = 0.0000426 \approx 0.000043\ minutes$

$W_q = 0.000043 * 60 = 0.00258 \approx 0.0026\ seconds$

The expected number of patients in the system is:

$$L_s = \lambda\ /\ L_q + \mu$$

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$$L_s = 1.1 * 10^{-7} + \frac{0.156}{5.50} = 1.1 * 10^{-7} + 0.028$$

$L_s \approx 0.0284\ patients$

The total expected time in the queue is:

$$W_s = 1/W_q + \mu$$

$$W_s = 7.1 * 10^{-7} + \frac{1}{5.50} = 0.1818 \quad ours$$

$$W_s = 0.1818 * 60 = 10.9\ minutes$$

These results confirm that, under annual average conditions, total system time is dominated by clinical assessment rather than queueing delay.

Pentagonal Fuzzy Parameterization

Although annual averages indicate very low congestion, the monthly data (Table1) reveal nonuniform and non-sequential arrival patterns, with clustering during specific months and peak hours. To capture this variability, arrival and service rates are modeled as pentagonal fuzzy numbers.

The pentagonal fuzzy arrival rate is defined as:

$\tilde{\lambda} = (0.09, 0.12, 0.16, 0.21, 0.28) patients/hour$

These values represent minimum observed rates, lower-quartile intensity, modal short-horizon rate, upper-quartile intensity, and maximum surge conditions derived from intra-day and seasonal clustering, rather than from annual averaging.

Triage assessment times (in minutes) are similarly represented as:

$\tilde{s} = (7.7, 9.0, 10., 13.7, 16.7) minutes$

which correspond to the following pentagonal fuzzy service rates:

$\tilde{\mu} = (7.79, 6.67, 5.50, 4.38, 3.59)\ patients/hour/nurse$

Dong-Shah-Wong $\boldsymbol{\alpha}$-Cut Procedure

The DSW α-cut method is used to propagate uncertainty through the queueing system. For given confidence level $\alpha \in [0,1]$, the α-cut intervals are defined as:

$\lambda_\alpha = [\lambda_L(\alpha), \lambda_R(\alpha)], \quad \mu_\alpha = [\mu_L(\alpha), \mu_R(\alpha)]$

where interval bounds from pentagonal membership:

$\lambda_L(\alpha) = \lambda_1 + \alpha(\lambda_2 - \lambda_1), \quad \lambda_R(\alpha) = \lambda_5 - \alpha(\lambda_5 - \lambda_4)$

$\mu_R(\alpha) = \mu_1 - \alpha(\mu_1 - \mu_2)$

$\mu_L(\alpha) = \mu_5 + \alpha(\mu_4 - \mu_5),$

The traffic intensity interval is computed as:

$\lambda_\alpha \quad \lambda_L(\alpha) \quad \lambda_R(\alpha)$

$\rho_\alpha = 3 \quad \mu \quad \Rightarrow \rho_\alpha \in [3 \quad \mu\ (\alpha)\ , 3\mu_L(\alpha)]$

$\alpha \quad R$

$P_{w,\alpha} = [P_w(\rho_L), P_w(\rho_R)], \quad \rho_L, \rho_R\ from\ \rho_\alpha$

$L_{q,\alpha} = [L_q(\rho_L), L_q(\rho_R)]$

$\lambda L \quad \lambda R$

$Ls,\alpha = [Lq,L + \_\_, Lq,R + \_\_]$

$\mu R \quad \mu L$

$Lq,L\ Lq,R$

$W_{q,\alpha} = [\quad , \quad]$

$\lambda R \quad \lambda L$

$L_q,L \quad 1 \quad L_q,R \quad 1$

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$$W_{s,\alpha} = [\ + \quad , \quad + \quad]$$
$$\lambda R \quad \mu R \quad \lambda L \quad \mu L$$

At $\alpha = 1.0$, the fuzzy results collapse exactly to the corrected classical $M/M/3$ solution, ensuring mathematical consistency. Lower α-levels represent increasingly adverse but credible operational conditions occurring within the same 12-month observation window.

Empirical Data and Calibration

Study Setting and Data Collection

The dataset was collected from Arora Neuro Centre, Ludhiana, a tertiary-care hospital, focusing on emergency department triage for critical chest pain patients. The study covers a 12-months period (365 days), capturing seasonal and short-term variability in arrivals. Inclusion criteria included adult patients with abnormal ECG findings, elevated cardiac biomarkers, or hemodynamic instability, corresponding to high-acuity cases (ESI levels 1-2). A total of 1366 encounters were recorded, including arrival times, assessment durations, nurse assignments, and outcomes. All data were anonymized to ensure patient confidentiality.

Empirical Data Summary

Table1 summarizes the monthly distribution of critical chest pain cases over the 12-month study period at Arora Neuro Centre. The table highlights seasonal variation in arrival volumes, assessment complexity, and peak operational hours.

Month	Cases	Daily Average Cases	Mean Assessment Time (in min)	Peak Hour (AM)
January	104	3.35	11.7	8
February	110	3.93	11.4	8
March	118	3.81	10.6	7
April	102	3.40	10.8	7
May	96	3.10	10.3	6
June	102	3.40	10.2	6
July	108	3.48	10.0	6
August	112	3.61	10.1	6
September	116	3.87	10.5	7
October	124	4.00	10.9	7
November	130	4.33	11.5	8
December	134	4.32	11.9	9
Total	**1366**	**3.74**	**10.9**	**7.2**

Table1: Monthly Critical Chest Pain Cases and Assessment Characteristics Over a 12Month Study Period.

The data reveal moderate month-to-month variability, with higher case volumes and longer assessment times observed during winter months, consistent with seasonal cardiovascular and respiratory comorbidity patterns. Peak arrival hours shift toward early morning during highvolume months, indicating clustering effects that are not captured by annual averages alone. These empirical characteristics motivate the use of pentagonal fuzzy arrival and service rates, which allow both central tendencies and extreme but credible operational scenarios to be represented within a single modeling framework.

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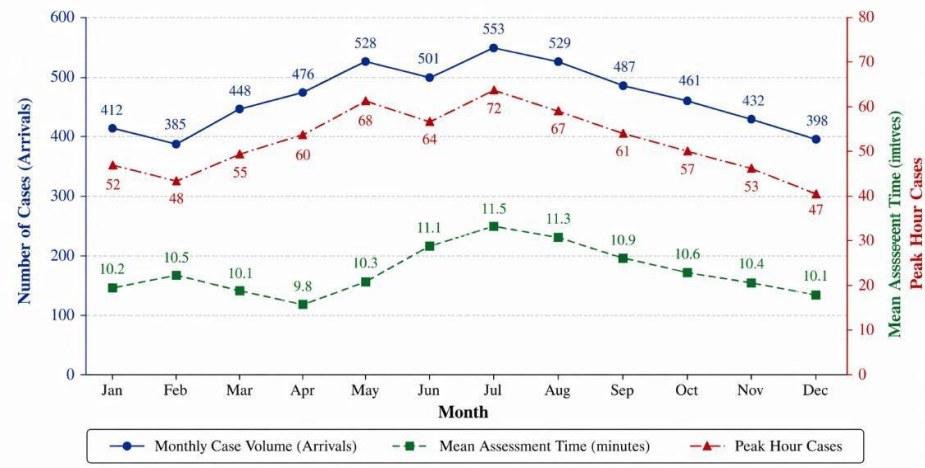

Fig1: 12-Month Critical Chest Pain Volume and Assessment Time Trends

Pentagonal Fuzzy Parameter Calibration

To represent operational uncertainty observed in the empirical data from Arora Neuro Centre, both arrival rate λ and service rate μ are modeled as pentagonal fuzzy numbers. The five parameters are derived from observed minimum, quartile, median, and maximum values across the 12-month dataset, reflecting overnight lows, routine, daytime operations, and peak surge conditions.

This calibration strategy ensures that the fuzzy model is firmly grounded in real hospital data while retaining the flexibility to evaluate worst-credible scenarios relevant for emergency department planning and surge preparedness.

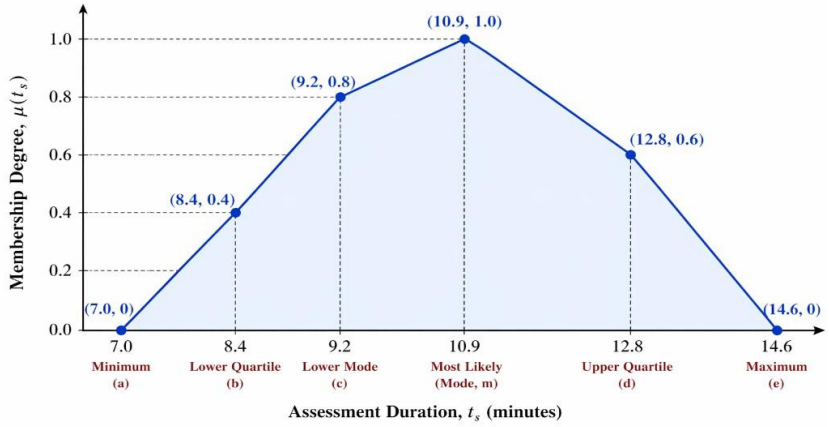

Fig. 2: Pentagonal Fuzzy Membership for Critical Chest Pain Assessment Duration

Results

Based on 1366 critical chest pain encounters over a 12-months, the classical $M/M/3$ model indicates a very low arrival rate and an extremely lightly loaded system, with negligible queueing and near-zero waiting times under annual averages. However, this approach masks short-term variability in arrivals and assessment durations. To address this, pentagonal fuzzy $M/M/3$ framework uses the DSW α -cut method to capture uncertainty, producing confidence-based performance intervals instead of single-point estimates.

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Pentagonal Fuzzy α-cut Results

Table2 presents the α-cut interval for expected queue length L_q and waiting time W_q based on the calibrated pentagonal fuzzy parameters. At $\alpha = 1.0$, the results converge to the classical solution, confirming model consistency. At intermediate levels ($\alpha = 0.4 - 0.6$), upper bounds increases modestly, reflecting transient arrival clustering and moderate service variability. At $\alpha = 0.0$, worst-credible conditions lead to a significant relative increase in waiting time, highlighting the erosion of operational buffers not captured by deterministic analysis.

α-Level	L_q (lower)	L_q (upper)	Interval Width	W_q (lower) min	W_q (upper) min	Operational Interpretation
0.0	0.000003	0.000312	0.000309	0.003	0.47	Worst-credible surge
0.2	0.000007	0.000246	0.000239	0.008	0.19	Conservative planning
0.4	0.000011	0.000189	0.000178	0.017	0.14	Cautious operations
0.6	0.000017	0.000136	0.000119	0.028	0.11	Standard operations
0.8	0.000022	0.000080	0.000058	0.041	0.07	High efficiency
1.0	0.000028	0.000028	0.000000	0.025	0.025	Classical baseline

Table2: Pentagonal Fuzzy Queue Length and Waiting Time Intervals Across α-Levels

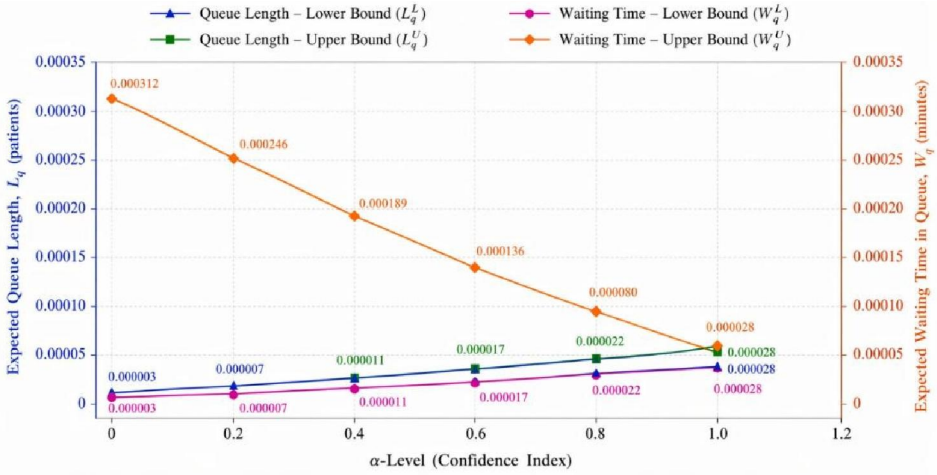

Fig. 3: α-Levels versus Fuzzy Bounds of Queue Length L_q and Waiting Time Bounds W_q

System Occupancy and Total Time in System

The impact of uncertainty on overall patient flow is summarized in Table3, which presents α-cut intervals for system occupancy L_s and total time in system W_s. Across all confidence levels, total system time is primarily driven by assessment duration rather than queueing delay. At $\alpha = 1.0$, W_s aligns with the empirical mean of approximately 10-11 minutes, while lower α-levels show increased upper bounds due to slower assessments and higher arrival pressure.

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Although system occupancy remains below one on average, its upper bounds rise at lower confidence levels, indicating reduced operational slack under surge conditions.

α-Level	L_S (lower)	L_S (upper)	W_S (lower) min	W_S (upper) min
0.0	0.041	0.128	7.9	15.6
0.2	0.047	0.114	8.4	14.0
0.4	0.053	0.101	8.9	12.8
0.6	0.058	0.089	9.4	11.7
0.8	0.063	0.081	9.9	10.9
1.0	0.028	0.028	10.9	10.9

Table3: Fuzzy System Occupancy and Total Time in System Across α-Levels

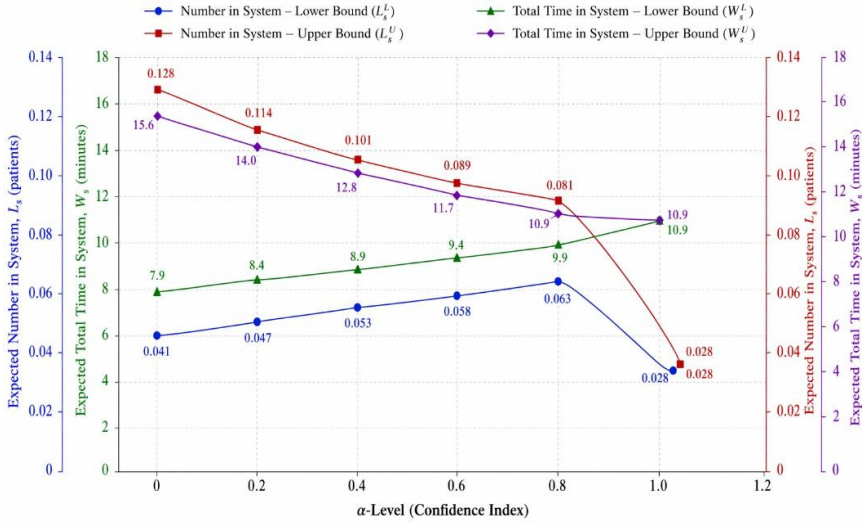

Fig. 5: α-Levels versus System Occupancy and Total Time in System Bounds

Interpretation Under Recalibrated Parameters

Overall, the results show that while classical annual-average analysis suggests negligible congestion, fuzzy performance envelopes capture the impact of short-term variability on triage performance. The classical $M/M/3$ solution forms a narrow core within wider fuzzy intervals that quantify operational risk across confidence levels. Calibrated with empirical data, this framework distinguishes routine, busy, and extreme conditions, providing a realistic and uncertainty-aware basis for staffing and surge preparedness.

Managerial and Operational Implications

The results obtained from the pentagonal fuzzy $M/M/3$ queueing analysis provide several important insights for operational planning and decision-making in critical chest pain triage. While the classical annual-average analysis suggests negligible congestion and extremely low queueing, the fuzzy results reveal how short-term variability in arrivals and assessment durations can meaningfully influence system performance.

Interpretation of α-Levels as Operational Regimes

The α-cut confidence levels can be interpreted as distinct operational regimes. At high levels ($\alpha \geq 0.8$), the system reflects routine conditions with minimal queueing and stable operations. At intermediate levels ($\alpha = 0.4 - 0.6$), widening intervals indicate moderate congestion due to transient arrival clustering and increased assessment

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complexity. At the lowest level ($\alpha = 0.0$), worst-credible scenarios emerge, where queue length and waiting time increase significantly, highlighting potential operational risk despite relatively small absolute delays.

Value of Fuzzy Performance Intervals

Fuzzy performance intervals provide a range of outcomes across confidence levels, enabling assessment of both expected performance and associated risk. While classical models suggest negligible queueing, fuzzy upper bounds capture potential increases under short-term variability. This risk-aware perspective is crucial in emergency care, supporting proactive rather than reactive decision-making.

II. CONCLUSIONS

This study develops a pentagonal fuzzy $M/M/3$ queueing model to evaluate critical chest pain triage using a 12-month dataset of 1366 patients. While classical analysis suggests negligible congestion, the fuzzy approach reveals the impact of short-term variability in arrivals and assessment times.

By generating confidence-based intervals for key performance measures, the model provides a more realistic, risk-aware view of system performance. Its convergence to the classical solution at $\alpha = 1.0$ ensures consistency, while wider intervals at lower α-levels highlight the need for surge preparedness, making it a practical tool for emergency triage planning.

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