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Analysis of Finite Source Machine Repair Systems with Fuzzy Priority Discipline

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Abstract: *This paper studies a finite source machine repair system incorporating a priority-based service discipline within a fuzzy environment. In many real-life industrial and healthcare systems, system parameters such as failure rates, repair rates, and service priorities are often imprecise due to uncertainty and lack of complete information. To address this issue, the proposed model represents these parameters using triangular fuzzy numbers. Machines in the system are classified into different priority categories, ensuring that higher-priority failed machines receive service preference over lower-priority ones. The system is analyzed using a fuzzy queueing framework, and steady-state performance measures such as system size, queue length, waiting time, and server utilization are derived in fuzzy form. The α-cut approach is employed to transform fuzzy measures into interval-valued functions, and defuzzification is carried out using Simpson's 1/3 rule to obtain crisp results. Numerical illustration is provided to demonstrate the applicability and effectiveness of the model. The results highlight how priority mechanisms and uncertainty influence system performance, offering valuable insights for decision-making in complex repair and maintenance systems...*

Keywords: Finite Source Model, Machine Repair System, Priority Discipline, Triangular Fuzzy Numbers, Simpson's 1/3 Rule

I. INTRODUCTION

Queueing theory is a significant area of applied probability and operations research that focuses on the analysis of waiting lines, congestion, and service processes. It offers mathematical frameworks to study systems in which customers, jobs, or machines compete for limited service resources. The scope of queueing theory is wide, encompassing applications in manufacturing systems, service industries, healthcare operations, transportation networks, telecommunication systems, and computer networks. Among these, machine interference problems are particularly important for optimizing production efficiency and maintenance management in industrial settings. However, traditional queueing models often assume precise system parameters, which may not be realistic in practical situations where uncertainty and vagueness are inherent. To address this limitation, fuzzy set theory, introduced by Lotfi A. Zadeh, is integrated into queueing analysis. Fuzzy set theory provides a powerful framework to model imprecise and uncertain parameters, such as arrival rates and service rates, using fuzzy numbers. By combining queueing theory with fuzzy set concepts, more realistic and flexible models can be developed, leading to better decisionmaking in complex and uncertain environments.

In modern production and service systems, machines often differ in their importance, requiring differentiated service mechanisms. Critical machines demand immediate repair to minimize production losses, making priority-based service disciplines essential. Traditional queueing models assume precise system parameters; however, real-life systems are characterized by uncertainty in failure and repair processes. To address this limitation, fuzzy set theory is incorporated into queueing analysis. This study focuses on a finite source machine repair system with a pre-emptive priority discipline under a fuzzy environment. The model captures uncertainty in breakdown and repair rates using triangular

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fuzzy numbers and evaluates system performance using fuzzy analytical techniques. Priority systems are broadly classified into:

1.1 Non-pre-emptive priority

Pre-emptive priority

Priority queueing systems have been widely studied due to their practical relevance in manufacturing, communication networks, and healthcare systems. Studies have shown that priority mechanisms improve system performance by differentiating service among customer classes. Researchers such as Jouini and Roubos (2014) emphasized the importance of priority-based service differentiation. Stecke and Aronson (1985) and Haque and Armstrong (2007) explored service discipline strategies in machine repair systems. Pre-emptive and nonpre-emptive priority mechanisms have been extensively analyzed, with applications in machine interference problems and production systems. Recent studies incorporate uncertainty using fuzzy set theory, where system parameters are represented as fuzzy numbers. However, limited work has been done on integrating fuzzy modeling with prioritybased finite source repair systems, which motivates this study.

1.2 Model description

The system consists of a finite population of N identical machines and a group of W repairmen (servers)**.** Each machine operates independently and may fail randomly, thereby requesting repair service. The system includes multiple types of services, each assigned a specific priority level. All repairmen are capable of handling every type of service; however, they provide service according to the established priority structure, ensuring that higherpriority service requests are attended to before lower-priority ones.

Assumptions of the model are as follows:

: The breakdown rate per machine is denoted by (arrival rate) and is represented as a triangular fuzzy number.

$= (_1, _2, _3)$

: The repair rate per repairman is denoted by (service rate) and is also represented as a triangular fuzzy number.

$= (_1, _2, _3)$

The operating (up-time) and repair (down-time) durations are assumed to follow exponential distributions with fuzzy rates a, respectively.

= (Fuzzybusy time of the server)

'k' be the number of failed machines (customers) in the system at time t.

1.2.1 Arrival (Breakdown) Process

Only the machines that are currently operating can fail. If 'k' machines are already failed, then the remaining (N − k) machines are operational. Therefore, the effective breakdown rate becomes;

$\lambda_{-1} = (N - k)\lambda$ (1.1)

Thus, the breakdown rate decreases as the number of failed machines increases.

1.2.2 Service (Repair) Rate

The repair process depends on the number of available repairmen.

$$\left\{ \begin{array}{ll} , & < \\ =, & \geq \end{array} \right\} \quad (1.2)$$

or equivalently, = min {k, W} (1.3)

If the number of failed machines is less than or equal to the number of repairmen, all failed machines are repaired simultaneously. If the number of failed machines exceeds the number of repairmen, then only W machines receive service where k > W and the rest wait in the queue.

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1.2.3 Priority Discipline

The system follows a pre-emptive priority discipline. If a higher-priority service request arrives while a lower-priority service is in progress, the ongoing service is interrupted immediately. The repairman begins servicing the higher-priority machine. The interrupted service resumes later from the point where it was stopped without loss of work.

Let the service types be

1, 2, 3, ……,

with priorities

$_1 > _2 > _3 >$ .... > meaning that $_1$ has the highest priority.

1.2.4 Transient State Analysis

Let (t) denote the possibility measure that there are 'k' failed machines in the system at time 't'. The system behavior is governed by the following fuzzy differential–difference equations.

Case 1: k = 0

$$0() = - 0( ) + 1 ( ) \quad (1.4)$$

Case 2: $1 \leq \leq -1$

$$0 = (N - k + ( ) 1)_{-1} - [(-) +] + ( ) (k+1)_{+1} ( ) \quad (1.5)$$

Case 3: $W \leq \leq -1$

$$0 = (N - k + ( ) 1)_{-1} - [(-) +] + ( ) W_{+1} ( ) \quad (1.6)$$

Case 4: =

$$() = _{-1} ( ) ( ) \quad (1.7)$$

Initial Condition:

At time, t = 0, the system is assumed to be empty:

$$_0 0 ( ) = 1 a \quad 0 \quad ( ) = 0, f \geq 1 \quad (1.8)$$

1.2.5 Steady- State Analysis

In steady state, the system probabilities become independent of time.

Hence,

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() (1.9)

= 0 , = 0,1,2, …….

and, = lim () (1.10)

→∞

1.2.6 Steady state balance equations

= 0 =1 −1

, < { where, =, } ≥

!

= { −!! 0 , 0 ≤ < ; ! (1.11); (−)! ! − 0, ≤ ≤ }

where, = −!!! ,for k< W (1.14)

and, = −!!! = ,for k≥ W (1.15)

The normalization condition is: = 1

$\left(\sum_{k=0}^{W} \frac{N!\ \tilde{\rho}^k}{(N-k)!k!}\right.$

rule, $P = \frac{T}{\Sigma}$

=0 (1.16)

Where, $_0$=+ =+1 −!!! − −1 (1.12)

In Simpson's 1/3 (1.13)

1.2.7 Performance Measures

Average Number of FailedMachines in the system:

L = Nk=0 kPk

Average Number of FailedMachines in queue:

L = (−) Pk

Effective arrival rate:

= = (−) Pk Average Waiting Time:

W= L*e*

Average Waiting Time in queue:

L W =

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1.3 Numerical Analysis Using Simpson's Rule

In fuzzy queueing models, the system parameters such as breakdown rate and repair rate are represented as triangular fuzzy numbers. Using the α–cut approach, the fuzzy performance measures become functions of α.

Where, 0 ≤ α ≤ 1

To obtain crisp values, we evaluate

F = $_0{}^1$F α d(α)

To solve this, we apply Simpson's 1/3 rule.

F = $^h3$ [$_0$ + 4 $_1$ + 2 $_2$ + 4 $_3$ + $_4$]

Here, h $\frac{1-0}{4}$ = b___−a = = 0.25

n

Given Data

Total machines: N = 10

Number of repairmen: W = 2 Fuzzy breakdown rate:

= (1, 2, 3)

Fuzzy repair rate:

= (4, 5, 6)

Pre-emptive priority rule: high priority = 60 % and low priority = 40 %

1.3.1 α-cut Representation: using mid-point representation, the α-cutof fuzzy numbers are; (α) = [1+α, 3−α]

() = [4 + α, 6 − α]

For α = 0, (α) = [1, 3] and () = [4 , 6]

Case 1: for k < 2

!

=( ) − !!

0 =( ) 10!10−0.250 !0!0 = 1

10! 0.25^1

1 (=) 10− 1 !1! = 10 × 0.25 = 2.5

Case 2: for k ≥ W

10! 0.25^2

2 (=) 10− 2 !2! 20 = 2.8125

10! 0.25^3

3 (=) 10− 3 !2! 23−2 = 2.8125

Similarly, $_4$ = 2.4609, $_5$ = 1.230, $_6$ = 0.462, $_7$ ≈ 0.153, $_8$ ≈ 0.048, $_9$ ≈ 0.014, $_{10}$ ≈ 0.0036

Thus, $\sum_{k=0}^{10} T_k \approx 13.495$

From (1.13), all the probabilities are:

$_0$ = 0.074

$_1$ = 0.185

$_2$ = 0.208

$_3$ = 0.208

$_4$ = 0.182

$_5$ = 0.091(1.17)

$_6$ = 0.034

$_7$ = 0.011

$_8$ = 0.0035

$_9$ = 0.001

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$_{10} \approx 0$

Case 1: for $\alpha = 0$

Using equations (1.17), we get all the measures as;

Average number of machine in system is $L = \sum^{N}_{k=0} kP_k$

$L = 0(0.074) + 1(0.185) + 2(0.208) + 3(0.208) + 4(0.182) + 5(0.091) + 6(0.034) + 7(0.011) + 8(0.0035) + 9(0.001) + 0$

$L \approx 2.61$ (1.18)

Average Number of Failed Machines in queue is;

$L = 10=2(-2\ )$

Queue exits only when $k \geq W$

$L \approx 0.62$ (1.19)

Effective arrival rate is $= {}_{=}(-)P_k \approx 7.39$

Average Waiting Time is $W = {}^{L} \approx s$ 0.353

e

Average Waiting Time in queue is $W = {}^{L} \approx 0.084$

Table – 1.1: (performance measures for all α)

	L	L	W	W
0	2.61	0.62	0.353	0.084
0.25	2.89	0.78	0.366	0.099
0.5	3.18	0.98	0.381	0.118
0.75	3.49	1.21	0.397	0.138
1	3.82	1.47	0.415	0.160

Now using Simpson's 1/3 rule:

Average number of machines in the system is given by;

$L = \frac{h}{3}[\ _0 + 4\ _1 + 2\ _2 + 4\ _3 + \ _4]$

$= \frac{0.25}{3}[2.61 + 4(2.89) + 2(3.18) + 4(3.49) + 3.82]$

$L \approx 3.21$

Average number of machines in queue is given by;

$L = \frac{0.25}{3}[0.62 + 4(0.78) + 2(0.98) + 4(1.21) + 1.47]$

$L \approx 1.01$

Average waiting time in system is given by;

$W = \frac{0.25}{3}[0.353 + 4(0.366) + 2(0.381) + 4(0.397) + 0.415]$

$W \approx 0.385$

Average waiting time in the queue is given by;

$W = \frac{0.25}{3}[0.084 + 4(0.099) + 2(0.118) + 4(0.138) + 0.160]$

$W \approx 0.125$

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Table: 1.2: Average number of machines in the system (Ḷ)

	Lower bound	Upper bound
0.0	2.61	3.82
0.2	2.89	3.49
0.4	3.18	3.18
0.6	3.49	2.89
0.8	3.65	2.75
1.0	3.82	3.82

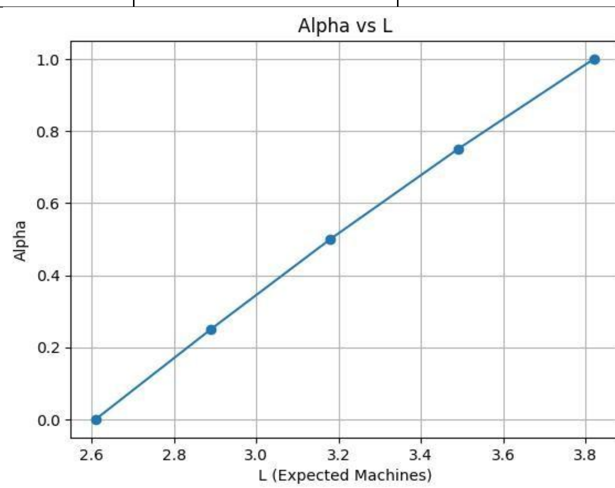

Figure: 1.2: Average number of machines in the system (Ḷ)

Table: 1.3: Average queue length (Ḷ)

	Lower bound	Upper bound
0.0	2.62	1.47
0.2	0.78	1.21
0.4	0.98	0.98
0.6	1.21	0.78
0.8	1.34	0.70
1.0	1.47	1.47

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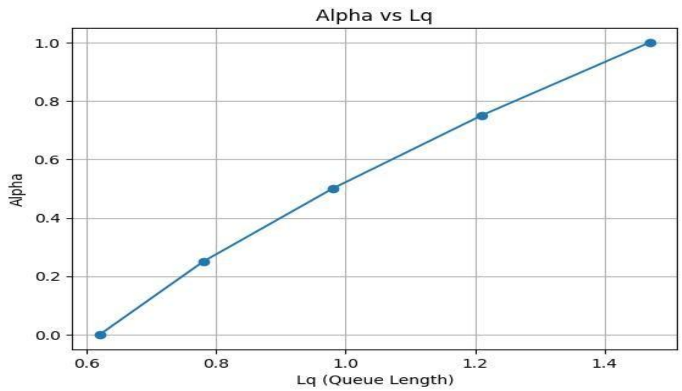

Figure: 1.3: Average queue length (Ḷ)

Table: 1.4: Waiting time in queue (Ẉ)

	Lower bound	Upper bound
0.0	0.084	0.160
0.2	0.099	0.138
0.4	0.118	0.118
0.6	0.138	0.099
0.8	0.149	0.090
1.0	0.160	0.160

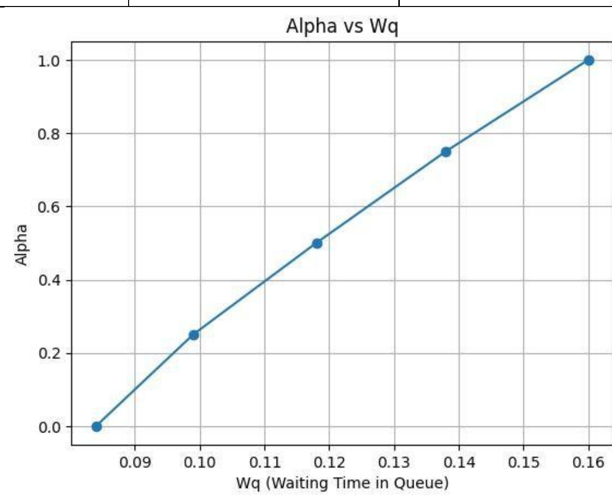

Figure: 1.4: Waiting time in queue (Ẉ)

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1.4 Conclusion

The results demonstrate that all performance measures increase with α, indicating that as uncertainty decreases, the system operates under higher and more realistic congestion levels. The α-cut tables show that lower bounds increase while upper bounds decrease, converging to crisp values at $\alpha = 1$. The graphical analysis confirms that queue formation becomes significant when the number of failed machines exceeds the number of repairmen. The fuzzy approach effectively captures uncertainty and provides a more flexible and realistic evaluation of system performance.

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