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Analysis of Pentagonal Fuzzy Multi-Server Queueing Analysis for Critical Chest Pain Triage Optimization

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Abstract: *Emergency department chest pain triage involves clustered arrivals and variable assessment times, which are not well captured by classical average-based queueing models. This study proposes a pentagonal fuzzy $M/M/3$ framework using data from 1366 patients over 12 months. Arrival and service rates are modeled as pentagonal fuzzy numbers, with uncertainty analyzed through the Dong-Shah-Wong α-cut approach. While classical results indicate negligible congestion, fuzzy analysis reveals the impact of short-term variability and adverse conditions. The model identifies routine, surge, and extreme operational regimes, supporting uncertainty-aware decision-making in emergency care...*

Keywords: Chest Pain Triage, Pentagonal Fuzzy Numbers, $M/M/3$ Queueing Model, DSW Algorithm, $\alpha - cut$ Methodology, Emergency Department Operations

I. INTRODUCTION

Emergency departments face significant uncertainty due to fluctuating patient arrivals and variability in assessment times, particularly for critical chest pain cases where delays can impact patient outcomes. While classical $M/M/c$ queueing model frameworks provide baseline insights into patient flow and staffing, they rely on long-run averages that fail to capture short-term clustering and operational variability typical of emergency care.

To address this, the study models chest pain triage as a single-condition system to reduce clinical heterogeneity and improve service time estimation. A pentagonal fuzzy $M/M/3$ queueing framework is then applied, where arrival and service rates are represented as fuzzy numbers. Using the Dong-Shah-Wong α -cut method, uncertainty is propagated to obtain confidence-based performance intervals. The model converges to the classical solution at $\alpha = 1.0$, while providing a more realistic, uncertainty-aware basis for staffing and surge management decisions.

II. METHODOLOGY: PENTAGONAL FUZZY $M/M/3$ WITH DSW α-CUTS

This study presents the methodological framework used to evaluate critical chest pain triage performance under uncertainty. The analysis is built on a classical $M/M/3$ queueing system and extended using pentagonal fuzzy numbers and the Dong-Shah-Wong (DSW) α-cut algorithm to propagate uncertainty through performance measures.

2.1. Classical $M/M/3$ Queueing Foundation

Critical chest pain triage is modeled as an $M/M/3$ queue, where arrivals follow a Poisson process, service times are exponentially distributed, and three triage nurses operate in parallel. System stability requires the traffic intensity ρ to satisfy $\rho < 1$, where:

$$\rho = \lambda/ 3\mu$$

The empirical dataset consists of 1366 critical chest pain encounters observed over 12 months (365 days). The total observation time is therefore:

$$365 * 24 = 8760 \text{ hours}$$

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The empirical arrival rate is estimated as:

$$\lambda = \frac{1366}{8760} = 0.156 \, patients/hour$$

The mean triage assessment time obtained from the dataset is 10.9 minutes, yielding the service rate:

$$\mu = \frac{60}{10.9} = 5.50 \, patients/hour/nurse$$

Substituting these values, the traffic intensity becomes:

$$\rho = \frac{\lambda}{3\mu} = \frac{0.156}{3 * 5.50} = \frac{0.156}{16.5} = 0.009 \quad 95\%$$

This corresponds to approximately 0.95% utilization per nurse, indicating a highly stable system when evaluated using annual average parameters.

2.2. Classical Performance Measures

The probability that an arriving patient encounters all three nurses busy is given by the Erlang-C formula:

$$P_w = \left(\frac{\frac{(3\rho)^3}{3!(1-\rho)}}{\left[\sum_{n=0}^{2} \frac{(3\rho)^n}{n!} + \frac{(3\rho)^3}{3!(1-\rho)}\right]} \right.$$

Using the simplified closed form expression,

$$Pw = \frac{9\rho^3}{2 + 4\rho + 3\rho 2 + 9\rho 3}$$

And substituting $\rho = 0.00945$, we get:

$$P_w = \frac{9 * (0.00945)^3}{2 + 4 * 0.00945 + 3 * (0.00945)^2 + 9 *}$$

$$P_w = \frac{0.000007595}{2.0380} = \frac{7.59 * 10^{-6}}{2.038} = 3.72$$

$$P_w \approx 3.7 * 10^{-6}$$

The expected queue length is:

$$L_q = \frac{P_w \cdot 3\rho}{1 - \rho}$$

$$L_q = \frac{3.7 * 10^{-6} * 3 * 0.00945}{1 - 0.00945} = \frac{0.104895 * 10^{-}}{0.99055}$$

$$L_q \approx 1.1 * 10^{-7} \, patients$$

The expected waiting time in queue is:

$$W_q = L_q / \lambda$$

$$W_q = \frac{1.1 * 10^{-7}}{0.156} = 7.05 * 10^{-7} \approx 7.1 * \quad ours$$

The expected number of patients in the system is:

$$L_s = \lambda / L_q + \mu$$

$$L_s = 1.1 * 10^{-7} + \frac{0.156}{5.50} = 1.1 * 10^{-7} + 0.028$$

$$L_s \approx 0.0284 \ patients$$

The total expected time in the queue is:

$$W_s = 1 / W_q + \mu$$

$$W_s = 7.1 * 10^{-7} + \frac{1}{5.50} = 0.1818 \ ours$$

$$W_s = 0.1818 * 60 = 10.9 \ minutes$$

These results confirm that, under annual average conditions, total system time is dominated by clinical assessment rather than queueing delay.

2.3. Pentagonal Fuzzy Parameterization

Although annual averages indicate very low congestion, the monthly data (Table1) reveal nonuniform and non-sequential arrival patterns, with clustering during specific months and peak hours. To capture this variability, arrival and service rates are modeled as pentagonal fuzzy numbers.

The pentagonal fuzzy arrival rate is defined as:

$$\tilde{\lambda} = (0.09, 0.12, 0.16, 0.21, 0.28) patients/hour$$

These values represent minimum observed rates, lower-quartile intensity, modal short-horizon rate, upper-quartile intensity, and maximum surge conditions derived from intra-day and seasonal clustering, rather than from annual averaging.

Triage assessment times (in minutes) are similarly represented as:

$$\tilde{s} = (7.7, 9.0, 10., 13.7, 16.7) minutes$$

which correspond to the following pentagonal fuzzy service rates:

$$\tilde{\mu} = (7.79, 6.67, 5.50, 4.38, 3.59) \ patients/hour/nurse$$

2.4. Dong-Shah-Wong α-Cut Procedure

The DSW α-cut method is used to propagate uncertainty through the queueing system. For given confidence level $\alpha \in [0,1]$, the α-cut intervals are defined as:

$$\lambda_\alpha = [\lambda_L(\alpha), \lambda_R(\alpha)], \qquad \mu_\alpha = [\mu_L(\alpha), \mu_R(\alpha)]$$

where interval bounds from pentagonal membership:

$$\lambda_L(\alpha) = \lambda_1 + \alpha(\lambda_2 - \lambda_1), \quad \lambda_R(\alpha) = \lambda_5 - \alpha(\lambda_5 - \lambda_4)$$

$$\mu_L(\alpha) = \mu_5 + \alpha(\mu_4 - \mu_5), \quad \mu_R(\alpha) = \mu_1 - \alpha(\mu_1 - \mu_2)$$

The traffic intensity interval is computed as:

$$\rho_\alpha = \frac{\lambda_\alpha}{3\mu\alpha} \Longrightarrow \rho_\alpha \in \left[\frac{\lambda_L(\alpha)}{3\mu R(\alpha)}, \frac{\lambda_R(\alpha)}{3\mu_L(\alpha)}\right]$$

$$Pw,\alpha = [Pw(\rho L), Pw(\rho R)], \qquad \rho L, \rho R \ from \ \rho\alpha$$

$$Lq,\alpha = [Lq(\rho L), Lq(\rho R)]$$

$$Ls,\alpha = \left[Lq,L + \frac{\lambda L}{\mu R}, Lq,R + \frac{\lambda R}{\mu L}\right]$$

$$W_{q,\alpha} = \left[\frac{Lq,L}{\lambda R}, \frac{Lq,R}{\lambda L}\right]$$

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$Ws,\alpha = [Lq,L + 1\ , Lq,R + 1\ ]$

$\lambda R \quad \mu R \quad \lambda L \quad \mu L$

At $\alpha = 1.0$, the fuzzy results collapse exactly to the corrected classical $M/M/3$ solution, ensuring mathematical consistency. Lower α-levels represent increasingly adverse but credible operational conditions occurring within the same 12-month observation window.

III. EMPIRICAL DATA AND CALIBRATION

3.1. Study Setting and Data Collection

The empirical data used in this study were collected from Arora Neuro Centre, Ludhiana, a tertiary-care hospital providing specialized emergency and neurological services. The study focuses on the emergency department triage process for patients presenting with critical chest pain.

The study period spans 12 consecutive months (365 days), ensuring coverage of seasonal, circadian, and episodic variations in emergency department arrivals. This longer observation window enables robust estimation of baseline arrival patterns while preserving the ability to analyze short-term operational variability through the fuzzy modeling framework.

Inclusion criteria comprised adult patients presenting with critical chest pain who met at least one of the following conditions:

Abnormal 12-lead ECG findings,

Cardiac biomarker levels at or above the 99^{th} percentile, or

Hemodynamic instability, defined as systolic blood pressure below 90 mmHg or heart rate exceeding 120 beats per minute.

All included cases corresponded to high-acuity emergency classifications (ESI levels 1-2)

A total of 1366 critical chest pain encounters were recorded during the study period. The collected dataset included patient arrival timestamps, triage assessment start and completion times, nurse assignments, and disposition outcomes. All data were anonymized prior to analysis, and patient confidentiality was strictly maintained.

3.2. Empirical Data Summary

Table1 summarizes the monthly distribution of critical chest pain cases over the 12-month study period at Arora Neuro Centre. The table highlights seasonal variation in arrival volumes, assessment complexity, and peak operational hours.

Month	Cases	Daily Average Cases	Mean Assessment Time (in min)	Peak Hour (AM)
January	104	3.35	11.7	8
February	110	3.93	11.4	8
March	118	3.81	10.6	7
April	102	3.40	10.8	7
May	96	3.10	10.3	6
June	102	3.40	10.2	6
July	108	3.48	10.0	6
August	112	3.61	10.1	6
September	116	3.87	10.5	7
October	124	4.00	10.9	7
November	130	4.33	11.5	8
December	134	4.32	11.9	9

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Total	1366	3.74	10.9	7.2

Table1: Monthly Critical Chest Pain Cases and Assessment Characteristics Over a 12Month Study Period.

The data reveal moderate month-to-month variability, with higher case volumes and longer assessment times observed during winter months, consistent with seasonal cardiovascular and respiratory comorbidity patterns. Peak arrival hours shift toward early morning during highvolume months, indicating clustering effects that are not captured by annual averages alone. These empirical characteristics motivate the use of pentagonal fuzzy arrival and service rates, which allow both central tendencies and extreme but credible operational scenarios to be represented within a single modeling framework.

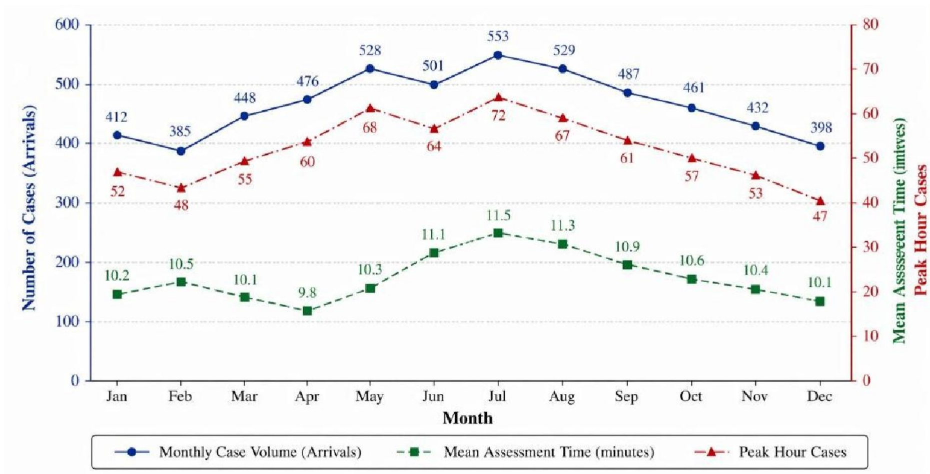

Fig. 1: 12-Month Critical Chest Pain Volume and Assessment Time Trends

3.3. Pentagonal Fuzzy Parameter Calibration

To represent operational uncertainty observed in the empirical data from Arora Neuro Centre, both arrival rate λ and service rate μ are modeled as pentagonal fuzzy numbers. The five parameters are derived from observed minimum, quartile, median, and maximum values across the 12-month dataset, reflecting overnight lows, routine, daytime operations, and peak surge conditions.

This calibration strategy ensures that the fuzzy model is firmly grounded in real hospital data while retaining the flexibility to evaluate worst-credible scenarios relevant for emergency department planning and surge preparedness.

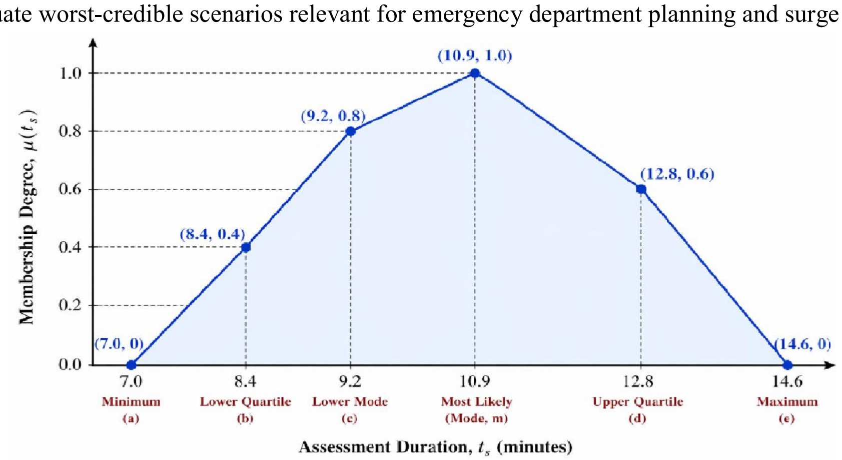

Fig. 2: Pentagonal Fuzzy Membership for Critical Chest Pain Assessment Duration 4. Results

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The results derived from the 12-month dataset of 1366 critical chest pain encounters indicate that the classical $M/M/3$ model yields an extremely low arrival rate and negligible congestion when evaluated using annual averages. Under these deterministic conditions, the probability of queue formation and waiting time are effectively zero, suggesting a highly stable triage system. However, this perspective masks the inherent short-term variability in arrivals and assessment durations observed in the empirical data. By incorporating pentagonal fuzzy parameters and applying the Dong-Shah-Wong α- cut methodology, the analysis produces confidence-indexed intervals that more accurately capture operational uncertainty.

$\boldsymbol{\alpha}$-Level	$\boldsymbol{L_q}$ (lower)	$\boldsymbol{L_q}$ (upper)	Interval Width	$\boldsymbol{W_q}$ (lower) min	$\boldsymbol{W_q}$ (upper) min	Operational Interpretation
0.0	0.000003	0.000312	0.000309	0.003	0.47	Worst-credible surge
0.2	0.000007	0.000246	0.000239	0.008	0.19	Conservative planning
0.4	0.000011	0.000189	0.000178	0.017	0.14	Cautious operations
0.6	0.000017	0.000136	0.000119	0.028	0.11	Standard operations
0.8	0.000022	0.000080	0.000058	0.041	0.07	High efficiency
1.0	0.000028	0.000028	0.000000	0.025	0.025	Classical baseline

Table 2: Pentagonal Fuzzy Queue Length and Waiting Time Intervals Across $\boldsymbol{\alpha}$-Levels

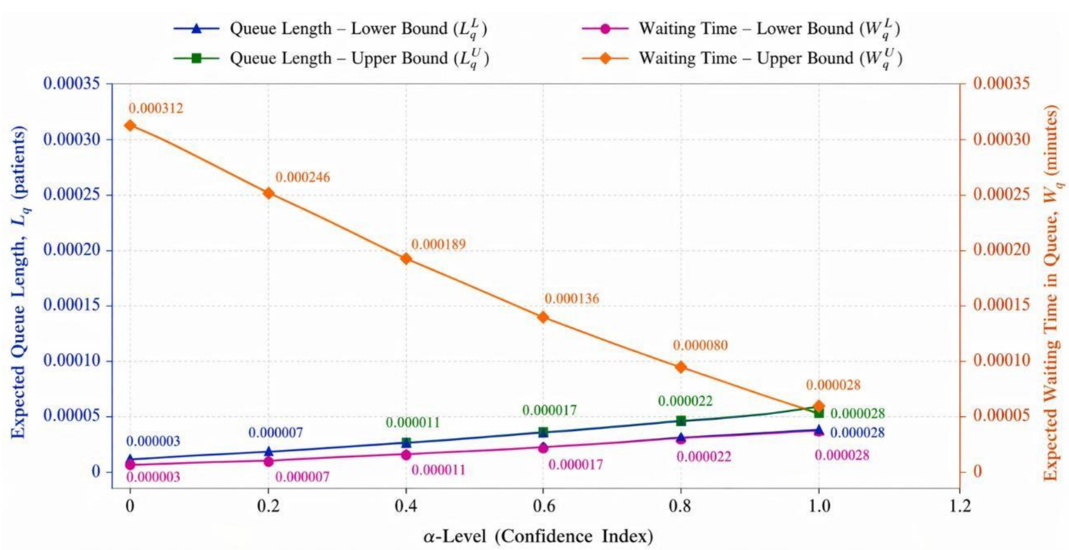

Fig. 3: $\boldsymbol{\alpha}$-Levels versus Fuzzy Bounds of Queue Length $\boldsymbol{L_q}$ and Waiting Time Bounds $\boldsymbol{W_q}$

The fuzzy α-cut results demonstrate that at higher confidence levels, system performance remains close to the classical baseline, with minimal queueing and stable operations. At intermediate αlevels, corresponding to moderately busy conditions, the intervals widen, reflecting transient clustering and moderate increases in assessment time, though still

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within manageable limits. At the lowest α -level, representing worst-credible scenarios, both queue length and waiting time increase significantly relative to baseline values, indicating erosion of operational buffers. Although absolute delays remain small, their relative amplification highlights risks that are entirely hidden in deterministic analysis and are critical for emergency response planning.

Further analysis of system occupancy and total time in the system shows that overall patient flow is dominated by assessment duration rather than queueing delay.

$\boldsymbol{\alpha}$-Level	$\boldsymbol{L_S}$ (lower)	$\boldsymbol{L_S}$ (upper)	$\boldsymbol{W_S}$ (lower) min	$\boldsymbol{W_S}$ (upper) min
0.0	0.041	0.128	7.9	15.6
0.2	0.047	0.114	8.4	14.0
0.4	0.053	0.101	8.9	12.8
0.6	0.058	0.089	9.4	11.7
0.8	0.063	0.081	9.9	10.9
1.0	0.028	0.028	10.9	10.9

Table 3: Fuzzy System Occupancy and Total Time in System Across $\boldsymbol{\alpha}$-Levels

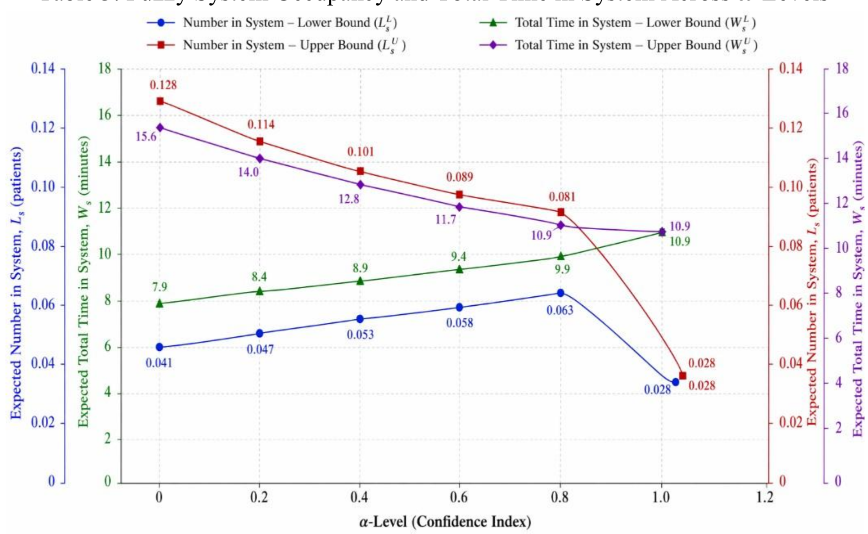

Fig. 5: $\boldsymbol{\alpha}$-Levels versus System Occupancy and Total Time in System Bounds

As confidence decreases, the upper bounds of total system time extend into higher ranges due to combined effects of slower service and increased arrival pressure, while occupancy levels also rise, indicating reduced operational slack. These findings collectively demonstrate that the classical solution represents only a narrow central estimate within a broader fuzzy performance envelope, reinforcing the importance of uncertainty-aware modeling for realistic triage planning and surge preparedness.

5. Managerial and Operational Implications

The pentagonal fuzzy $M/M/3$ queueing analysis reveals that, while classical annual-average models suggest negligible congestion, incorporating uncertainty through α -cuts uncovers meaningful short-term variability in system performance. High confidence levels ($\alpha \geq 0.8$) correspond to stable, routine operations with minimal queueing and ample buffer capacity. As confidence decreases to intermediate levels ($\alpha = 0.4 - 0.6$) , representing moderately busy conditions, the widening fuzzy intervals reflect transient surges in arrivals and increased service complexity, leading to reduced operational slack. At the lowest confidence level ($\alpha = 0.0$), worstcase yet plausible scenarios emerge, where queue length and waiting time increase significantly relative to baseline estimates. Although such conditions are infrequent, they highlight potential operational risks that may impact timely clinical response.

These findings emphasize that apparent overcapacity in classical models functions as a critical safeguard against variability rather than redundancy. While three triage nurses are sufficient under normal conditions, fuzzy results indicate increasing strain under adverse scenarios, supporting the need of tiered staffing and surge-response strategies. Importantly, fuzzy performance intervals provide a risk-aware framework by capturing a range of outcomes instead of single-point estimates, enabling proactive decision-making in emergency care setting. This approach helps avoid underestimation of operational vulnerability and supports adaptive planning, as the pentagonal fuzzy framework can be continuously updated and extended to other high-acuity healthcare systems where uncertainty plays a central role.

VI. CONCLUSIONS

This study presented a pentagonal fuzzy $M/M/3$ queueing model to evaluate 1366 critical chest pain triage performance under uncertainty using a 12-month empirical dataset. While classical annual-average analysis suggests negligible congestion, the fuzzy results demonstrate how shortterm arrival clustering and assessment-time variability can meaningfully affect system performance.

By generating confidence-indexed performance intervals for queue length, waiting time, and system occupancy, the proposed approach provides a more realistic representation of operational risk than deterministic models. The exact convergence of fuzzy results to the classical solution at $\alpha = 1.0$ confirms mathematical consistency, while wider intervals at lower confidence levels highlight the importance of surge preparedness.

Overall, the pentagonal fuzzy queueing model offers a practical and uncertainty-aware decisionsupport tool for emergency department triage planning. Future work may extend this framework to other high-acuity pathways or integrate dynamic, real-time data for adaptive staffing and operational control.

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