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On Distributional Sadik Transform and Boehmian Space

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Abstract: *In present article the author has tried to define Sadik Transform in distributional sense. Some of the properties of Sadik transform are proved. Boehmian Space for Sadik distributional transform is defined along with some basic properties.*

Keywords: Boehmians, Distributions, Sadik transform

Subject Classification: 33C99

I. Introduction

Convolution Function: The convolution of the function [3]$f(t) and g(t)$ is defined as

$$f(t) * g(t) = \int_0^t f(t-u)\, g(u) du.$$

Convolution Theorem: [4] If $S_a\{f(t)\} = F(v^\alpha, \beta) and S_a\{g(t)\} = G(v^\alpha, \beta)$. Then

$$S_a\{f(t) * g(t)\} = S_a\left\{\int_0^t f(t-u)\, g(u) du\right\}$$

$$= v^\beta S_a\{f(t)\} S_a\{g(t)\} = v^\beta F(v^\alpha, \beta) G(v^\alpha, \beta).$$

Properties: [1]

(1) If k_1, k_2 are non-negative real numbers then

$S_a\{k_1 f(t) + k_2 g(t)\} = k_1 S_a\{f(t)\} + k_2 S_a\{g(t)\}$.

(2) $\lim_{t \to 0} f(t) = \lim_{v \to 0} S_a\{f(v)\} = f(0)$.

The Sadik Transform of Distribution:

Let $E(R_+)$ be the space of smooth functions of an arbitrary support on R_+ and $E'(R_+)$be its strong dual of distributions of compact support. Denote $D(R_+)$ as subspace of $E(R_+)$ of test functions of compact support then its dual space $D'(R_+)$ consists of Schwartz distributions.

Clearly, $D \subset E$ and hence $E \subset E' \subset D'$.

The Kernel function $K(v,t) = \frac{e^{-tv^\alpha}}{v^\beta}$ of Sadik transform is a member of $E(R_+)$.

Hence, it will be suitable to define **distributional Sadik Transform** of $f(t) \in E'(R_+)$ as the adjoint operator

$\varphi(v) = \langle f(t), \frac{e^{-tv^\alpha}}{v^\beta} \rangle$ .... (1);

where$v$-complex variable, α-any non-zero real number and β-any real number, for every distribution

$f \in E'(R_+)$.

Theorem 1: φ is well defined mapping in the space$E(R_+)$. As $\frac{e^{-tv^\alpha}}{v^\beta} \in E(R_+)$.

Theorem 2: φ is infinitely smooth and $\frac{d^k}{dv^k}\{\varphi(v)\} = \langle f(t), \frac{d^k}{dv^k}\left(\frac{e^{-tv^\alpha}}{v^\beta}\right)\rangle$, for every$f(t) \in E^{'}(R_+)$.

To prove this theorem we can use Theorem 2.9.1 from [11]

Theorem 3: $\varphi$is linear operator.

Definition 1: Let$f, g \in E^{'}(R_+)$. The generalized convolution between $fandg$ is defined by

$\langle (f * g)(t), \emptyset(t)\rangle = \langle f(t), \langle g(x), \emptyset(t + x)\rangle\rangle$, for every $\emptyset \in E(R_+)$…….. (2)

Using equation (1) and (2), we will get $\varphi(f * g)(v) = v\varphi f(v).\varphi g(v)$.

Theorem 4: Let $f \in E^{'}(R_+)andg(t) = \begin{cases} f(t-a), & t \geq a \\ 0, & t < a \end{cases}$ then

$$S_a\{g(v)\} = e^{-av}\varphi\{f(v)\}$$

Proof: Here, clearly $g \in E^{'}(R_+)$.A translation property of distributions through a[11] implies that

$S_a\{g(v)\} = \langle f(t-a), ve^{-vt}\rangle = e^{-av}\varphi\{f(v)\}$.

{Put $t - a = v \therefore t = v + a,$

$$Now, if t = a then v = 0\ and t = \infty then v = \infty.$$

$$\therefore S_a\{g(v)\} = \langle f(v), ve^{-v(v+a)}\rangle = \langle f(v), ve^{-av}.e^{-v^2}\rangle$$

$$= e^{-av}\langle f(v), ve^{-v^2}\rangle = e^{-av}\varphi\{f(v)\}\}$$

Theorem 5: Let $f \in E^{'}(R_+)$ then

(i) $S_a\big(t.f(t)\big) = -\frac{1}{\alpha v^{\alpha-1}}.\frac{d}{dv}\big(\varphi(v)\big) - \frac{\beta}{\alpha v^\alpha}.\varphi(v)$.

(ii) $S_a\big(t^2.f(t)\big) = (-1)^2\left(\frac{1}{\alpha v^{\alpha-1}}.\frac{d}{dv} + \frac{\beta}{\alpha v^\alpha}\right)^2 \varphi(v)$.

Proof: By using equation (1) and (2) above we have,

$$\frac{d}{dv}\{\varphi(v)\} = \frac{d}{dv}\langle f(t), \frac{e^{-tv^\alpha}}{v^\beta}\rangle = \langle f(t), \frac{d}{dv}\left(\frac{e^{-tv^\alpha}}{v^\beta}\right)\rangle$$

$$= \langle f(t), \frac{d}{dv}\big(v^{-\beta}e^{-tv^\alpha}\big)\rangle$$

$$= \langle f(t), \big(e^{-tv^\alpha}.(-\beta)v^{-\beta-1} + v^{-\beta}.e^{-tv^\alpha}.(-t)\alpha v^{\alpha-1}\big)\rangle$$

$$= \langle f(t), -\frac{\beta}{v^{\beta+1}}.e^{-tv^\alpha} - \alpha v^{\alpha-1}.\frac{1}{v^\beta}.t.e^{-tv^\alpha}\rangle$$

$$= \langle f(t), -\frac{\beta}{v}.\frac{1}{v^\beta}.e^{-tv^\alpha} - \alpha v^{\alpha-1}.\frac{1}{v^\beta}.t.e^{-tv^\alpha}\rangle \text{ [10, 26(2)]}$$

$$= \langle t.f(t), -\alpha v^{\alpha-1}.\frac{1}{v^\beta}.t.e^{-tv^\alpha}\rangle - \langle f(t), \frac{\beta}{v}.\frac{1}{v^\beta}.e^{-tv^\alpha}\rangle$$

$$= -\alpha v^{\alpha-1}\langle t.f(t), \varphi(v)\rangle - \frac{\beta}{v}\langle f(t), \varphi(v)\rangle.$$

Divide by $\alpha v^{\alpha-1}$, we will get

$\frac{1}{\alpha v^{\alpha-1}}.\frac{d}{dv}\{\varphi(v)\} = -S_a\big(t.f(t)\big) - \frac{\beta}{v}S_a\big(f(t)\big).\frac{1}{\alpha v^{\alpha-1}}.$

$\therefore S_a\{t.f(t)\} = -\frac{1}{\alpha v^{\alpha-1}}.\frac{d}{dv}\{\varphi(v)\} - \frac{\beta}{\alpha v^{\alpha}}\{\varphi(v)\}.$

(ii) $S_a\{t^2.f(t)\} = S_a\{t.(t.f(t)\} = \frac{d}{dv}\langle f(t), \frac{d}{dv}\big(v^{-\beta}e^{-tv^{\alpha}}\big)\rangle$

$$= \langle f(t), \frac{d}{dv}\left(\frac{d}{dv}\big(v^{-\beta}e^{-tv^{\alpha}}\big)\right)\rangle$$

$$= \langle f(t), -\frac{1}{\alpha v^{\alpha-1}}.\frac{d}{dv}\left(-\frac{1}{\alpha v^{\alpha-1}}.\frac{d}{dv}\{\varphi(v)\} - \frac{\beta}{\alpha v^{\alpha}}\{\varphi(v)\}\right) - \frac{\beta}{\alpha v^{\alpha}}\left(-\frac{1}{\alpha v^{\alpha-1}}.\frac{d}{dv}\{\varphi(v)\} - \frac{\beta}{\alpha v^{\alpha}}\{\varphi(v)\}\right)\rangle$$

$$= \langle f(t), -\frac{1}{\alpha v^{\alpha-1}}\left[-\frac{1-\alpha}{\alpha v^{\alpha}}.\frac{d}{dv}\{\varphi(v)\} - \frac{1}{\alpha v^{\alpha-1}}.\frac{d^2}{dv^2}\{\varphi(v)\} - \left(\frac{\beta(-\alpha)}{\alpha v^{\alpha+1}}\{\varphi(v)\} - \frac{\beta}{\alpha v^{\alpha}}.\frac{d}{dv}\{\varphi(v)\}\right)\right]$$

$$- \frac{\beta}{\alpha v^{\alpha}}\left[-\frac{1}{\alpha v^{\alpha-1}}.\frac{d}{dv}\{\varphi(v)\} - \frac{\beta}{\alpha v^{\alpha}}\{\varphi(v)\}\right]\rangle$$

$$= \langle f(t), \frac{1-\alpha}{\alpha^2 v^{2\alpha-1}}.\frac{d}{dv}\{\varphi(v)\} + \frac{1}{\alpha^2 v^{2\alpha-2}}.\frac{d^2}{dv^2}\{\varphi(v)\} - \frac{\beta}{\alpha v^{2\alpha}}\{\varphi(v)\} + \frac{\beta}{\alpha^2 v^{2\alpha-1}}.\frac{d}{dv}\{\varphi(v)\} + \frac{d}{dv}\{\varphi(v)\}.\frac{\beta}{\alpha^2 v^{2\alpha-1}}$$

$$+ \frac{\beta^2}{\alpha^2 v^{2\alpha}}\{\varphi(v)\}\rangle$$

$= \langle f(t), \frac{1}{\alpha^2 v^{2\alpha-2}}.\frac{d^2}{dv^2}\{\varphi(v)\}\rangle + \langle f(t), \frac{1-\alpha+2\beta}{\alpha^2 v^{2\alpha-1}}.\frac{d}{dv}\{\varphi(v)\}\rangle + \langle f(t), \left(\frac{\beta^2}{\alpha^2 v^{2\alpha}} - \frac{\beta}{\alpha v^{2\alpha}}\right)\{\varphi(v)\}\rangle.$

By properties of distributions, we have

$$S_a\{t^2.f(t)\} = \frac{1}{\alpha^2 v^{2\alpha-2}}\langle t^2.f(t), \varphi(v)\rangle + \frac{1-\alpha+2\beta}{\alpha^2 v^{2\alpha-1}}\langle t.f(t), \varphi(v)\rangle + \left(\frac{\beta^2}{\alpha^2 v^{2\alpha}} - \frac{\beta}{\alpha v^{2\alpha}}\right)\langle f(t), \varphi(v)\rangle$$

$$= (-1)^2\langle f(t), \left[\frac{1}{\alpha v^{\alpha-1}}.\frac{d}{dv} + \frac{\beta}{\alpha v^{\alpha}}\right]^2 \varphi(v)\rangle$$

$$= (-1)^2\left(\frac{1}{\alpha v^{\alpha-1}}.\frac{d}{dv} + \frac{\beta}{\alpha v^{\alpha}}\right)^2 \varphi(v).$$

Theorem 6: (Shifting property) Let$f \in E'(R_+)$ then $S_a\{e^{at}.f(t)\} = \frac{(v^{\alpha}-a)^{\frac{\beta}{\alpha}}}{v^{\beta}}\varphi(v^{\alpha}-a)$

Proof: $S_a\{e^{at}.f(t)\} = \langle f(t), e^{at}.\frac{e^{-tv^{\alpha}}}{v^{\beta}}\rangle$

$$= \langle f(t), \frac{1}{v^{\beta}}e^{-t(v^{\alpha}-a)}f(t)\rangle$$

$$= \langle f(t), \int_0^t \frac{1}{v^{\beta}}e^{-t(v^{\alpha}-a)}f(t)dt\rangle.$$

Put $v^{\alpha} - a = u^{\alpha} \therefore v^{\alpha} = u^{\alpha} + a \Rightarrow v = (u^{\alpha}+a)^{\frac{1}{\alpha}}$

$\therefore v^{\beta} = (u^{\alpha}+a)^{\beta/\alpha}.$

$$\therefore S_a\{e^{at}.f(t)\} = \langle f(t), \frac{1}{v^{\beta}}\int_0^t e^{-tu^{\alpha}}.f(t)dt\rangle$$

$$= \frac{1}{v^{\beta}}\langle f(t), \int_0^t e^{-tu^{\alpha}}.f(t)dt\rangle$$

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$$= \frac{1}{v^\beta}\langle f(t), u^\beta \varphi(u)\rangle$$
$$= \frac{u^\beta}{v^\beta}\langle f(t), \varphi(u)\rangle$$
$$= \frac{(v^\alpha - a)^{\frac{\beta}{\alpha}}}{v^\beta}\varphi(v^\alpha - a)\text{is proved.}$$

Boehmians:

Consider a linear space V and U a subspace of V. For each pair of members, $u \in V and$ $v \in U$, define a product $u * v$ satisfying the conditions

(i) $v, w \in U \Rightarrow v * w \in U and v * w = w * v$.

(ii) $u \in V and v, w \in U \Rightarrow (u * v) * w = u * (v * w)$

(iii) $If u_1, u_2 \in V, v \in U and k \in \mathbb{R}$.Then$(u_1 + u_2) * v = u_1 * v + u_2 * v$and $k(u_1 * v) = (ku_1) * v$.

Now, if Δ denotes the family of sequences from U which satisfy

(i) If $u_1, u_2 \in V, (x_n) \in \Delta and u_1 * x_n = u_2 * x_n, n \in \mathbb{N}$ then $u_1 = u_2$.

(ii) $(x_n), (y_n) \in \Delta \Rightarrow x_n * y_n \in \Delta$.Then here every member of Δ is called a δ-sequence.

Let $S = \{((x_n), (z_n)) : (x_n) \subseteq V^N, (z_n) \in \Delta\}$ be a class of pairs of sequences for each $n \in \mathbb{N}$.

Here, an element $((x_n), (z_n)) \in S$ denoted by $\frac{x_n}{z_n}$ is a quotient of sequences when

$x_i * z_j = x_j * z_i, \forall i, j \in \mathbb{N}$.

Two quotient of sequences $\frac{x_n}{z_n} and \frac{y_n}{\emptyset_n}$ are said to be equivalent if $x_i * z_j = y_j * \emptyset_i, \forall i, j \in \mathbb{N}$;

denoted by $\frac{x_n}{z_n} \sim \frac{y_n}{\emptyset_n}$.

Here, $\sim$ is an equivalence relation on S and hence induces the equivalence classes of S.

Denotes the equivalence class of $\frac{x_n}{z_n}$by$\left[\frac{x_n}{z_n}\right]$. These equivalence classes are called Boehmians.

We shall denote the space of all Boehmians byB_H.

Now, we can define the sum of two Boehmians and multiplication by a scalar as-

$\left[\frac{x_n}{z_n}\right] + \left[\frac{y_n}{\emptyset_n}\right] = \left[\frac{(x_n * \emptyset_n) + (y_n * z_n)}{z_n * \emptyset_n}\right]and\alpha \left[\frac{x_n}{y_n}\right] = \left[\frac{\alpha x_n}{y_n}\right], \alpha \in C$.

Also the product operation * and the differentiation can be defined by

$\left[\frac{x_n}{y_n}\right] * \left[\frac{z_n}{\emptyset_n}\right] = \left[\frac{x_n * z_n}{y_n * \emptyset_n}\right]$and $D^\alpha \left[\frac{x_n}{y_n}\right] = \left[\frac{D^\alpha x_n}{y_n}\right]$.

The linear space V has a notion of convergence. The natural relationship between the product * and the convergence is given by

1. If $x_n \to x as n \to \infty$ in V and for any fixed element u of U, $x_n * u \to x * u$ in V as $n \to \infty$.
2. If $x_n \to x as n \to \infty$ in V and $(\delta_n) \in \Delta$then$x_n * \delta_n \to x in V as n \to \infty$.

Definition 2: If $\left[\frac{x_n}{y_n}\right] \in B_H and u \in U$then$\left[\frac{x_n}{y_n}\right] * u = \left[\frac{x_n * u}{y_n}\right]$.

This is an extension of * operation to $B_H \times U$.

Two types of convergence $\delta and \Delta$ on $B_H$as

Definition 3: A sequence of Boehmians $(\alpha_n) \in B_H$ is said to be δ-convergent to α if there exists a δ- sequence $(\emptyset_n)$ such that $(\alpha_n * \emptyset_n), (\alpha * \emptyset_n) \in S, \forall n \in \mathbb{N} and$

$(\alpha_n * \emptyset_k) \to (\alpha * \emptyset_k) in S as n \to \infty, \forall k \in \mathbb{N}.$

It is denoted by $\alpha_n \xrightarrow{\delta} \alpha$.

Definition 4: A sequence of Boehmians $(\alpha_n) \in B_H$ is said to be Δ-convergent to α if there exists a δ- sequence $(\emptyset_n) \in \Delta$ such that$(\alpha_n - \alpha) * \emptyset_n \in S, \forall n \in \mathbb{N} and (\alpha_n - \alpha) * \emptyset_n \to 0 \; in S as n \to \infty$.

It is denoted by $\alpha_n \xrightarrow{\Delta} \alpha$.

The Sadik Transform of Boehmian:

Let$V = L^1(R_+) \; and \; U = D(R_+)$, and Δ be the collection of sequences (x_n) from$U = D(R_+)$ such that

1. $\int_{R_+} x_n(t)dt = 1$

2. $\|x_n\|_{L^1} < M$, for all $(x_n) \in \Delta$ and some constant $M > 0$.

3.$\int_{|x|>\varepsilon} |x_n(t)|dt \to 0 \; as \; n \to \infty, \varepsilon > 0.$

Then the space of Boehmians $B_H(L^1, D, *, \Delta)$ is convolution algebra with

(i) $\left[\frac{x_n}{z_n}\right] + \left[\frac{y_n}{\emptyset_n}\right] = \left[\frac{(x_n*\emptyset_n)+(y_n*z_n)}{z_n*\emptyset_n}\right]and\alpha \left[\frac{x_n}{y_n}\right] = \left[\frac{\alpha x_n}{y_n}\right], \alpha \in R$

(ii) $\left[\frac{x_n}{y_n}\right] * \left[\frac{z_n}{\emptyset_n}\right] = \left[\frac{x_n*z_n}{y_n*\emptyset_n}\right]$

(iii)$D^k \left[\frac{x_n}{y_n}\right] = \left[\frac{D^k x_n}{y_n}\right], k \in \mathbb{N}.$

If $(x_n) \in \Delta$ then $S_a x_n(t) \to t$ uniformly on compact subsets of $R_+ \; as \; n \to \infty$.

Theorem 7: If $f_n \in L^1$ such that $\left[\frac{f_n}{g_n}\right] \in B_H(L^1, \; D, \; *, \; \Delta)$ then $S_a f_n(v) = \int_0^\infty v e^{-\frac{t}{v}} f_n(t)\, dt$ converges uniformly on each compact set of R_+.

Proof: Since $S_a g_n \to v as n \to \infty$ on compact subsets of $R_+, S_a g_n > 0$ for almost all $k \in \mathbb{N}$ and hence

$S_a f_n(v) = S_a f_n(v).\frac{S_a g_k(v)}{S_a g_k(v)} = \frac{v S_a (f_n * g_k)(v)}{S_a g_k(v)} = \frac{v S_a (f_k * g_n)}{S_a g_n(v)} = \frac{S_a f_k(v)}{S_a g_k(v)}.S_a g_n(v)onK$;

where$K$ is some compact subset of R_+.

Now, taking limit as$n \to \infty$, we will get$S_a f_n(v) \to \frac{v S_a f_k(v)}{S_a g_k(v)}$.

We can define the Sadik transform of $\beta \in B_H(L^1, \; D, \; *, \; \Delta)$; $\beta = \left[\frac{f_n}{g_n}\right]$ by $\mathfrak{S}\beta = \lim_{n \to \infty} S_a f_n$ on compact subset of R_+.

Claim: To prove that the definition is well defined.

If possible let $\beta_1 = \beta_2$; where $\beta_1 = \left[\frac{f_n}{g_n}\right] and \beta_2 = \left[\frac{h_n}{\delta_n}\right]$ then$f_n * \delta_m = h_m * g_n = h_n * g_m$. Applying Sadik transform on both sides, we will get $v.S_a f_n(v) S_a \delta_m(v) = v.S_a h_n(v) S_a g_m(v)$.

Now, as $n \to \infty \lim_{n\to\infty} S_a f_n(v) = \lim_{n\to\infty} S_a h_n(v) \Rightarrow \mathfrak{S}\beta_1 = \mathfrak{S}\beta_2$.

Theorem 8: Let $u_1, u_2 \in B_H(L^1,\ D,\ *,\ \Delta)$ and $k \in \mathbb{C}$ then

(i) $\mathfrak{S}(u_1 + u_2) = \mathfrak{S}(u_1) + \mathfrak{S}(u_2)$

(ii) $\mathfrak{S}(ku_1) = k\mathfrak{S}(u_1)$

(iii) $\mathfrak{S}(u_1 * f_n) = \mathfrak{S}(f_n * u_1) = v.\mathfrak{S}(u_1)$

(iv)$\mathfrak{S}(u_1) = 0\ \Rightarrow u_1 = 0$.

(v) $f_n \to f \in B_H(L^1,\ D,\ *,\ \Delta) \Rightarrow \mathfrak{S}f_n \to \mathfrak{S}f \in B_H(L^1,\ D,\ *,\ \Delta)asn \to \infty$ on compact subsets.

Proof: We can prove (i), (ii) and (iv) using the linearity property of Sadik transform.

To prove (iii) Let$u_1 \in B_H(L^1,\ D,\ *,\ \Delta)$ such that $u_1 = \left[\frac{f_n}{g_n}\right]$ then $u_1 * g_n = \left[\frac{f_n * g_n}{g_n}\right]$.

Hence, $\mathfrak{S}(u_1 * g_n) = v.\lim_{n\to\infty} S_a f_n(v) = v.\mathfrak{S}(u_1)$.

Also we will prove (v) using proof of Theorem 2 (f) from[2].

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