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Magnetohydrodynamic Flow of an Upper-Convected Maxwell Fluid over a Porous Stretching Surface in Presence of Second-Order Slip

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Abstract: *This paper presents an analytical investigation of steady laminar magnetohydrodynamic (MHD) slip flow of an upper-convected Maxwell (UCM) viscoelastic fluid over a porous isothermal stretching sheet. The formulation accounts for the combined effects of elasticity parameter, magnetic interaction, suction/injection, and velocity slip. The highly nonlinear governing equations are treated using a BVPh 2.0 package. The analytical results are validated through comparison with previously reported studies, showing excellent agreement. The BVPh 2.0 package demonstrates strong capability for solving nonlinear boundary-layer equations.*

Keywords: Magnetohydrodynamics; Upper-convected Maxwell fluid; Porous stretching sheet; second-order velocity slip

1. Introduction

The mechanical response of fluids is fundamentally governed by constitutive relations that link stress to deformation history. For classical Newtonian fluids, these relations reduce to the Navier–Stokes equations, which adequately describe a wide range of simple liquids. However, numerous materials encountered in industrial processing and biological transport exhibit rheological behaviors that deviate substantially from Newtonian assumptions. Polymeric melts, drilling muds, lubricants, suspensions, and biofluids often display elasticity, memory effects, and shear-dependent viscosity, necessitating the use of non-Newtonian models.

Over the past several decades, various constitutive formulations have been proposed to capture such complex behavior. Models including second-grade, Maxwell, Oldroyd-B, and upper-convected Maxwell (UCM) fluids have been widely analyzed due to their mathematical tractability and physical relevance [1-9]. Among these, the UCM model is particularly significant because it incorporates stress relaxation and elastic memory while remaining sufficiently structured for analytical development.

Another physically realistic consideration is second-order velocity slip at the solid boundary. In micro-scale systems, polymer processing, and rarefied gas dynamics, the classical no-slip condition becomes inadequate. Instead, partial slip proportional to shear stress is often more appropriate [10]. The combined inclusion of slip, suction/injection, porosity, and magnetic field effects significantly enriches the mathematical structure of the governing equations.

Motivated by these considerations, the present work study an analytical solution for MHD flow of a UCM fluid over a porous stretching sheet in presence of second-order slip condition using Mathematica-based package BVPh 2.0 [11], which is specifically designed for nonlinear boundary-value and eigenvalue problems involving coupled ODE systems. The computational framework is built upon the homotopy analysis method (HAM) [12, 13], an established analytical technique for handling strongly nonlinear differential equations.

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The BVPh 2.0 package [11] represents a HAM-based computational platform that facilitates the analytical treatment of nonlinear boundary-value and eigenvalue problems described by coupled ordinary differential equations. The software is freely accessible online (http://numericaltank.sjtu.edu.cn/BVPh.htm).

2. Problem Formulation

We consider steady, two-dimensional boundary-layer flow of an incompressible, electrically conducting upper-convected Maxwell (UCM) fluid induced by a linearly stretching porous sheet. The fluid is initially quiescent. The constitutive relation for the UCM model is defined through the Cauchy stress tensor as follows:

$$T = -pI + S \tag{1}$$

Here, p denotes pressure, I is the identity tensor, and S represents the extra stress tensor.
For an upper-convected Maxwell fluid, the extra stress tensor satisfies:

$$S + \lambda\left(\frac{dS}{dt} - LS - SL^T\right) = \mu A_1 \tag{2}$$

In the above relation, λ is the relaxation time parameter, μ is the dynamic viscosity, L is the velocity gradient tensor, and A_1 denotes the first Rivlin–Ericksen tensor. The material derivative $\frac{dS}{dt}$ accounts for convected transport effects in the viscoelastic fluid.
Here, $L = \nabla V$ represents the velocity gradient tensor, where V denotes the velocity vector and $|V|$ its magnitude. The parameter μ is the dynamic viscosity, λ is the relaxation time, and A_1 is the first Rivlin–Ericksen tensor, defined as follows:

$$A_1 = L + L^T \tag{3}$$

The governing boundary-layer equations are formulated by incorporating magnetic body forces and porous medium resistance. Appropriate slip boundary conditions are imposed at the stretching surface, while ambient quiescent conditions are enforced far from the wall.

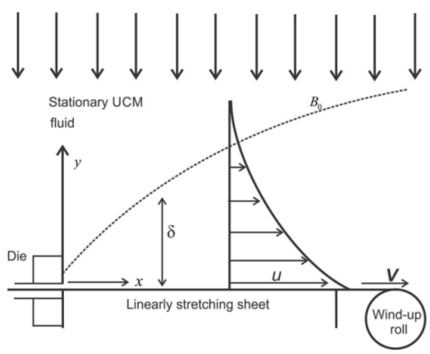

Fig. 1: Physical Model

For steady two-dimensional magnetohydrodynamic flow, the governing continuity and momentum equations are expressed as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x}\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y} - \sigma {B_0}^2 u \tag{5}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y}\frac{\partial S_{yx}}{\partial x} + \frac{\partial S_{yy}}{\partial y} \tag{6}$$

In these expressions, ρ denotes fluid density, σ represents electrical conductivity, B_0 is the imposed transverse magnetic field, and S_{xx}, S_{xy}, S_{yx}, and S_{yy} are components of the extra stress tensor.

Under standard boundary layer scaling assumptions [1], the following order-of-magnitude relations are introduced:

$$u = O(1), v = O(\delta), x = O(1), y = O(\delta) \tag{7}$$

$$\frac{T_{xx}}{\rho} = O(1), \frac{T_{xy}}{\rho} = O(\delta), \quad \frac{T_{yy}}{\rho} = O(\delta^2) \tag{8}$$

Neglecting pressure gradient effects, the reduced boundary-layer equation becomes:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left[u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right] = \nu\frac{\partial^2 u^2}{\partial y} - \left(\frac{\sigma B_0^2}{\rho}\right)u \tag{9}$$

Here, δ denotes the boundary layer thickness.

The corresponding boundary conditions are given by:

$$u = Bx + l_1(x)\frac{\partial u}{\partial y} + l_2(x)\frac{\partial^2 u}{\partial y^2}, \quad v = -V_0 \ at \ y = 0; \ u \to 0 \ as \ y \to \infty \tag{10}$$

The parameter V_0 represents the normal mass flux at the wall. Positive values ($V_0 > 0$) correspond to suction, whereas negative values ($V_0 < 0$) indicate injection. The function $l_1(x)$ and $l_2(x)$ denotes the slip coefficient.

To transform the governing partial differential equations into an ordinary differential equation, we introduce the following similarity variables:

$$\eta = \sqrt{\frac{B}{\nu}}\, y, \quad u = Bx\, f'(\eta), \quad v = -\sqrt{\nu B}\, f(\eta) \tag{11}$$

Substituting these similarity transformations into the boundary-layer model leads to the following nonlinear ordinary differential equation:

$$f''' - M^2 f' - (f')^2 + f f'' + \beta\,(2 f f' f'' - f^2 f''') = 0 \tag{12}$$

The corresponding boundary conditions are:

$$f(0) = R, \quad f'(0) = 1 + L_1 f''(0) + L_2 f'''(\eta), \quad f'(\eta) \to 0 \ as \ \eta \to \infty \tag{13}$$

Here, $M^2 = (\sigma B_0{}^2)/(\rho B)$ denotes the magnetic parameter, $\beta = \lambda B$ is the elasticity number, and B is the stretching rate. The dimensionless slip parameter is $L_1 = l(x)\sqrt{(B/\nu)}$, while $R = V_0 / \sqrt{(\nu B)}$ represents the wall mass transfer parameter. Positive R corresponds to suction and negative R to injection. The similarity formulation is consistent provided that the slip parameter does not explicitly depend on x, implying that $l(x)$ must be proportional to x.

The resulting equation forms a highly nonlinear similarity boundary value problem, which is solved analytically using BVPh 2.0.

3. Analytical Solution via the Homotopy Analysis Method (HAM)

The Homotopy Analysis Method (HAM) provides a powerful semi-analytical framework for solving nonlinear boundary-value and eigenvalue problems. In 2012, Liao introduced the computational package BVPh 2.0, designed for treating coupled nonlinear ordinary differential equations arising in boundary-layer theory. The present nonlinear system governing the flow problem can be systematically handled within this framework. Only the essential steps relevant to the current formulation are outlined below.

Within BVPh 2.0, it is first necessary to define appropriate auxiliary linear operators corresponding to each governing equation, together with the associated boundary conditions. The software allows the use of high-order deformation equations and provides flexibility in selecting convergence-control parameters to ensure series convergence. These parameters are typically determined through minimization of the residual errors of the governing equations.

Following the standard HAM procedure, the solutions for the dimensionless velocity function $f(\eta)$ and temperature (or auxiliary) function $\theta(\eta)$ are expressed in the form of infinite series expansions:

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$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \tag{22}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) \tag{23}$$

The higher-order components $f_m(\eta)$ and $\theta_m(\eta)$ are obtained from the m-th order deformation equations. In accordance with the boundary conditions at infinity, the solution structure is chosen as:

$$f(\eta) = \sum_{m=0}^{\infty} \sum_{k=1}^{\infty} a_{m,k}\, \eta^k e^{m\eta} \tag{24}$$

$$\theta(\eta) = \sum_{m=0}^{\infty} \sum_{k=1}^{\infty} b_{m,k}\, \eta^k e^{m\eta}, \tag{25}$$

Here $a_{m,k}$ and $b_{m,k}$ denote unknown coefficients computed iteratively. These expressions guide the selection of suitable auxiliary linear operators.

We select the following linear operators:

$$\mathcal{L}_1(f) = \frac{d^3 f}{d\eta^3} + \frac{d^2 f}{d\eta^2} \tag{26}$$

$$\mathcal{L}_2(\theta) = \frac{d^2\theta}{d\eta^2} - \theta \tag{27}$$

These operators satisfy the properties:

$$\mathcal{L}_1(c_1 + c_2\eta + c_3 e^{-\eta}) = 0 \tag{28}$$

$$\mathcal{L}_2(c_4 e^{\eta} + c_5 e^{-\eta}) = 0 \tag{29}$$

where c_1, c_2, c_3, c_4, and c_5 are arbitrary constants. To initiate the deformation process, the following initial approximations are selected in agreement with the boundary conditions:

$$f_0(\eta) = (1 - \alpha_1)(e^{-\eta} - 1) + \eta + s \tag{30}$$

$$\theta_0(\eta) = e^{-\eta} \tag{31}$$

The auxiliary functions are chosen as:

$$H_f(\eta) = e^{-\eta} \tag{32}$$

$$H_\theta(\eta) = e^{-\eta} \tag{33}$$

With these definitions of auxiliary operators, initial guesses, and auxiliary functions, the BVPh 2.0 package systematically constructs the convergent analytical series solution of the coupled nonlinear system.

4. Results and Discussion

The skin-friction coefficient $f''(0)$ has been computed for different values of the elasticity parameter (β), magnetic parameter (M), and mass transfer parameter (R) using BVPh 2.0 package. To assess the reliability of the present analytical solution, the results have been compared with previously published data reported by Raftari and Yildirim [2] and Saadatmandi and Sanatkar [3] for the case $L_1 = L_2 = 0$.

The comparison confirms that the present results are in very close agreement with existing benchmark solutions.

Table 1: **Comparison of $f''(0)$ values obtained using BVPh 2.0 package with HPM, and RLC methods ($R = 0.3, L_1 = L_2 = 0$)**

β	M	Present Values	HPM	RLC
0	0	-1.161176	-1.161187	-1.16142682
	5	-5.25124	-5.251225	-5.25123645
5	0	-7.634	-7.633621	-7.0732289
	5	-10.979	-11.22855	-10.8421099
10	0	-64.4364	-64.418655	-64.0715002
	5	-67.8152	-68.023447	-67.6691104

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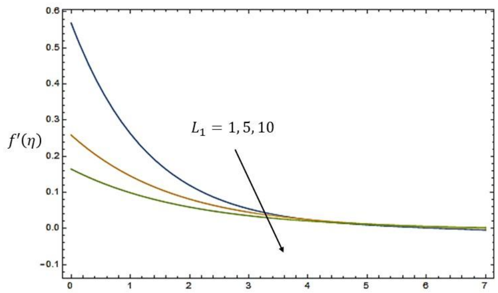

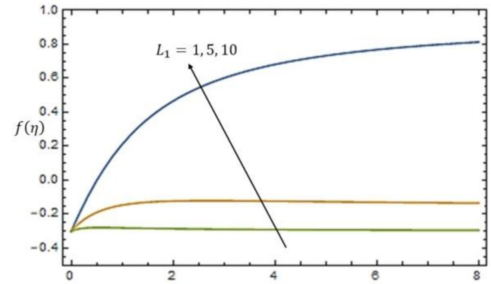

Fig. 2: Effect of slip parameter on vertical velocity **Fig. 3:** Effect of slip parameter on horizontal velocity

The graphical illustrations (Figures 2-3) further demonstrate the influence of the governing parameters on the non-dimensional velocity profiles. The variations of first-order slip parameter (L_1) and second-order slip parameter (L_2) are systematically analyzed.

Figure 2 demonstrate the effect of the slip parameter (L_1) on the velocity components while keeping $M, \beta,$ and R fixed. For both Newtonian and UCM fluids, increasing L_1 reduces the magnitude of velocity throughout the boundary layer. A higher slip parameter indicates weaker momentum transfer from the stretching surface to the fluid, resulting in lower fluid acceleration. Additionally, for a given value of L_1, velocity components are generally smaller for UCM fluids compared to Newtonian fluids. The reduction in horizontal velocity due to slip is more pronounced under injection conditions than suction.

5. Concluding Remarks

In this study, the HAM based BVPh 2.0 package approach has been employed to derive approximate analytical solutions of the nonlinear similarity equations governing magnetohydrodynamic (MHD) flow of an upper-convected Maxwell (UCM) fluid over a porous stretching surface. The principal conclusions of the present investigation are summarized as follows:

The BVPh 2.0 package results exhibit excellent agreement with previously published analytical and semi-analytical solutions obtained using HPM, and RLC methods.

Larger slip parameter values reduce the momentum transfer from the stretching surface to the fluid, thereby lowering the velocity profiles under both suction and injection conditions.

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