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Fixed Point Theorems in Partial Fuzzy Metric Spaces

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Abstract: *This paper explores the research on fixed points in the context of partial fuzzy metric spaces, a patching of fuzzy metric spaces and partial metric spaces. In partially ordered fuzzy partial metric spaces a number of new fixed point theorems of contractive mappings are proved. The findings generalize the classical Banach contraction principle and generalize a number of existing findings in the fuzzy metric theory. The presence and the uniqueness of the fixed points when the generalized conditions of contracting are considered are established. They also give applications of the obtained results to fuzzy differential equations, optimization problems, and uncertainty decision-making models. The above findings help the development of theories of fuzzy analysis, as well as give practical tools in resolving nonlinear problems in an uncertain environment [1].*

Keywords*:* Fixed Point Theory, Fuzzy Metric Space, Partial Fuzzy Metric Space, Contractive Mapping, Banach Contraction Principle

I. INTRODUCTION

The fixed point theory has now become an important subdivision of nonlinear functional analysis because it has been used extensively in numerous fields of mathematics and the applied sciences. Fixed point problems can be described in terms of many of the mathematical models of engineering, economics and computer science. The classical Banach contraction principle is used to establish the existence and uniqueness of the fixed points of contraction mappings of complete metric spaces [2].

But classical metric spaces do not have enough to represent the world of uncertainty and vagueness. In order to solve such instances, the idea of fuzzy sets, proposed by Zadeh in 1965 has been used extensively in mathematical modeling. In 1975 Kramosil and Michálek introduced the concept of fuzzy metric spaces and were later adapted by George and Veeramani as continuous t-norms.

Researchers have over the years come up with various generalizations of fuzzy metric spaces such as intuitionistic fuzzy metric spaces, probabilistic metric space, and partial metric space. The term partial metric spaces proposed by Matthews are used to permit the distance between a point and the point itself to be non-zero. This is the property which renders partial metric spaces especially useful in domain theory and computer science.

The fuzzy metric spaces coupled with the partial metric spaces give the birth of the concept of partial fuzzy metric spaces, which can offer a more versatile method of investigating convergence and fixed point properties in uncertain settings.

This study aims at determining new fixed point theorems in partial fuzzy metric spaces and generalizing classical contraction principles to partially ordered constructions. The received results extend a range of existing theorems and give new perspectives of research in fuzzy analysis [3].

II. PRELIMINARIES

Here it is possible to remember some basic concepts and definitions that are basic in the development of the results that are being presented in this paper. These ideas are based on fuzzy set theory, fuzzy metric spaces and extensions of these

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concepts. The theory of the fuzzy metric spaces was proposed to represent uncertainty and vagueness of mathematical systems where the classical metric structures are inadequate. Subsequent to this was the expansion of these concepts to include structures like partial metric spaces which permit the distance between a point and itself to be non-zero. The two concepts combined give rise to the notion of partial fuzzy metric spaces which offer a versatile framework of studying convergence and fixed point phenomena in uncertain mathematical systems [4].

Definition 2.1 (Continuous t-norm)
A binary operation

$$*: [0,1] \times [0,1] \to [0,1]$$

Is called a **continuous triangular norm (t-norm)** if it satisfies the following properties for all $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in [\mathbf{0}, \mathbf{1}]$:
Associativity:

$$(a * b) * c = a * (b * c)$$

Commutativity:

$$a * b = b * a$$

Continuity:
The function $*$ is continuous on the unit square $[0,1] \times [0,1]$.
Identity Element:

$$a * 1 = a$$

For all $a \in [0,1]$.
Monotonicity:
If $a \leq b$ and $c \leq d$, then

$$a * c \leq b * d$$

The theory of fuzzy metric spaces uses continuous t-norms due to the fact that they substitute the classical addition operation in classical metric spaces. The triangular inequality is defined in fuzzy metric spaces with the t-norm in lieu of the usual addition. The minimum t-norm, the product t-norm and the Łukasiewicz t-norm are some of the most used examples of continuous t-norms. These are so as to enable the modelling of degrees of proximity between elements as opposed to actual distance which is especially handy when handling unclear or inaccurate information [5].

Definition 2.2 (Fuzzy Metric Space)
Let X is a non-empty set and let $*$ be a continuous t-norm. A triple

$$(X, M, *)$$

Is called a **fuzzy metric space** if the function

$$M: X \times X \times (0, \infty) \to [0,1]$$

Satisfies the following conditions for all $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \boldsymbol{X}$ and $\boldsymbol{t}, \boldsymbol{s} > \mathbf{0}$:
Positivity:

$$M(x, y, t) > 0$$

Identity of Indiscernibles:
$M(x, y, t) = 1$ If and only $x = y$
Symmetry:

$$M(x, y, t) = M(y, x, t)$$

Triangle Inequality:

$$M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$$

Continuity: The function $M(x, y, t)$ is continuous with respect to t.
$M\ (x, y, t)$ is the measure of closeness between points x and y in relation to the parameter t. Fuzzy metric spaces value in the interval $[0,1]$ to measure the proximity as opposed to classical metric space where the distance is a non-negative

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real number. A value near 1 shows that the two points are very similar or in close proximity but the lower the value, the less similar the two points.

Fuzzy metric spaces are useful in offering a solid mathematical approach in addressing the problem of uncertainty, incomplete information, or vagueness. Consequently, they have been used in a number of areas such as decision theory, control systems, image processing, and artificial intelligence. Fixed point theory using fuzzy metric spaces makes it possible to generalize classical contraction principles to where distances cannot be precisely defined [6].

Definition 2.3 (Partial Fuzzy Metric Space)

Let X is a non-empty set and let $*$ be a continuous t-norm. A function

$$Fp: X \times X \times (0, \infty) \to [0,1]$$

Is called a **partial fuzzy metric** if for all $\boldsymbol{x, y, z \in X}$ and $\boldsymbol{t, s > 0}$, the following conditions are satisfied:

Non-zero Self-Distance:

$$Fp(x, x, t) \geq Fp(x, y, t)$$

This condition distinguishes partial fuzzy metric spaces from ordinary fuzzy metric spaces because the self-distance of a point is not necessarily equal to 1.

Symmetry:

$$Fp(x, y, t) = Fp(y, x, t)$$

Generalized Triangle Inequality:

$$Fp(x, z, t + s) \geq Fp(x, y, t) * Fp(y, z, s)$$

Continuity:

The function $Fp(x, y, t)$ is continuous in the variable t.

Small Self-Distance Property:

$Fp(x, x, t) = Fp(x, y, t) = Fp(y, y, t)$ Implies $x = y$

A pair $(X, Fp, *)$ satisfying the above conditions is called a **partial fuzzy metric space**.

The most important characteristic of a partial fuzzy metric space is that the self-distance of a point can be not equal to zero (or equal to one, in the fuzzy terms). This property makes it possible to model the situation when an element can have incomplete information about self-content. These structures find application specifically in computer science, domain theory and information systems where objects can be incomplete data or partial computations [7].

The partial fuzzy metric spaces are extensions of the classical fuzzy metric space and offer a more productive framework in which to analyze convergence, completeness and fixed point properties. The fact that the distance between a point and the point it has to be zero or one may not be realistic in many mathematical models which deal with uncertainty. The partial fuzzy metric spaces address this shortcoming and allow more general fixed point results to be developed.

The latter theories in this section will be applied in the following sections in proving new fixed point theorems and exploring the convergence property of mappings on partial fuzzy metric spaces [8].

III. CONVERGENCE AND COMPLETENESS

Convergence and completeness are concepts that are pivotal in understanding of the fixed point theory and fuzzy metric space. In the classical metric spaces the notion of convergence is established in terms of the distance between the points that tend to zero. But in fuzzy metric spaces and partial fuzzy metric spaces, convergence is defined in terms of the extent of proximity of elements nearing the top value, which typically is 1. The notions are required to define the existence of fixed points since numerous fixed point theorems are based on the convergence of iterative sequences produced by a mapping [9].

The concept of convergence in fuzzy partial metric space is a bit different than in classical metric space as there is a fuzzy distance function, which is an evaluation of the similarity between objects in terms of membership values in the

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range $[0, 1]$. The greater the resemblance or the proximity of the two elements the closer the value of the fuzzy metric to 1. In turn, convergence of a sequence is determined by the fact that the fuzzy metric values are close to 1.

Definition 3.1 (Convergence)

Let $(X, Fp,*)$be a partial fuzzy metric space and let $\{x_n\}$ be a sequence in X. The sequence $\{x_n\}$ is said to **converge** to a point $x \in X$ if

$$\lim_{n\to\infty} \mathrm{Fp}(x_n, x, t) = 1$$

for every t>0.

That is, a sequence $[x_n]$ approaches x when the level of proximity between the components x_n and the point x becomes more like 1 when n approaches infinity. This definition is based on the fact that the components of the sequence get closer to the limit point that is more and more closely related in the context of the fuzzy proximity.

It should be mentioned that, convergence in partial fuzzy metric spaces can have some features that are not similar to classical convergence. As an example, unlike in the case of standard fuzzy metric spaces, sequences may show a slightly different behavior due to the existence of non-zero self-distance in partial fuzzy metric spaces. However, the overall intuition is the same: the further the sequence moves the closer the sequence elements and the limit point are to each other [10].

Convergence is a basic concept used in the study of the behavior of iterative sequences produced by mappings. Iterative processes like this are frequently used in constructing sequences in the fixed point theory.

$x_{n+1} = T(x_n)$

T is a self-mapping of the space, X. When such a sequence ends in a point, x, and some conditions are met, it is frequently possible to prove that x is a fixed point of the mapping, T. Thus computing convergence properties is an important step of fixed point theorems.

Definition 3.2 (Cauchy Sequence)

Let $(X, Fp,*)$ be a partial fuzzy metric space and let $\{x_n\}$ be a sequence in X. The sequence $\{x_n\}$ is said to be a **Cauchy sequence** if

$$\lim_{m,n\to\infty} \mathrm{Fp}(x_m, x_n, t) = 1$$

For every $t > 0$.

The definition implies that the closeness of the terms x_m and x_n is close to 1 as both mmm and n are inclined to infinity. In an intuitive sense this suggests that the terms of the sequence get arbitrarily nearer to one another as the sequence unfolds.

Cauchy sequences are a critical concept especially with respect to completeness. The convergence of sequences is in many mathematical structures (such as fuzzy metric spaces) intimately associated with the behavior of Cauchy sequences. A Cauchy sequence is not always convergent; nevertheless, in spaces that are complete, every Cauchy sequence has a limit in the **space** [11].

A version of the Cauchy property of sequences, typically applied in fixed point theory, is that the existence of limits of iterative sequences of contraction mappings. Numerous fixed point theorems proofs are of the type that a sequence or sequence are constructed by repeated use of a mapping in such a way that they are a Cauchy sequence. After this property is formed, the completeness of this space ensures that there is a limit point.

Definition 3.3 (Complete Partial Fuzzy Metric Space)

A partial fuzzy metric space $(X, Fp,*)$ is said to be **complete** if every Cauchy sequence in X converges to a point in X.

One of the most significant properties that fixed point results need is completeness. A large number of classical theorems such as the Banach contraction principle are dependent on the completeness of the space involved. Without completeness, although a sequence may be satisfying the Cauchy condition, it may tend towards a point that is not in the space, and it would then give no fixed point in the space.

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In partially fuzzy metric spaces, completeness means that sequences of iterative sequences of contraction mappings are contained in the space, and converge to limit points which are well-defined. This fact allows defining the presence and uniqueness of standard points of different kinds of mappings [12].

Convergence, Cauchy sequences, and completeness are all concepts that are necessary to provide the foundational framework needed to develop the fixed point theorems in fuzzy partial metric spaces. These concepts will be applied in the following parts of the paper to obtain various fixed points results with varying conditions of contractive.

IV. AUXILIARY LEMMAS

Here, we include some auxiliary lemmas which are significant in the proving of the main fixed point theorems of this paper. These findings give the required theoretical basis to demonstrate the convergence of sequence of iterative operations in semi-fuzzy metric spaces. The lemmas are primarily concerned with the behavior of sequences under contractive conditions and are concerned with limits on sequences produced by continuous mappings. These are the results that are typically used in fixed point theory so that iterative sequences are able to attain a well-defined point in the space [13].

Lemma 4.1

Let $(X, Fp,*)$ be a fuzzy partial metric space and let $\{x_n\}$ be a sequence in X. Suppose that the sequence satisfies a **contractive condition** of the form

$Fp(x_{n+1}, x_n,t) \geq k\, Fp(x_n, x_n-1,t)$

For all $n \geq 1, t > 0$, and some constant $0 < k < 1$.

Then the sequence $\{x_n\}$ is a **Cauchy sequence** in X.

Proof.

Assume that the sequence $\{x_n\}$ satisfies the given contractive condition. By applying the contractive inequality repeatedly, we obtain

$$Fp(x_{n+1}, x_{n,t}) \geq k\, Fp(x_n, x_{n-1}, t)$$

And

$$Fp(x_n, x_{n-1}, t) \geq k\, Fp(x_{n-1}, x_{n-2}, t)$$

Continuing this process, we obtain

$$Fp(x_{n+1}, x_n, t) \geq knFp(x_1, x_0, t)$$

Since $0 < k < 1$, the sequence k^n converges to zero as n→∞. Consequently, the differences between successive terms of the sequence become arbitrarily small in terms of fuzzy proximity [14].

Using the generalized triangle inequality property of the fuzzy partial metric space, we can estimate the nearness between any two terms x_m and x_n of the sequence. As $m, n \to \infty$, the fuzzy metric $Fp(xm, x_n, t)$ approaches 1. Therefore,

$$\lim_{m,n\to\infty} Fp(x_m, x_n, t) = 1$$

For every t>0. Hence, the sequence $\{x_n\}$ is a **Cauchy sequence** in the fuzzy partial metric space X.

Lemma 4.2

Let $(X, Fp,*)$ be a **complete fuzzy partial metric space**, and let

$T: X \to X$ be a continuous mapping. Suppose that a sequence $\{x_n\}$ in X is defined iteratively by

$x_{n+1} = T(x_n)$

For $n = 0,1,2, \ldots$ Then the sequence $\{x_n\}$ converges to a point in X.

Proof.

Let x0∈X be an arbitrary initial point and define the sequence $\{x_n\}$ by the iterative rule

$x_{n+1} = T(x_n)$

For all $n \geq 0$. From Lemma 5.1, if the mapping T satisfies a suitable contractive condition, then the sequence $\{x_n\}$ forms a Cauchy sequence in the fuzzy partial metric space X.

Since the space $(X, Fp,*)$ is assumed to be **complete**, every Cauchy sequence in X converges to a limit that belongs to X. Therefore, there exists some point $x \in X$ such that

$$\lim_{n\to\infty} \mathrm{Fp}(\mathrm{x}_n, \mathrm{x}, \mathrm{t}) = 1$$

For every $t > 0$.

Now, since the mapping T is continuous, taking limits on both sides of the iterative relation $\mathrm{x}_{n+1} = T(\mathrm{x}_n)$ yields

$$\lim_{n\to\infty} \mathrm{x}_{n+1} = \lim_{n\to\infty} \mathrm{T}(\mathrm{x}_n)$$

which implies

$$x = T(x)$$

Therefore, the sequence (x_n) approaches the point x in the space X. This defines the presence of the limit of the iterative sequence of the mapping T.

The lemmas constitute vital instruments in the demonstration of the fixed point theorems that are given in the subsequent sections. Specifically, the Cauchy property of sequences of contracting conditions is ensured by Lemma 4.1, whereas the existence of limits of complete fuzzy partial metric spaces is guaranteed by Lemma 4.2. The combination of these two is the basis of deciding the existence and uniqueness of fixed points of many different classes of mappings [15].

V. MAIN FIXED POINT THEOREMS

Here we define the major fixed point outcomes of mappings that are defined on complete partial fuzzy metric areas. Fixed point theorems are of basic nature in nonlinear analysis as they ensure everything has a solution and a unique solution to different mathematical models. The classical Banach contraction principle has been generalized extensively to other more generalized metric frameworks such as fuzzy metric spaces and partial fuzzy metric spaces. The findings of this section augment the classical principles of contraction and give conditions when a mapping has a unique fixed point in a partial fuzzy metric space [16].

Theorem 5.1 (Banach Type Fixed Point Theorem)

Let $(X, Fp,*)$ be a **complete fuzzy partial metric space** and let

$$T: X \to X$$

be a mapping satisfying the contractive condition

$$Fp(Tx, Ty, t) \geq k\, Fp(x, y, t)$$

For all $x, y \in X$ and$t > 0$, where $0 < k < 1$.

Then the mapping T has a **unique fixed point** in X.

Proof.

Let $\mathrm{x}_0 \in X$ be an arbitrary point and construct a sequence $\{\mathrm{x}_n\}$ in X defined by

$\mathrm{x}_{n+1} = T(\mathrm{x}_n),\ n = 0,1,2, \ldots$

Using the given contractive condition, we obtain

$$Fp(x_{n+1}, x_n, t) = Fp(Tx_n, Tx_n - 1, t) \geq kFp(x_n, x_n - 1, t).$$

By repeated application of this inequality, it follows that the sequence $\{\mathrm{x}_n\}$ satisfies a contractive relation. Consequently, the sequence becomes progressively closer in terms of the fuzzy metric and forms a **Cauchy sequence** in X.

Since $(X, Fp,*)$ is complete, the sequence $\{\mathrm{x}_n\}$ converges to some point $x \in X$. Taking limits in the relation $\mathrm{x}_{n+1} = T(\mathrm{x}_n)$ and using continuity arguments, we obtain

$$Tx = x.$$

Thus, x is a fixed point of T.

To prove uniqueness, suppose that y is another fixed point of T. Then

$Tx = x$ And $Ty = y$.

Using the contractive condition, we obtain

$$Fp(x,y,t)=Fp(Tx,Ty,t)\geq kFp(x,y,t).$$

Since $0<k<1$, the above relation implies that $x=y$. Hence, the fixed point is unique.

Theorem 5.2 (Generalized Contractive Mapping)

Let $(X,Fp,*)$ be a complete fuzzy partial metric space and let

$$T:X\rightarrow X$$

be a mapping satisfying a generalized contraction condition of the form

$$Fp(Tx,Ty,t)\geq \phi(Fp(x,y,t)$$

for all $x,y\in X$ and $t>0$, where

$$\phi:[0,1]\rightarrow[0,1]$$

is a continuous function satisfying

$\phi(s)>s$ For $0<s<1$.

Then the mapping T admits a **unique fixed point** in X.

Proof.

Let $x0\in X$ be an arbitrary point and define a sequence $\{x_n\}$ by

$x_{n+1}=T(x_n)$.

Using the generalized contraction condition, we obtain a sequence of fuzzy distances that increases toward 1. This implies that the sequence $\{x_n\}$ becomes increasingly close in terms of the fuzzy metric [17].

Using arguments similar to those in Theorem 5.1, the sequence $\{x_n\}$ can be shown to be a Cauchy sequence. Since the space X is complete, the sequence converges to some point $x\in X$.

By taking limits and using the continuity of the mapping T, we obtain

$$Tx=x.$$

Hence x is a fixed point of the mapping T. Uniqueness follows from the generalized contraction condition.

Theorem 5.3 (Partially Ordered Case)

Let $(X,Fp,*)$ be a complete fuzzy partial metric space and suppose that X is equipped with a **partial order relation** $\leq$. Let

$$T:X\rightarrow X$$

Be a **monotone (order-preserving) mapping**, that is,

$x\leq y\ \Rightarrow\ T(x)\leq T(y)$.

Assume that T satisfies a contractive condition similar to that in Theorem 5.1 and that there exists an element $x_0\in X$ such that

$$x_0\leq T(x_0).$$

Then the mapping T has at least one fixed point in X.

Proof.

Starting from the initial point x_0, define the sequence

$x_{n+1}=T(x_n)$.

Since the mapping T is monotone and $x_0\leq T(x_0)$, it follows that

$$x_0\leq x_1\leq x_2\leq\cdots.$$

Thus, the sequence is increasing with respect to the partial order. Using the contractive condition and the completeness of the fuzzy partial metric space, the sequence $\{x_n\}$ can be shown to be Cauchy and hence convergent.

Let x be the limit of the sequence. Taking limits in the relation $x_{n+1}=T(x_n)$, we obtain

$$Tx=x.$$

Therefore, x is a fixed point of the mapping T.

VI. ILLUSTRATIVE EXAMPLE

We here give a typical example to show how useful the fixed point theorems developed in the last section are. And the point of this example is to demonstrate that a contraction mapping that is defined on a partial fuzzy metric space satisfies the hypotheses of the Banach-type fixed point theorem, and hence does have a unique fixed point[18].

Let

$$X = [0,1]$$

be the closed interval of real numbers between 0 and 1. Clearly, X is a non-empty set and forms a complete subset of the real numbers with respect to the usual topology.

Now define a function

$$Fp: X \times X \times (0,\infty) \to [0,1]$$

by

$$Fp(x,y,t) = \frac{t}{t + |\, x - y \,|}$$

For all $x, y \in X$ and $t > 0$.

We show that this function defines a partial fuzzy metric on the set X.

First, observe that for every $x, y \in X$ and $t > 0$,

$Fp(x,y,t) = \frac{t}{t+|x-y|} > 0$

Since $t > 0$ and the denominator is always positive.

Second, if $x = y$, then

$$Fp(x,x,t) = \frac{t}{t+0} = 1.$$

Conversely, if $Fp(x,y,t) = 1$, then it must be the case that $|\, x - y \,| = 0$, which implies $x = y$. Hence the identity condition is satisfied.

Third, the function is symmetric because

$Fp(x,y,t) = \frac{t}{t+|x-y|} = \frac{t}{t+|y-x|} = Fp(y,x,t)$.

Fourth, the function holds a generalized triangular inequality condition with respect to fuzzy metric spaces with an appropriate continuous t-norm. Moreover, the function is also continuous in the variable t, as it is made of continuous functions. Accordingly, the couple $(X, Fp, *)$ is a partial fuzzy metric space [19].

Next, consider the mapping

$$T: X \to X$$

Defined by

$$T(x) = \frac{x}{2}.$$

It is clear that T maps the interval [0,1] into itself because if $x \in [0,1]$, then $0 \le \frac{x}{2} \le \frac{1}{2}$, which also belongs to the interval [0,1].

Now we verify that the mapping T satisfies the contractive condition. Let $x, y \in X$. Then

$$|\, T(x) - T(y) \,| = \left|\, \frac{x}{2} - \frac{y}{2} \,\right| = \frac{|\, x - y \,|}{2}.$$

Substituting this into the fuzzy partial metric expression, we obtain

$$Fp(Tx, Ty, t) = \frac{t}{t + |\, T(x) - T(y) \,|} = \frac{t}{t + \frac{|x-y|}{2}}.$$

Since

$$\frac{t}{t + \frac{|x-y|}{2}} \ge \frac{t}{t + |\, x - y \,|},$$

it follows that

$$Fp(Tx,Ty,t) \geq Fp(x,y,t).$$

Thus the mapping T satisfies a contractive-type condition in the fuzzy partial metric space.
Now construct the iterative sequence

$$x_{n+1} = T(x_n)$$

starting from an arbitrary initial point $x_0 \in X$. This gives

$$x_1 = \frac{x_0}{2}, x_2 = \frac{x_0}{4}, x_3 = \frac{x_0}{8}, \ldots.$$

Hence,
$x_n = \frac{x_0}{2^n}$.
As $n \to \infty$,

$$x_n \to 0.$$

Therefore, the sequence converges to the point x=0.
Finally, observe that

$$T(0) = \frac{0}{2} = 0.$$

Hence 000 is a fixed point of the mapping T. Moreover, by Theorem 5.1, the contraction condition guarantees that the fixed point is unique.
Thus, the mapping $T(x) = \frac{x}{2}$ defined on the partial fuzzy metric space $(X, Fp, *)$ admits a unique fixed point, which is $x = 0$. This example illustrates the practical application of the fixed point theorems established in this study [20].

VII. APPLICATIONS

The fixed point theorems in the fuzzy partial metric spaces are significantly used in a range of areas of mathematics and applied sciences that use uncertainty and imprecision. These findings give mathematical instruments of demonstrating the existence and stability of solution in complex systems.

7.1 Fuzzy Differential Equations

There is a large variety of applications of the fixed point techniques to determine the existence and uniqueness of solutions of fuzzy differential equations. There are a lot of dynamic systems which have uncertain initial conditions or parameters. With the help of converting such problems into fixed point problems, researchers are able to establish that solution exists and converges within a fuzzy metric framework [21].

7.2 Fuzzy Integral Equations

Fuzzy integral equations are commonly used in physics, engineering, and control systems whereby data in a system might not be exactly known. Fixed point theorems are properties that guarantee that iterative processes will arrive at a solution despite the imprecision or fuzziness of the system.

7.3 Optimization Problems

Parameters in real-world optimization problems might be uncertain (cost, demand, resource availability etc). The fixed point techniques in fuzzy metric spaces can be used to find the best solutions to multi-criteria decision making.

7.4 Computer Science

Fixed point computations are applied in fuzzy automata, artificial intelligence, and network stability analysis, where systems tend to be run under uncertain information, or incomplete information [22].

VIII. DISCUSSION

The obtained findings in this work generalize a number of classical fixed point theorems which had been formulated earlier on the standard metric spaces and subsequently extrapolated to fuzzy metric spaces. The current work gives a more relaxed mathematical framework of the study of the convergence of sequences and fixed point existence under uncertainty by proposing the framework of partial fuzzy metric spaces. In contrast with classical metric spaces, partial

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fuzzy metric spaces permit the self-distance of a point to be non-zero hence being well adapted to the modeling of incomplete or partial information.

The theorems developed in this paper have shown that contractive mappings as defined in these spaces can still be used to ensure that fixed points do exist and are also unique, given the right conditions. In addition, the contraction conditions that are generalized in the current research expand the applicability of the fixed point theory. The outcomes of these works lead to the evolution of the fuzzy analysis and open up new opportunities of using fixed point methods in the mathematical models with uncertainty, especially in optimization, different equations and the systems of computations.

IX. CONCLUSION

This paper has fixed point results which have been proven within a framework of partial fuzzy metric spaces. The project is a continuation of classical fixed point theory since it involves the use of fuzzy structures that enable the modeling of uncertainty and imprecision in mathematical systems. Existence and uniqueness of fixed points have been established by establishing appropriate contractive conditions to the mappings on complete fuzzy partial metric spaces. These findings are generalizations of the famous Banach contraction principle and lead to a further evolution of the fuzzy metric theory.

The terms convergence, Cauchy sequences and completeness were examined in the arena of partial fuzzy metric spaces which prove useful in the study of nonlinear problems in a more flexible setting. The derived theorems can be applied in resolving different problems that occur in applied mathematics, especially the ones that involve uncertainty or incomplete information. Future studies can be aimed at generalizing these findings to multivalued mappings, intuitionistic fuzzy metric spaces, and nonlinear fuzzy differential equations and at investigating their use in optimization theory and computational models.

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