

Model Order Reduction to Linear Time Invariant System: A Critical Comparative Approach

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Abstract: *Model Order Reduction (MOR) techniques play a crucial role in simplifying complex systems while preserving their essential dynamics. This paper presents a critical analysis of conventional MOR methods applied to linear time-invariant (LTI) systems. The objective is to compare and evaluate the effectiveness of different reduction approaches in terms of accuracy, computational efficiency, and applicability across various system sizes and complexities. The paper first reviews fundamental concepts of MOR and discusses common reduction methods such as balanced truncation, modal truncation, and system approximation via projection. It then introduces a comparative framework that systematically evaluates these techniques based on their theoretical foundations, numerical stability, and practical limitations. Through numerical experiments and case studies, we demonstrate how different MOR methods perform under varying conditions, including high-dimensional systems, parameter variations, and input-output responses. Our analysis highlights the trade-offs between accuracy and computational cost inherent in each method and provides insights into selecting appropriate reduction techniques based on specific system requirements.*

Keywords: ISE minimize approach, Linear Time Integral System, Model Order Reduction to using conventional Method, MATLAB

I. INTRODUCTION

Model Order Reduction (MOR) techniques are indispensable tools in the field of control systems and computational science, offering a means to simplify complex dynamical systems while retaining their essential behaviour. For Linear Time-Invariant (LTI) systems [1,4], MOR plays a pivotal role in reducing computational complexity and enhancing system understanding. This paper delves into a critical examination of conventional MOR methods as applied to LTI systems [1,9,10,16,21], aiming to provide a comparative framework that elucidates their strengths, weaknesses, and applicability across various scenarios. The complexity of real-world systems often necessitates the use of reduced-order models for efficient analysis [10,12], design, and simulation. Traditional methods such as balanced truncation [1], Pade approximation method [21], Differentiation method [16], Routh Approximation method [9] and Stability Preservation method [10] have been extensively studied and applied in MOR. These methods offer different trade-offs between accuracy and computational cost [23], making their comparative analysis crucial for selecting the most suitable approach based on specific system requirements. The objective of this study is to scrutinize and compare these conventional MOR techniques [9,10] within a unified framework. By critically evaluating their theoretical foundations, numerical stability [23], and practical limitations, we aim to provide insights into their performance across a spectrum of system sizes, complexities, and input-output scenarios. Through numerical experiments and case studies [19,23], we seek to highlight the comparative advantages and disadvantages of each method [4], thereby aiding researchers and practitioners in making informed decisions when choosing an appropriate MOR strategy for their applications [25]. This paper is structured as follows: provides a brief overview of MOR fundamentals and introduces the conventional reduction methods [1,9,10,16,21] under consideration. outlines the proposed comparative framework and evaluation criteria [1]. The present numerical [19,23] experiments and case studies, respectively, showcasing the comparative performance of the MOR methods. Finally, five conventional methods [1,9,10,16,21] conclude with a summary of key

findings and implications for future research in MOR for LTI systems [25]. Sinha et.al [2] A new interpretation has been given to model reduction by the method of balanced realizations, which shows that it can be regarded as a type of aggregation method. This enables a user to retain the conceptual advantage of balanced realization, as well as those of aggregation. Thus, an important advantage of the latter, that the eigenvalues placed by feeding back the states of the reduced model are retained for the original system, can be fully exploited with the balanced matrix method as well. Sambariya et.al [17] a large order system is reduced by using the Cuckoo Search Algorithm (CSA) to a reduced order approximate model. The denominator coefficients of a desired reduced order system are determined by Routh approximation method while the numerator coefficients are determined using CSA based on integral square error minimization as an objective function pertaining to a unit step as input. The efficacy of the proposed method is tested with three SISO test systems to get a corresponding reduced order system and extended to a MIMO system. The results are 4 satisfactory in terms of minimum error with the proposed method as compared to Routh Pade approximation and weighted sum multi-objective harmony search based reduced models. Duddet et.al [4] This paper presents a novel hybrid model reduction method to simplify large-scale continuous-time single-input–single-output and multi-input–multi-output dynamic systems using the advantages of the balanced truncation method and the particle swarm optimization (PSO) algorithm. The balanced truncation method obtains the reduced model denominator coefficients to ensure stability. The PSO algorithm minimizes the integral square error between the step responses of the original system and the reduced model as much as possible. It leads to the optimal 5 numerator coefficients. The advantage of the proposed approach is that, for optimizing reduced model numerator coefficients, the search space boundaries of the PSO algorithm are not entirely random. They are selected using the balanced truncated reduced model numerator. So, the suggested method avoids two significant problems with evolutionary algorithms: the arbitrary choice of search space and the longer simulation time. Four power system models and four numerical examples from the literature are considered to assess the effectiveness of the proposed reduction method.

II. PROBLEM STATEMENT

Consider a higher order system of order n and is represented by equation.

$$G(s) = \frac{Y(s)}{R(s)} = \frac{a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n}{s^n + b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_n} \quad (1)$$

Where, a_i and b_i are constants for $i=1, 2, \dots, n$.

If r represents a reduced order as of lesser order than n , then, the reduced order model of the system in equation (1) is represented as in equation (2). The important and principal requirement of the reduced order model is to possess all important specifications of the original system.

$$G_r(s) = \frac{Y_r(s)}{R_r(s)} = \frac{c_1 s^{r-1} + c_2 s^{r-2} + \dots + c_r}{s^r + d_1 s^{r-1} + d_2 s^{r-2} + \dots + d_r} \quad (2)$$

Where, c_r and d_r are unknown constants and are to be determined by using cuckoo search algorithm subjected to minimization of integral square error defined in equation (3). Error denotes difference of the unit step responses by original system and the reduced model to a unit-step function.

Consider an n th-order complex multivariable LTI model with u inputs and v outputs characterized in the form of a transfer matrix as

$$[G(s)] = \frac{1}{D(s)} \begin{bmatrix} A_{11}(s) & A_{12}(s) & \dots & A_{1u}(s) \\ A_{21}(s) & A_{22}(s) & \dots & A_{2u}(s) \\ \vdots & \vdots & \ddots & \vdots \\ A_{v1}(s) & A_{v2}(s) & \dots & A_{vu}(s) \end{bmatrix} = [g_{ij}(s)]_{v \times u} \quad (3)$$

where $i = 1, 2, 3, \dots, v$; $j = 1, 2, 3, \dots, u$. Hence $g_{ij}(s)$ can be written as

$$g_{ij}(s) = \frac{A_{ij}(s)}{D(s)} \quad (4)$$

where $g_{ij}(s)$ is the numerous elements of the higher-order and the reduced-order transfer function matrices, respectively. The desired lower-order model preserves the essential features of the HOS and its response matches as closely as possible with the response of the original system for the same type of inputs.

III. RESEARCH METHODOLOGY

There are various conventional techniques available for order reduction of LTI systems. The various techniques are discussed as:

Pade Approximation Method [21]: The Pade approximation method is a powerful technique used in mathematics and engineering, particularly in the context of model order reduction (MOR) and approximation of transfer functions. Here's an overview of the Pade approximation method.

Differentiation Method [16]: Differentiation methods in the context of model order reduction (MOR) involve techniques that focus on capturing the dynamics and behaviour of a system through differentiation operations. These methods are particularly useful when dealing with systems characterized by differential equations or transfer functions.

Routh Approximation Method [9]: Routh Approximation in the context of model order reduction (MOR), as Routh's stability criterion is typically associated with control systems and stability analysis rather than model reduction.

Balanced Truncation Method [1]: Balanced Truncation (BT) is a widely used and effective model order reduction (MOR) method, especially for linear time-invariant (LTI) systems. The balanced truncation method is a powerful technique used for model order reduction (MOR) in control theory and system analysis. It aims to reduce the complexity of a high-dimensional system while preserving its essential dynamic behaviour and characteristics. Here's a structured conclusion about the balanced truncation method for MOR.

Stability Preservation Method [10]: The Preservation method in model order reduction (MOR) refers to a class of techniques and considerations aimed at maintaining key system properties and behaviours during the reduction process. All these conventional methods are applied to some standard numerical examples shown in next section. The minimum value of Integral Square Error showcases the best approximation techniques discussed among all the methods taken into considerations.

IV. NUMERICAL EXAMPLES

Example 1: Consider a fifth order system described by the transfer function [19]

$$G_5(s) = \frac{156 + 396s + 264s^2 + 82s^3 + 10s^4}{40 + 148s + 173s^2 + 84s^3 + 21s^4 + s^5} \quad (5)$$

Table 1: Qualitative comparison of performance analysis of the proposed reduced models and other reduced models in the literature discussed for test system

Reduction Process	Reduced Order Model	IAE	ISE	ITAE	ITSE	t_s (Sec)	t_r (Sec)
Pade Approximation Method [21]	$\frac{-3.301s + 0.2079}{s^2 - 0.7845028s + 0.0533153}$	8.3156	2.4628	7.1982	2.3014	5.45	00
Differentiation method [16]	$\frac{1056s + 2376}{504s^2 + 2076s + 1776}$	221.9605	519.0229	1.2381	3.0598	3.04	1.65
Routh Approximation Method [9]	$\frac{2.6347s + 1.03787}{s^2 + 0.9847s + 0.265869}$	14.4138	3.2718	50.4399	6.6953	3.97	2.67
Stability Preservation	$\frac{336.67s + 156}{137.359s^2 + 106.51s + 40}$	35.9188	16.9119	161.2672	72.9875	9.28	1.98

Method [10]							
Balanced Truncation Method [1]	$\frac{0.01845s^2 + 8.985s + 45.13}{s^2 + 15.27s + 11.57}$	2.5547	0.0825	10.9585	0.3173	4.75	2.68

For getting the reduced model using various conventional methods of order reduction is showcased in tabular form as illustrated in Table-1 for this original system shown in equation (5).

Example 2: Considering the 4th order system presented in [17] as:

$$G(s) = \frac{24 + 24S + 7S^2 + S^3}{24 + 50S + 35S^2 + 10S^3 + S^4} \quad (6)$$

By using the Pade approximation method, the Reduced Order Model is as:

$$R_{PA2}(s) = \frac{2.5142 + 0.7287s}{2.5142 + 3.4516s + s^2} \quad (7)$$

The ROM of this system using differentiation method is found as:

$$R_{DM2}(s) = \frac{144 + 48s}{288 + 300s + 70s^2} \quad (8)$$

And the other reduced model as computed using Routh approximation is as shown below:

$$R_{RA2}(s) = \frac{0.7946 + 0.7946s}{0.7946 + 1.6556s + s^2} \quad (9)$$

The time responses of the original system and the reduced models obtained by the proposed algorithm and some other standard order reduction techniques are depicted. It can be observed that the response of the proposed reduced system is closely matched to the outcome of the actual system. It is observed that the proposed reduced model is giving the lowest value of error and closest approximation as compared to the other standard and recently proposed MOR methods.

Table 2: Qualitative comparison of performance analysis of the proposed reduced models and other reduced models in the literature discussed for test system

Reduction Process	Reduced Order Model	IAE	ISE	ITAE	ITSE	t_s (Sec)	t_r (Sec)
Pade Approximation Method [21]	$\frac{2.5142 + 0.7287s}{2.5142 + 3.4516s + s^2}$	0.1542	9.4043	0.1904	7.7850	3.92	2.25
Differentiation method [16]	$\frac{144 + 48s}{288 + 300s + 70s^2}$	42.956	20.0192	244.080	119.306	2.73	1.54
Routh Approximation Method [9]	$\frac{0.7946 + 0.7946s}{0.7946 + 1.6556s + s^2}$	0.4689	0.0038	1.6362	0.0099	3.71	2.32
Stability Preservation Method [10]	$\frac{24 + 20.571s}{24 + 42s + 30s^2}$	2.2742	0.0974	8.1678	0.3371	5.59	1.93
Balanced Truncation Method [1]	$\frac{2.501 + 0.62925s + 0.026s^2}{2.501 + 3.398s + s^2}$	0.1131	0.0012	0.191	2.2721	3.91	2.26

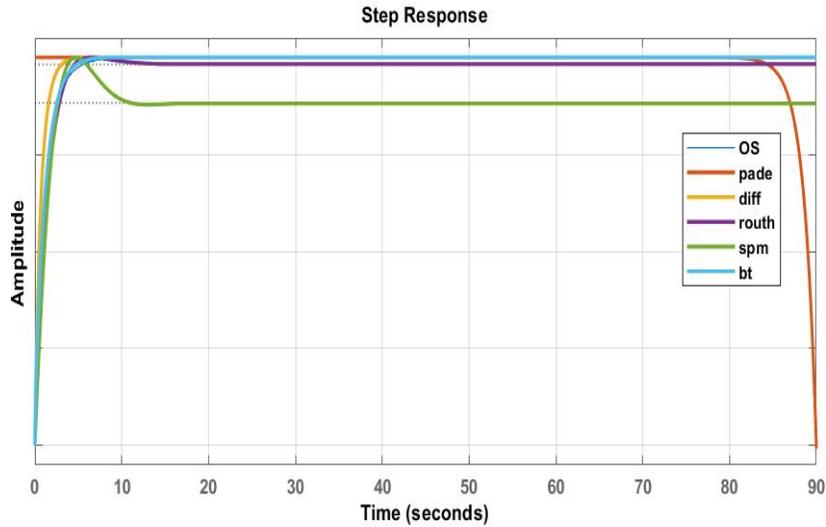


Fig. 1: Step Response of Original System and its Reduced Models

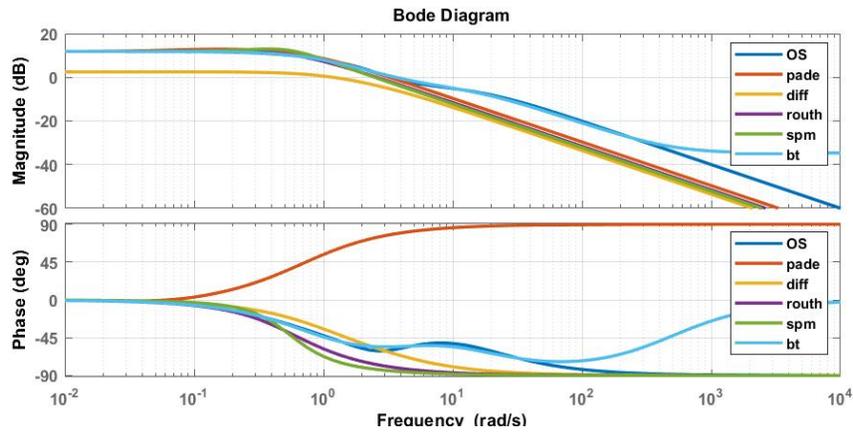


Fig. 2: Bode Plot of Original System and its Reduced Models

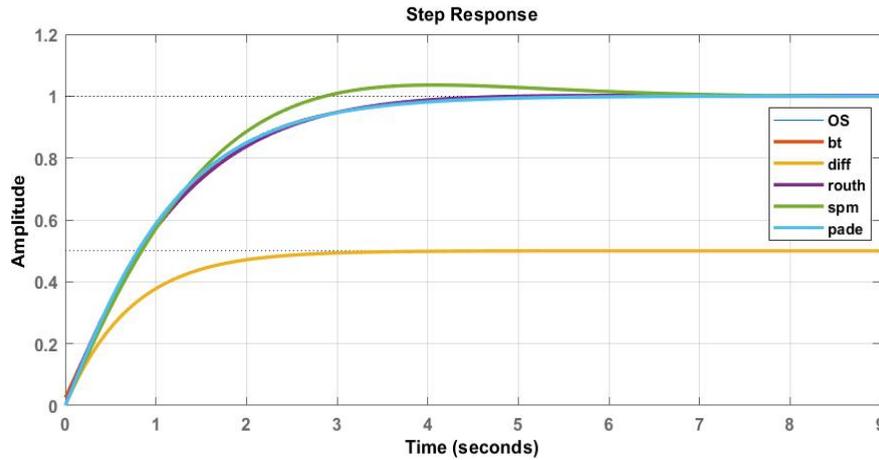


Fig. 3: Step Response of Original System and its Reduced Models

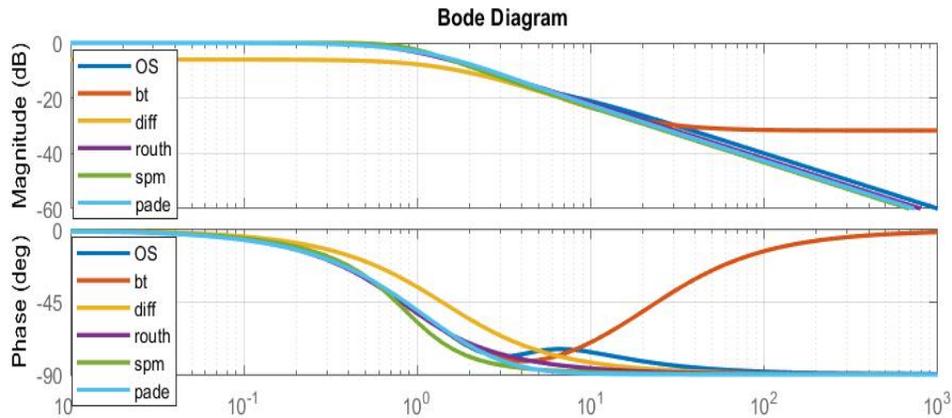


Fig. 4: Bode Plot of Original System and its Reduced Models

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Validation and Discussion: To validate the significance and the superiority of the presented methodology with few other general model diminution methodologies, integral square error (ISE) and relative integral square error (RISE) of their lower-order systems are computed as given in [5,8].

$$ISE = \int_0^t [y(t) - r(t)]^2 dt \quad (10)$$

$$ITSE = \int_0^t t[y(t) - r(t)]^2 dt \quad (11)$$

To observe the validity of the recommended technology, another two performance indices called an integral absolute error (IAE) and an integral time-weighted absolute error (ITAE) in between the HOS and the ROM are also computed, which are discussed in [6,7].

$$IAE = \int_0^t |y(t) - r(t)| dt \quad (12)$$

$$ITAE = \int_0^t t|y(t) - r(t)| dt \quad (13)$$

where $y(t)$ and $r(t)$ are step responses of HOS and ROM, respectively at t time and $\hat{y}(t)$ is the impulse behaviour of the HOS.

V. CONCLUSION

Model order reduction (MOR) plays a critical role in transforming complex systems into manageable linear time-invariant (LTI) models. Through a conventional comparative approach, MOR techniques enable us to analyse, simulate, and control large-scale systems efficiently. By reducing the complexity of mathematical models while preserving important system characteristics, MOR techniques facilitate faster simulations, control design, and optimization tasks. The best proposed method is the Balanced Truncation Method because this Integral squirrel error (ISE) is the very less in other conventional method. This approach enables Model Order Reduction to LTI system a critical conventional method.

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