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Performance Analysis of Queueing Network Models with Fuzzy Arrival and Service Rates

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Abstract: *Queueing network models are used as basic tools in analyzing performance in systems whose performance is of significant concern in the form of congestion and wait time, including telecommunications, manufacturing and computer systems. Nevertheless, the classical queueing models assume crisp (deterministic or well-defined stochastic) arrival rates and service rates. These parameters in real-life situations are also uncertain in nature since human behavior, changes in environment, or incomplete information are inherently uncertain. The fuzzy set theory offers a solid mathematical model of such uncertainties. The performance analysis of the queueing network models in which arrival and/or service rates are expressed as fuzzy numbers is explored in this paper. We overview the current approaches to fuzzy queueing model and derive analytical performance indication of common models of fuzzy queueing and validate them using numerical examples. The most important Key Performance Indicators like mean queue length, waiting time, and use of the system are calculated on the basis of fuzzy parameterizations. It is compared to the conventional stochastic models to show the advantages of the fuzzy approach and its drawbacks. The future direction of research in incorporating fuzzy logic with advanced queueing networks is also discussed in the paper.*

Keywords*:* Queueing Networks, Fuzzy Arrival Rate, Fuzzy Service Rate, Fuzzy Queueing Theory, Performance Analysis, Uncertainty Modeling

I. INTRODUCTION

Many processes of operations and service, such as telecommunications services, manufacturing, health services, transportation, and computer networks use queueing systems. They simulate scenarios in which things, like customers, data packets, or tasks, come in and wait, and get served by the scarce resources. The study of these systems makes it possible to distribute resources more effectively, minimize waiting time, and increase the overall performance. Classical queueing theory relies on models such as M/M/1, M/M/c and Jackson networks and assumes that the system parameters are known exactly, in particular, arrival rates (λ) and service rates (μ) as well as gives exact analytical results on the performance measures, e.g., queue length, waiting time, and system utilization.

But in real-life systems, the arrivals and service rates are usually uncertain with changing demand, changing processing capacities and external environmental dynamics. These uncertainties render the classical models inadequate in terms of capturing the performance of a system. In order to eliminate this shortcoming, a powerful approach to model imprecision is the fuzzy set theory proposed by Zadeh (1965). Fuzzy queueing models make fuzzy numbers of the arrival and service rate parameters, which introduces vagueness, and gives performance measures as fuzzy numbers, indicating the possible range of values and not just a deterministic value.

The use of fuzzy queueing models is especially relevant in the queueing networks, in which nodes are connected to one another in many aspects. A failure in one node can spread to the rest of the network affecting system-level. Fuzzy steady-state probabilities and metrics of network performance can be computed in these networks using techniques like α-cut decomposition and interval arithmetic.

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This study aims at producing a systematic model of the analysis of queueing networks with uncertain arrival and service rates of the fuzzy type, deriving fuzzy performance indicators, and demonstrating the impact of uncertainty in parameters on the dynamics of the systems. The research is more realistic and informing as compared to classical models and contributes to sound decision-making and optimization of resources under uncertain operational conditions.

II. LITERATURE REVIEW

Jean Pierre, et.al. (2015). this article reveals that the method presented in this paper, the L-R, is among the valid methods used to calculate the performance measures of fuzzy queues. Based on this calculation method, we determine the number of customers and waiting time of a simple queue M/M/1 in fuzzy environment. The benefit of L-R method is its short method, is convenient and flexible as opposed to the popular and referred to alpha-cuts method[1].

Mwangi, Sammy. (2015). The university has over the years experienced an increase in the number of people with introduction of a double intake system that has consequently resulted into long waiting time and long queues in the students finance department where there are limited service stations, inefficiencies in the payment system used and students being disorderly. An adequate queuing system is required in order to improve service delivery. This is done through the installation of appropriate mechanisms to guarantee a proper flow of students at the service counters. When analyzing the model in question, we only consider the main queue and empirically analyze it. The application of the principles and equations of the queuing theory demonstrated the following results: the average number of customers is 22 per hour; the service rate is 23.7 customers per hour. The system utilization was 92.95, probability of a zero customer waiting is 7.05, number of customers waiting on average is 12.252 and the waiting time is 33.415 min. The research contrasted the single server model to the multi-server model and found out that M/M/1 was not the optimal model to the Finance department. The study conducted on a questionnaire of 384 respondents revealed that nearly the entire number of customers are not satisfied regarding the nature of the waiting lines and some of the students have already left at normal occasions because of the long queues. The waiting time of the students should not be ignored and the study has highlighted the need to constantly check their evolving needs and the enhancement in the wait period when attending to the students. In the current competitive business world, the contemporary society is increasingly becoming a service dominated one. The capability of customer satisfaction and service operation has provided an organization with an edge in the market and has thus resulted in a growing significance of service operations management. Consequently, waiting has attracted the interest of all business operation management specialists immensely[2].

Ehsanifar, Mohammad (Arash) et.al. (2017). To forecast the performance indices like waiting time, this paper features a simulation model developed in an attempt to understand the elements of queue in the real world in the context of uncertain and subjective situation. This paper aims at estimating the waiting time of every customer in an M/M/C queuing model. In that sense, to make decision makers access useful results with sufficient knowledge on the behavior of system, the queuing system is taken into account in the fuzzy setting whereby the arrival and service times are discussed as fuzzy variables. The suggested method of vague systems would be able to reflect the system more realistically, and more details would be presented to design queueing systems in practice. Moreover, complex systems modeling and the study of the behavior of queuing are successfully modeled by the method of simulation. Lastly, a case

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[1] Jean Pierre, MUKEBA & Mabela, Rostin & Lukata, B.. (2015). Computing Fuzzy Queueing Performance Measures by L-R Method. Journal of Fuzzy Set Valued Analysis. 2015. 57-67. 10.5899/2015/jfsva-00226.

[2] Mwangi, Sammy. (2015). An Empirical Analysis of Queuing Model and Queuing Behaviour in Relation to Customer Satisfaction at Jkuat Students Finance Office. American Journal of Theoretical and Applied Statistics. 4. 233. 10.11648/j.ajtas.20150404.12.

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study of the numerical example in a banking system is also solved to reveal justifiability of developed model in the actual scenario[3].

Sujatha, N. et.al. (2017). The current research paper works with the fuzzy queueing models with single and multi-servers using the triangular fuzzy numbers with the assistance of alpha -cut method. The arrival rate as well as the service rate are expected to be of fuzzy nature. It is also assumed that the arrival follows Poisson distribution and the service rate follows Erlang -k (Ek) distribution. The discrete queues that are normally taken into consideration in reality are not bright enough as compared to fuzzy queues. Moreover, queuing which concerns fuzzy logic extension makes them more practicable. The queueing models are interpreted in triangular fuzzy numbers various performance measures of the queueing models. The effectiveness of the model is also estimated due to the application of DSW algorithm in different scenarios[4].

Zaki, Noor et.al. (2019). a queuing system is the process that aims to test the efficiency of a model based on the concepts of the queue models: arrival and service time distributions, queue disciplines and queue behaviour. The primary objective of this paper is to compare the behaviour of a queuing system at check-in counters in reference to Queuing Theory Model and Fuzzy Queuing Model. The Queuing Theory Model provides performance measures of a single value as compared to the Fuzzy queuing model which provides a variety of figures. The Fuzzy Queuing Model defines the membership function of performance measures in Dong, Shah and Wong (DSW) algorithm. According to the observation, it is common when the customers have to spend too much time in the queue, thereby implying that the systems of the services are not efficient. The variables were gathered in the form of data, including arrival time in the line (server) and service time. Findings indicate that performance measures of the Queuing Theory Model are within the performance measures of the Fuzzy Queuing Model which is computed. Therefore, the findings of the Fuzzy Queuing Model can be used consistently to quantify the queuing efficiency of an airline business to resolve the issue in the waiting line and will enhance the quality of service offered by Airline Company[5].

III. PRELIMINARIES

In this section; the basic mathematical concepts and tools necessary in the modeling of the queueing networks with fuzzy arrival and service rate are presented. The two major components of this modeling framework are the description of numbers with the fuzzy concept and the arithmetic operation which are done on such numbers hence extending classical queueing theory to take care of the uncertainty.

3.1. Fuzzy Number Representation

Fuzzy numbers give a mathematical system of describing uncertain or imprecise parameters. Fuzzy numbers are expressed in the form of degrees of membership to a set unlike crisp numbers which form a precise value? Triangular fuzzy numbers (TFNs) are popular because they are simple and computationally feasible among other classes of the fuzzy numbers[6].

A triangular fuzzy number $\tilde{x}$ is defined by a triplet (a,b,c), where . Here, aaa $a \leq b \leq c.$ represents

[3] Ehsanifar, Mohammad (Arash) & Hamta, Nima & Hemesy, Mahshid. (2017). A Simulation Approach to Evaluate Performance Indices of Fuzzy Exponential Queuing System (An M/M/C Model in a Banking Case Study). Business Process Management Journal. 10.22116/JIEMS.2017.54604.

[4] Sujatha, N. & Murty, Akella & Gvsr, Deekshitulu. (2017). Analysis of multiple server fuzzy queueing model using α – CUTS. International Journal of Mechanical Engineering and Technology. 8. 35-41.

[5] Zaki, Noor & Saliman, Aqilah & Abdullah, Nur & Hussain, Nur & Amit, Norani. (2019). Comparison of Queuing Performance Using Queuing Theory Model and Fuzzy Queuing Model at Check-in Counter in Airport. Mathematics and Statistics. 7. 17-23. 10.13189/ms.2019.070703.

[6] Kanyinda, J. P., Matendo, R. M. M., & Lukata, B. U. E. (2015). Computing fuzzy queueing performance measures by L-R method. *Journal of Fuzzy Sets Valued Analysis*, 1, *57–67.*

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the lower bound, b represents the most likely or modal value, and ccc represents the upper bound of the fuzzy number.

The membership function of $\tilde{x}$ is defined as:

$$\mu_{\tilde{x}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & \text{otherwise} \end{cases}$$

This membership function assigns to each value xxx a degree of membership $\mu_{\tilde{x}}(x) \in [0, 1]$, indicating how strongly x belongs to the fuzzy number $\tilde{x}$ The degree of membership is 0 outside the interval [a,c] and reaches 1 at the modal value b. This structure captures both the uncertainty (range between a and c) and the most probable value (at b)[7].

An important concept in fuzzy arithmetic is the **α-cut**. An α-cut of a fuzzy number is an interval representing all values that have a membership degree greater than or equal to a chosen level α∈[0,1]. For a triangular fuzzy number, the α-cut x~α is expressed as:

x~α=[xαL,xαU]=[(1−α)a+αb,(1−α)c+αb].

This is a useful interval representation that is especially helpful to use with computations; it transforms the fuzzy number into a crisp interval at any given level corresponding to 0 -value in -values, which means that common interval arithmetic can be used. Through the consideration of several α-levels, it is possible to create the fuzzy number entirely and calculate performance measures at previous levels of confidence.

3.2. Fuzzy Arithmetic

The operation of addition, subtraction, multiplication, and division of fuzzy numbers are defined by fuzzy arithmetic. Arithmetic operations are typically performed through alpha cuts in case of triangular fuzzy numbers.

The sum of two triangular fuzzy numbers is defined $\tilde{x} = (a_1, b_1, c_1)$ and $\tilde{y} = (a_2, b_2, c_2)$ **as:**

$$\tilde{x} + \tilde{y} = (a_1 + a_2, b_1 + b_2, c_1 + c_2),$$

while scalar multiplication by a positive constant k>0 is:

$$k \cdot \tilde{x} = (ka_1, kb_1, kc_1).$$

$$\tilde{x}^{\alpha} = [x_{\alpha}^{L}, x_{\alpha}^{U}] \text{ and } \tilde{y}^{\alpha} = [y_{\alpha}^{L}, y_{\alpha}^{U}],$$

These are related to preserving the shape of the triangle and keeping the computational simplicity. In non-linear operations, like division or functions of ratios of fuzzy numbers (used in queueing measures like utilization or waiting time), fuzzy arithmetic is based on interval computation using α-cut. The standard interval arithmetic rules are applied at every α-level to calculate the corresponding interval of the output. For example, if then the division $\tilde{x}/\tilde{y}$ at level α is computed as:

$$\left[\min\left\{\frac{x_{\alpha}^{L}}{y_{\alpha}^{L}}, \frac{x_{\alpha}^{L}}{y_{\alpha}^{U}}, \frac{x_{\alpha}^{U}}{y_{\alpha}^{L}}, \frac{x_{\alpha}^{U}}{y_{\alpha}^{U}}\right\}, \max\left\{\frac{x_{\alpha}^{L}}{y_{\alpha}^{L}}, \frac{x_{\alpha}^{L}}{y_{\alpha}^{U}}, \frac{x_{\alpha}^{U}}{y_{\alpha}^{L}}, \frac{x_{\alpha}^{U}}{y_{\alpha}^{U}}\right\}\right].$$

This type of approach which is an alpha cut guarantees that the uncertainty of the inputs is always translated to the outputs making fuzzy arithmetic a strong tool in analysing queueing networks where the arrival and service rates are imprecise[8].

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[7] Revathi, S., & Selvakumari, K. (2021). Cost analysis of fuzzy queueing models with priority disciplines. *Advances and Applications in Mathematical Sciences*, 21(2), 743–747.

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IV. FUZZY QUEUEING NETWORK MODEL FORMULATION

In practice, where there is a networked structure many service nodes tend to be networked so that customers, jobs, or data packets are routed between nodes based on some routing probabilities. To model such networks in an uncertain situation one needs to generalize classical queueing theory, with fuzzy arrival and service rates. This section is the formulation of the fuzzy queueing network models, its assumptions, network structure and mathematical representation.

4.1. Assumptions

A number of assumptions are made in order to come up with a tractable fuzzy queueing network model:

Network Type: It is assumed that it is an open queueing network, i.e., jobs are allowed to enter the system and leave the system possibly via one or more service nodes. This study does not take into consideration closed networks, where jobs are in constant circulation.

Node Capacity: The node service capacity can be finite or infinite. At nodes with finite capacity, any incoming job can be rejected or blocked when the queue reaches its maximum length, but not at the nodes with infinite capacity. These are very flexible assumptions in modeling real systems like call centers, production lines or computer network[9]s.

Uncertain Arrivals and Service Rates: Interarrival times and service times in each node are uncertain and it is modeled as triangular fuzzy numbers (TFNs). Where iii implies the fuzzy arrival and service rates of node iii, respectively. The fuzziness absorbs uncertainty because of the changing demand, processing speeds or the environment. As an illustration, the service rate of a node can be most probable at 15 jobs an hour but can be within the range of 12-18 jobs an hour.

Queueing Discipline: The queueing discipline used is that of First-Come-First-Served (FCFS) which means that jobs are served in the order of arrival. Other algorithms like priority-based or round-robin are potentially applicable, but FCFS is a simple and universally applicable base on which to perform an analysis.

Routing Probabilities: Jobs flow between nodes based on crisp routing probabilities pij, which are the probability of a job leaving node iii going into node j. The arrival and service rate are fuzzy although the routing probabilities are deterministic to simplify the calculation of network flow.

Independence: The processes of interarrival and service in various nodes are independent. The model can be complicated with dependencies or correlations between nodes, which are to be considered in the future[10].

These assumptions can be used to construct a fuzzy network model that strikes a balance between realism and analytical tractability and which can be used to evaluate performance measures in the presence of uncertainty.

4.2. Network Structure

Fuzzy queueing network is a set of service nodes, which are interconnected. Jobs come into the network via external sources, are served at one or more nodes and go out of the network once they are finished. The figure 1 depicts a generic three node open queueing network with fuzzy parameters[11].

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[8] IJCRT Authors (2022). Fuzzy analysis of queuing models with C servers using fuzzy numbers. *International Journal of Creative Research Thoughts*, 10(4).

[9] Alonge w'Omatete, J., Matendo, R. M., & Okenge, D. L. (2025). Performance parameters of fuzzy Markovian queueing system FM/FM/1 in transient regime using flexible α-cuts. *Journal of Computing Research and Innovation, 8(1).*

[10] Aarthi, S. (2023). Comparative fuzzy queueing performance for single server queues. *Journal of Intelligent & Fuzzy Systems* (SAGE), illustrating fuzzy arrival and service parameter impacts on performance.

[11] *Implementation of queue models with fuzzy and stochastic components (2025). Journal of Advances in Science and Technology.*

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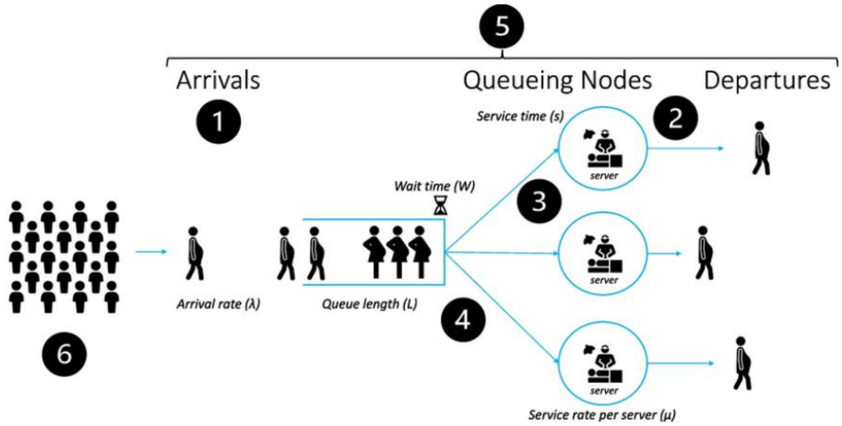

Figure 1. General Fuzzy Queueing Network

In this network

Node 1 represents the entry point where jobs arrive externally with fuzzy rate $\tilde{\lambda}_0$.

Node 2 and Node 3 represent intermediate service stations, each with fuzzy service rates $\tilde{\mu}_2$ and $\tilde{\mu}_3$. Jobs routed between nodes follow the deterministic probabilities pij.

Branching and merging is possible in the network, corresponding with real-world systems e.g. manufacturing lines with parts being fed in a certain order or in parallel, or a network of computers with multiple servers[12].

Fuzzy traffic equations are used to model the mathematical form of the network. For a node iii,

$\tilde{\lambda}_i$

the fuzzy arrival rate is given by:

$$\tilde{\lambda}_i = \tilde{\lambda}_0 p_{0i} + \sum_{j=1}^{N} \tilde{\lambda}_j \, p_{ji},$$

where p_{0i} is the external arrival probability of node i and pji is the routing probability of node j to node i. The successful solution of these equations with the use of fuzzy arithmetic gives the effective fuzzy arrival rates at each node which are used as inputs in the further performance measure calculations like fuzzy utilization, average queue length and waiting time.

To conclude, a flexible and analytically solvable fuzzy queueing network model is characterized by the network structure and assumptions, which can reflect the uncertainty in the arrivals and services and preserve the structure of classical queueing networks. In the subsequent sections, this framework will be used to derive the fuzzy performance measures[13].

V. PERFORMANCE MEASURES WITH FUZZY PARAMETERS

The classical queueing theory offers conveniently developed formulae of essential performance indicators in the form of utilization, average number in system, and waiting time. Such steps are normally communicated in the form of sharp arrival and service rates. But in cases where these rates are not known and are represented by fuzzy numbers, there is a

[12] Kanyinda, J. P., Matendo, R. M. M., & Lukata, B. U. E. (2015). Computing fuzzy queueing performance measures by L-R method. Journal of Fuzzy Sets Valued Analysis, 1, 57–67.

[13] Revathi, S., & Selvakumari, K. (2021). Cost analysis of fuzzy queueing models with priority disciplines. Advances and Applications in Mathematical Sciences, 21(2), 743–747.

need to redefine the classical measures to include the uncertainty that may occur. In this section, we will have a methodology of computing fuzzy performance measures of queueing networks[14].

5.1. Fuzzy System Utilization

The utilization of a service node represents the fraction of time the server is busy. For node iii in a classical M/M/1 system, utilization is defined as ρi=λi/μi. In the fuzzy framework, the arrival and service rates are replaced by triangular fuzzy numbers λ~iand μ~i, leading to the

fuzzy utilization:

$$\tilde{\rho}_i = \frac{\tilde{\lambda}_i}{\tilde{\mu}_i}.$$

The division of fuzzy numbers is not linear and thus it is normally computed by using α-cuts. The utilization of fuzzy is expressed as an interval:

$$\tilde{\rho}_i^{\alpha} = \left[\frac{\lambda_{i,\alpha}^L}{\mu_{i,\alpha}^U}, \frac{\lambda_{i,\alpha}^U}{\mu_{i,\alpha}^L}\right].$$

where λi,αL and λi,αU are the lower and upper bounds of the α-cut of λ~i , and μi,αL and μi,αU are similarly defined for μ~i. This approach ensures that the full range of possible utilization values under uncertainty is captured.

5.2. Average Number in System

The average number of jobs in the system (queue plus server) is a fundamental performance metric. In classical M/M/1 systems, it is given by Li=ρi/(1−ρi).

Extending this to the fuzzy environment, the fuzzy average number in the system is:

$$\tilde{L}_i = \frac{\tilde{\rho}_i}{1 - \tilde{\rho}_i}.$$

Because the operation involves division by a fuzzy number, α-cut based interval arithmetic is employed to compute the bounds at each confidence level:

$$\tilde{L}_i^{\alpha} = \left[\frac{\rho_{i,\alpha}^L}{1 - \rho_{i,\alpha}^U}, \frac{\rho_{i,\alpha}^U}{1 - \rho_{i,\alpha}^L}\right].$$

This interval representation captures the best-case and worst-case estimates of the number of jobs in the system, allowing system designers to evaluate both optimistic and conservative scenarios[15].

The fuzzy waiting time W~iat node i is computed from Little's Law as the ratio of the average number in the system to the arrival rate:

$$\tilde{W}_i = \frac{\tilde{L}_i}{\tilde{\lambda}_i}$$

Using α-cut intervals:

$$\tilde{W}_i^{\alpha} = \left[\frac{L_{i,\alpha}^L}{\lambda_{i,\alpha}^U}, \frac{L_{i,\alpha}^U}{\lambda_{i,\alpha}^L}\right].$$

This measure provides a range of plausible waiting times under parameter uncertainty.

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[14] Fuzzy analysis of queuing models with C servers. (2022). International Journal of Creative Research Thoughts, 10(4).

[15] Alonge w'Omatete, J., Matendo, R. M., & Okenge, D. L. (2025). Performance parameters of fuzzy Markovian queueing system FM/FM/1 in transient regime using flexible α-cuts. Journal of Computing Research and Innovation, 8(1).

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5.3. Network-Level Metrics

In case of a network of multiple service nodes, performance measurements can be summarized to derive network level statistics.

The fuzzy average number of jobs in the network is the sum of the fuzzy average numbers in each node:

$$\tilde{L}_{\text{total}} = \sum_{i=1}^{N} \tilde{L}_i,$$

where N is the number of nodes. Likewise, the overall waiting time may be defined as the maximum fuzzy waiting time of all the nodes, as the network bottleneck:

$$\widetilde{W}_{\text{total}} = \max_{i=1,\dots,N} \widetilde{W}_i.$$

These fuzzy metrics at network level enable the system planners to access the overall network performance in the presence of uncertainties, and also to indicate those nodes that can result in congestion or delays.

VI. NUMERICAL EXAMPLE

In order to explain how the proposed fuzzy queueing network methodology can be applied in practice, we will take a simplistic two node open network. The example illustrates the calculation of fuzzy traffic flows, fuzzy utilizations and the performance measures such as average number in the system and waiting times.

6.1. Parameter Specification

The network is made up of two service nodes which are in series and the jobs come in to the system at Node 1 and may be redirected to the second node before being out of the system at node 2. The network parameters such as arrival and service rates are represented as fuzzy numbers in the form of triangular fuzzy numbers (TFNs) to reflect the existing uncertainty. The values of the respective parameters can be seen in the table below.

Table 1. Parameter Specification

Parameter	Type	Fuzzy Number
Arrival rate to Node 1	λ~0	(8, 10, 12) jobs/hour
Service rate Node 1	μ~1	(12, 15, 18) jobs/hour
Service rate Node 2	μ~2	(10, 12, 14) jobs/hour
Routing p(1→2)	Crisp	0.4
Routing p(1→exit)	Crisp	0.6
Routing p(2→exit)	Crisp	1.0

The arrival rate to Node 1 is represented as λ~1=λ~0, while Node 2 receives a fraction of traffic from Node 1 based on the deterministic routing probability p12=0.

Fuzzy Traffic Equations

The fuzzy arrival rate at Node 1 is straightforward:

λ~1=λ~0=(8,10,12)

For Node 2, the fuzzy arrival rate is obtained by multiplying Node 1's fuzzy arrival rate by the routing probability:

λ~2=λ~1·0.4=(3.2,4,4.8)

These computations utilize scalar multiplication on triangular fuzzy numbers, preserving their triangular shape.

6.3. Fuzzy Utilizations

Using the fuzzy arrival and service rates, fuzzy utilization at each node is calculated as:

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$$\tilde{\rho}_i = \frac{\tilde{\lambda}_i}{\tilde{\mu}_i}$$

For α = 0 (representing the widest range of uncertainty):

$$\tilde{\rho}_1^0 = \left[\frac{8}{18}, \frac{12}{12}\right] = [0.444, 1.0]$$

$$\tilde{\rho}_2^0 = \left[\frac{3.2}{14}, \frac{4.8}{10}\right] = [0.229, 0.48]$$

At α = 1 (representing the core or most likely values):

$$\tilde{\rho}_1^1 = \frac{10}{15} = 0.667, \quad \tilde{\rho}_2^1 = \frac{4}{12} = 0.333$$

Thus, the fuzzy utilizations of the nodes are:

$$\tilde{\rho}_1 = (0.444, 0.667, 1.0), \quad \tilde{\rho}_2 = (0.229, 0.333, 0.48)$$

Node 1 exhibits a maximum utilization of 1.0 at α = 0, indicating that under worst-case conditions the server may become saturated, highlighting the practical advantage of fuzzy modeling in capturing potential instability.

6.4. Fuzzy Performance Measures

Using the classical M/M/1 formulas extended to fuzzy parameters, the average number of jobs in the system at each node is:

$$\tilde{L}_i = \frac{\tilde{\rho}_i}{1 - \tilde{\rho}_i}$$

For Node 1 at α = 0, the lower bound is $\frac{0.444}{1-0.444} \approx 0.8,$ and the upper bound approaches infinity as $\rho_1 \to 1$. At α = 1, $\tilde{L}_1 = 2.0$. For Node 2, L~2 ranges from 0.3 to 0.923 at α = 0, and 0.5 at α = 1.

The fuzzy waiting times are calculated via:

$$\tilde{W}_i = \frac{\tilde{L}_i}{\tilde{\lambda}_i}$$

yielding ranges of [0.08,∞][0.08, \infty][0.08,∞] hours for Node 1 at α = 0 and 0.2 hours at α = 1. For Node 2, the waiting time ranges from 0.075 to 0.192 hours at α = 0 and 0.083 hours at α = 1.

Table 2. Fuzzy Performance Measures (α = 0, 1)

Measure	Node 1 (α=0)	Node 1 (α=1)	Node 2 (α=0)	Node 2 (α=1)
ρ	[0.444,1.0]	0.667	[0.229,0.48]	0.333
L	[0.8, ∞]	2.0	[0.3,0.923]	0.5
W (hrs)	[0.08, ∞]	0.2	[0.075,0.192]	0.083

The findings obviously indicate the capability of fuzzy queueing models in the context of uncertainty and instability that may occur in the system, unlike classical crisp models. Specifically, the unlimited capacity ceiling at Node 1 signals the danger of congestion in extreme cases.

VII. RESULTS AND DISCUSSION

7.1. Impact of Fuzzy Uncertainty

The introduction of fuzzy arrival and service rates magnifies the variability of system performance. For instance, in Node 1, the average number of jobs in the system $\tilde{L}_1$ exhibits an unbounded upper limit when considering $\alpha = 0$, reflecting the worst-case scenario where the arrival rate is at its maximum and the service rate at its minimum. This illustrates how classical crisp models, which provide only a single-point estimate, can significantly underestimate the risk of congestion. Similarly, the fuzzy waiting time $\tilde{W}_1$ shows a wide range from 0.08 hours to infinity, highlighting the potential for severe delays under extreme conditions.

The fuzzy intervals thus provide a richer picture of system behavior, allowing decision-makers to evaluate both optimistic and pessimistic scenarios. The core values at $\alpha = 1$ represent the most likely outcomes, while the lower and upper bounds at $\alpha = 0$ and intermediate α-cuts capture variability due to uncertainty.

7.2. Parameter Sensitivity

The findings state that service rate uncertainty positively affects utilization and waiting times as compared to arrival rate uncertainty. This is visible in Node 1 where the upper limit of utilization is 1.0 in the worst-case scenario whereas in Node 2, the utilization range is comparatively moderate. The importance of monitoring and control of service capacity in uncertain environment is due to the increased likelihood of the node with stronger service rate variation to be congested.

By changing the form of the triangular fuzzy numbers, sensitivity analysis could also be done using the fuzzy framework. The decision-makers are able to evaluate the impact that variation in the most likely, minimum and maximum values of the arrival and service rates have on the overall performance of the networks. This type of analysis is important in systems where the historical data is not dense or where the conditions under which the operations take place are changing.

7.3. Comparison with Crisp Models

To benchmark the fuzzy model, the system is analyzed using crisp mean values: $\lambda=10$ jobs/hour, $\mu_1=15$ jobs/hour, and $\mu_2=12$jobs/hour. The resulting performance measures are:

$$\rho_1 = 0.667, \quad \rho_2 = 0.333$$

$$L_1 = 2.0, \quad L_2 = 0.5$$

$$W_1 = 0.2 \text{ hours}, \quad W_2 = 0.083 \text{ hours}$$

These values match the $\alpha = 1$ core values of the fuzzy model, demonstrating consistency between fuzzy and crisp analyses under nominal conditions. However, the crisp model fails to capture extreme scenarios, such as the potential for Node 1 to reach full utilization ($\rho_1=1$) or the unbounded waiting time, which are clearly revealed by the fuzzy intervals. This highlights the advantage of fuzzy modeling in risk-aware planning and robust decision-making.

7.4. Computational Challenges

Many α-cuts of interval arithmetic are needed to compute fuzzy performance measures. The analysis is computationally intensive as the size and complexity of the network increases, which in turn increases the number of α-level

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calculations. The overhead can be decreased by techniques like fewer α-cuts, or more efficient defuzzification techniques, which are based on optimization. Future research can involve effective algorithms of large scale fuzzy queueing network.

7.5. Practical Implications

The fuzzy queueing models find application especially in cases where:

Past data is scarce or inaccurate and accurate parameter estimation is challenging.

System parameters are subjective or expert dependent like service rate variations which are caused by human operators.

Risk-averse decisions. To avoid system overload or too much delays, one needs to know the best-case and the worst-case performance.

Figure 2 illustrates the fuzzy arrival and service rates for the numerical example. The triangular membership functions clearly depict the range of possible values for λ~ and μ~ providing a visual representation of uncertainty.

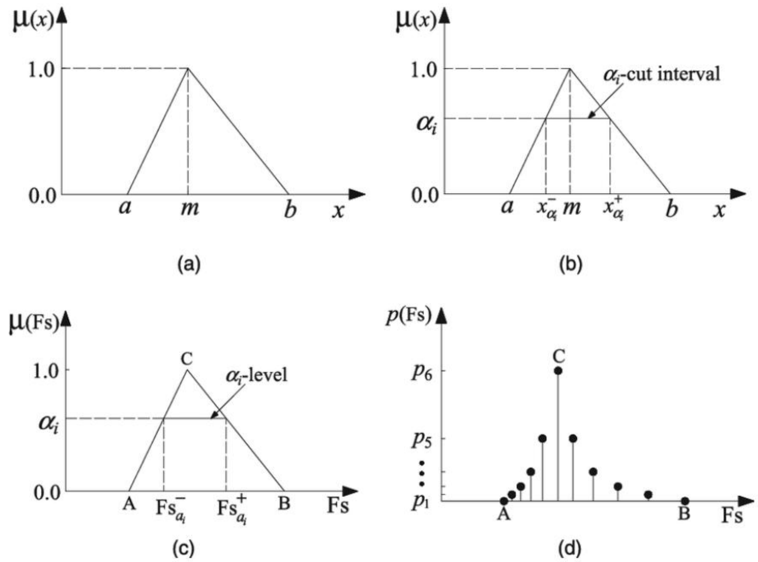

Figure 2. Fuzzy Arrival & Service Rate Membership Functions

VIII. CONCLUSION

This paper constructs a methodology of systematic analysis of the performance of a queueing network considering uncertain arrival and service rates that are represented in the form of triangular fuzzy numbers. The important performance indicators, including utilization, average number of jobs, and the waiting times, are modeled through extension of the classical queueing theory to the use of fuzzy arithmetic and alpha cut interval analysis in order to capture the inherent variability in the real world systems. The example of the two node network indicates the practicality of the approach as it presents the range of possible outcomes and indicates possible extremes, namely, congestion or unlimited waiting time. Although it offers powerful information to decision-making, the technique creates computational problems to large networks. The computational efficiency and further expansion of the framework to more complex, multi-class, or closed networks should be addressed in the future work. Fuzzy queueing modeling therefore improves reliability, planning and operational resiliency.

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