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Parametric Solutions of Ternary Quadratic Diophantine Equations

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Abstract: *This study investigates a generalized quadratic Diophantine equation $s^2 + t^2 - r^2 = (\mathcal{K} + \mathcal{P})^2$ with three unknowns s, t, r, extending classical techniques to explore connections with Krishnamurthy number $\mathcal{K}$ and Leyland primes $\mathcal{P}$ with values up to 5-digits. Distinct solution strategies are presented, and integer solutions satisfying the equation are explicitly computed with MATLAB scripts.*

Keywords: Diophantine equation, Ternary quadratic Diophantine equation, Krishnamurthy numbers, Leyland primes, Parametric solutions

I. INTRODUCTION

Diophantine equations require integer solutions to polynomial expressions. These equations range from simple linear forms to complex nonlinear relationships, and are closely tied to many classic and modern mathematical problems. For instance, Diophantine equations underpin the search for Pythagorean triples, the investigation of perfect numbers, and studies of prime distributions. Furthermore, a wide range of number motifs and sequences including squares, cubes, polygonal numbers, and prime numbers are inherently linked to Diophantine equations, which serve as a foundation for exploring varied sets of integer solutions [1-4].

Within this broad context, Quadratic Diophantine equations in multiple variables occupy a part. Solving these equations often reveals deep connections between algebraic properties and arithmetic phenomena, and leads to practical methods for generating integer solutions or discovering number patterns that follow quadratic relationships [5]. A significant area of current research focuses on ternary quadratic Diophantine equations, where a wide array of solution patterns can arise according to the chosen parameters and the structural properties of the equation. A fundamental aspect of this study involves the classification of quadratic Diophantine equations into homogeneous and non-homogeneous types, as this distinction plays a determinant role in determining suitable methods for analysis and the nature of the integer solutions obtained [6-12].

This present study builds upon these foundational works, presenting methods to solve a generalized quadratic Diophantine equation $s^2 + t^2 - r^2 = (\mathcal{K} + \mathcal{P})^2$ in three variables with $\mathcal{K}$ being Krishnamurthy number, $\mathcal{P}$ corresponds to a Leyland prime with values up to 5-digit. The approach utilizes reductions to the Pellian form and the application of Brahmagupta's lemma to construct families of integer solutions. Additionally, computational algebra technique, particularly MATLAB, is employed to systematically enumerate and verify these solutions.

II. RESULTS AND DISCUSSIONS

This section demonstrates the integer solutions identified for

$$s^2 + t^2 - r^2 = (\mathcal{K} + \mathcal{P})^2 \qquad (1)$$

under three patterns.

Pattern I:

Postulate $s = at$, for any integer $a > 1$,substituting into the original equation it follows that

$$t^2(a^2 + 1) - r^2 = (\mathcal{K} + \mathcal{P})^2$$

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This can be rearranged as

$$r^2 = t^2(a^2+1) - (\mathcal{K}+\mathcal{P})^2 \quad (2)$$

For the least positive integer solution, set $t_0 = \mathcal{K}+\mathcal{P}$ and $r_0 = a(\mathcal{K}+\mathcal{P})$, verifying that $r_0^2 = (a(\mathcal{K}+\mathcal{P}))^2$.

To generate further integer solutions, the equation (2) is associated with the Pell type equation

$$r^2 = t^2(a^2+1) + 1$$

Whose general solution $(\tilde{t}_n, \tilde{r}_n)$ is given by

$$\tilde{r}_n = \frac{f_{n,a}}{2}$$

$$\tilde{t}_n = \frac{g_{n,a}}{2\sqrt{a^2+1}}$$

where,

$$f_{n,a} = \left(2a^2+1+2a\sqrt{a^2+1}\right)^{n+1} + \left(2a^2+1-2a\sqrt{a^2+1}\right)^{n+1}$$

$$g_{n,a} = \left(2a^2+1+2a\sqrt{a^2+1}\right)^{n+1} - \left(2a^2+1-2a\sqrt{a^2+1}\right)^{n+1}$$

Applying Brahmagupta lemma between the solutions (t_0, r_0) and $(\tilde{t}_n, \tilde{r}_n)$ yields a parametric description of all integer solutions to the original equation (1)

The solution set is therefore,

$$t_{n+1} = \frac{(\mathcal{K}+\mathcal{P})f_{n,a}}{2} + \frac{a(\mathcal{K}+\mathcal{P})g_{n,a}}{2\sqrt{a^2+1}}$$

$$r_{n+1} = \frac{a(\mathcal{K}+\mathcal{P})f_{n,a}}{2} + \frac{(a^2+1)(\mathcal{K}+\mathcal{P})g_{n,a}}{2\sqrt{a^2+1}} n = 0,1,2, \ldots ..$$

In accordance with the parameterization $s = at$, the value of s_{n+1} is explicitly expressed as

$$s_{n+1} = \frac{a(\mathcal{K}+\mathcal{P})f_{n,a}}{2} + \frac{a^2(\mathcal{K}+\mathcal{P})g_{n,a}}{2\sqrt{a^2+1}}$$

Thus, the above solutions collectively define the full parametric family of integer solutions for the Diophantine equation $s^2 + t^2 - r^2 = (\mathcal{K}+\mathcal{P})^2$.

For each choice of the parameters $\mathcal{K} \in \{145, 40585\}$ and $\mathcal{P} \in \{17, 593, 32993\}$, the equation $s^2 + t^2 - r^2 = (\mathcal{K}+\mathcal{P})^2$ admits a corresponding set of integer solutions, which is computed and tabulated.

TABLE I

$\mathcal{K}$	$\mathcal{P}$	Ternary Quadratic Diophantine Equation	Solution
145	17	$s^2 + t^2 - r^2 = 26244$	$s_{n+1} = \frac{164af_{n,a}}{2} + \frac{164a^2g_{n,a}}{2\sqrt{a^2+1}}$ $t_{n+1} = \frac{164f_{n,a}}{2} + \frac{164ag_{n,a}}{2\sqrt{a^2+1}}$ $r_{n+1} = \frac{164af_{n,a}}{2} + \frac{164(a^2+1)g_{n,a}}{2\sqrt{a^2+1}}$
145	593	$s^2 + t^2 - r^2 = 544644$	$s_{n+1} = \frac{738af_{n,a}}{2} + \frac{738a^2g_{n,a}}{2\sqrt{a^2+1}}$ $t_{n+1} = \frac{738f_{n,a}}{2} + \frac{738ag_{n,a}}{2\sqrt{a^2+1}}$ $r_{n+1} = \frac{738af_{n,a}}{2} + \frac{738(a^2+1)g_{n,a}}{2\sqrt{a^2+1}}$
145	32993	$s^2 + t^2 - r^2 = 1098127044$	$s_{n+1} = \frac{33138af_{n,a}}{2} + \frac{33138a^2g_{n,a}}{2\sqrt{a^2+1}}$ $t_{n+1} = \frac{33138f_{n,a}}{2} + \frac{33138ag_{n,a}}{2\sqrt{a^2+1}}$ $r_{n+1} = \frac{33138af_{n,a}}{2} + \frac{33138(a^2+1)g_{n,a}}{2\sqrt{a^2+1}}$

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40585	17	$s^2+t^2-r^2=1648522404$	$s_{n+1}=\frac{40602af_{n,a}}{2}+\frac{40602a^2g_{n,a}}{2\sqrt{a^2+1}}$ $t_{n+1}=\frac{40602f_{n,a}}{2}+\frac{40602ag_{n,a}}{2\sqrt{a^2+1}}$ $r_{n+1}=\frac{40602af_{n,a}}{2}+\frac{40602(a^2+1)g_{n,a}}{2\sqrt{a^2+1}}$
40585	593	$s^2+t^2-r^2=1695627684$	$s_{n+1}=\frac{41178af_{n,a}}{2}+\frac{41178a^2g_{n,a}}{2\sqrt{a^2+1}}$ $t_{n+1}=\frac{41178f_{n,a}}{2}+\frac{41178ag_{n,a}}{2\sqrt{a^2+1}}$ $r_{n+1}=\frac{41178af_{n,a}}{2}+\frac{41178(a^2+1)g_{n,a}}{2\sqrt{a^2+1}}$
40585	32993	$s^2+t^2-r^2=5413722084$	$s_{n+1}=\frac{73578af_{n,a}}{2}+\frac{73578a^2g_{n,a}}{2\sqrt{a^2+1}}$ $t_{n+1}=\frac{73578f_{n,a}}{2}+\frac{73578ag_{n,a}}{2\sqrt{a^2+1}}$ $r_{n+1}=\frac{73578af_{n,a}}{2}+\frac{73578(a^2+1)g_{n,a}}{2\sqrt{a^2+1}}$

Pattern II:

Introduce the parameterization $r=at$, where $a>1$ is an integer. Substitution into equation (1) yields

$$s^2-t^2(a^2-1)=(\mathcal{K}+\mathcal{P})^2$$

Rearranging terms gives

$$s^2=t^2(a^2-1)+(\mathcal{K}+\mathcal{P})^2 \qquad (3)$$

The least positive integer solution is $t_0=(\mathcal{K}+\mathcal{P})$ and $s_0=a(\mathcal{K}+\mathcal{P})$, since $s_0^2=(a(\mathcal{K}+\mathcal{P}))^2$

Further integer solutions can be systematically generated by associating equation (3) with the Pell type equation

$$s^2=t^2(a^2-1)+1$$

The general solution $(\tilde{t}_n,\tilde{s}_n)$ for the Pell type equation is determined as

$$\tilde{s}_n=\frac{f_{n,a}}{2}$$

$$\tilde{t}_n=\frac{g_{n,a}}{2\sqrt{a^2-1}}$$

where,

$$f_{n,a}=\left(a+\sqrt{a^2-1}\right)^{n+1}+\left(a-\sqrt{a^2-1}\right)^{n+1}$$

$$g_{n,a}=\left(a+\sqrt{a^2-1}\right)^{n+1}-\left(a-\sqrt{a^2-1}\right)^{n+1}$$

By applying Brahmagupta lemma to the solutions (t_0,s_0) and $(\tilde{t}_n,\tilde{s}_n)$, a parametric family of integer solutions to the equation (1) is obtained.

The solution set is therefore,

$$t_{n+1}=\frac{(\mathcal{K}+\mathcal{P})f_{n,a}}{2}+\frac{a(\mathcal{K}+\mathcal{P})g_{n,a}}{2\sqrt{a^2-1}}$$

$$s_{n+1}=\frac{a(\mathcal{K}+\mathcal{P})f_{n,a}}{2}+\frac{(a^2-1)(\mathcal{K}+\mathcal{P})g_{n,a}}{2\sqrt{a^2-1}} n=0,1,2,\dots..$$

In accordance with the parameterization $r=at$, value ofr_{n+1} is explicitly expressed as

$$r_{n+1}=\frac{a(\mathcal{K}+\mathcal{P})f_{n,a}}{2}+\frac{a^2(\mathcal{K}+\mathcal{P})g_{n,a}}{2\sqrt{a^2-1}}$$

Thus, these solutions jointly represent the complete family of integer solutions for the equation $s^2+t^2-r^2=(\mathcal{K}+\mathcal{P})^2$.

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For each choice of the parameters $\mathcal{K} \in \{145, 40585\}$ and $\mathcal{P} \in \{17, 593, 32993\}$, the equation $s^2 + t^2 - r^2 = (\mathcal{K} + \mathcal{P})^2$ admits a corresponding set of integer solutions, which is computed and tabulated below.

TABLE II

$\mathcal{K}$	$\mathcal{P}$	Ternary Quadratic Diophantine Equation	Solution
145	17	$s^2 + t^2 - r^2 = 26244$	$s_{n+1} = \frac{164af_{n,a}}{2} + \frac{164a^2 g_{n,a}}{2\sqrt{a^2-1}}$ $t_{n+1} = \frac{164f_{n,a}}{2} + \frac{164ag_{n,a}}{2\sqrt{a^2-1}}$ $r_{n+1} = \frac{164af_{n,a}}{2} + \frac{164(a^2-1)g_{n,a}}{2\sqrt{a^2-1}}$
145	593	$s^2 + t^2 - r^2 = 544644$	$s_{n+1} = \frac{738af_{n,a}}{2} + \frac{738a^2 g_{n,a}}{2\sqrt{a^2-1}}$ $t_{n+1} = \frac{738f_{n,a}}{2} + \frac{738ag_{n,a}}{2\sqrt{a^2-1}}$ $r_{n+1} = \frac{738af_{n,a}}{2} + \frac{738(a^2-1)g_{n,a}}{2\sqrt{a^2-1}}$
145	32993	$s^2 + t^2 - r^2 = 1098127044$	$s_{n+1} = \frac{33138af_{n,a}}{2} + \frac{33138a^2 g_{n,a}}{2\sqrt{a^2-1}}$ $t_{n+1} = \frac{33138f_{n,a}}{2} + \frac{33138ag_{n,a}}{2\sqrt{a^2-1}}$ $r_{n+1} = \frac{33138af_{n,a}}{2} + \frac{33138(a^2-1)g_{n,a}}{2\sqrt{a^2-1}}$
40585	17	$s^2 + t^2 - r^2 = 1648522404$	$s_{n+1} = \frac{40602af_{n,a}}{2} + \frac{40602a^2 g_{n,a}}{2\sqrt{a^2-1}}$ $t_{n+1} = \frac{40602f_{n,a}}{2} + \frac{40602ag_{n,a}}{2\sqrt{a^2-1}}$ $r_{n+1} = \frac{40602af_{n,a}}{2} + \frac{40602(a^2-1)g_{n,a}}{2\sqrt{a^2-1}}$
40585	593	$s^2 + t^2 - r^2 = 1695627684$	$s_{n+1} = \frac{41178af_{n,a}}{2} + \frac{41178a^2 g_{n,a}}{2\sqrt{a^2-1}}$ $t_{n+1} = \frac{41178f_{n,a}}{2} + \frac{41178ag_{n,a}}{2\sqrt{a^2-1}}$ $r_{n+1} = \frac{41178af_{n,a}}{2} + \frac{41178(a^2-1)g_{n,a}}{2\sqrt{a^2-1}}$
40585	32993	$s^2 + t^2 - r^2 = 5413722084$	$s_{n+1} = \frac{73578af_{n,a}}{2} + \frac{73578a^2 g_{n,a}}{2\sqrt{a^2-1}}$ $t_{n+1} = \frac{73578f_{n,a}}{2} + \frac{73578ag_{n,a}}{2\sqrt{a^2-1}}$ $r_{n+1} = \frac{73578af_{n,a}}{2} + \frac{73578(a^2-1)g_{n,a}}{2\sqrt{a^2-1}}$

Pattern III

Introduce the bilinear parameterization defined by

$$s = \ell + m, \quad t = \ell - m,$$

where l, m are integers subject to $l \neq m$, and $l, m \neq 0$.

Substituting into the quadratic equation (1) yields

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$$(\ell + m)^2 + (\ell - m)^2 - r^2 = (\mathcal{K} + \mathcal{P})^2$$

which simplifies to

$$r^2 = 2\ell^2 + 2m^2 - (\mathcal{K} + \mathcal{P})^2$$

To construct integer solutions, select pairs (ℓ, m) such that $2\ell^2 + 2m^2 - (\mathcal{K} + \mathcal{P})^2$is a perfect square. That is there exists an integer r satisfying this equation. For each such choice, the corresponding values $s = l + m,\ \ t = l - m$, satisfy the original Diophantine equation (1).

For every pair $(\mathcal{K}, \mathcal{P})$ with $\mathcal{K} \in \{145, 40585\}$ and $\mathcal{P} \in \{17, 593, 32993\}$, the associated integer solution is determined and listed in the table below.

TABLE III

$\mathcal{K}$	$\mathcal{P}$	Ternary Quadratic Diophantine Equation	Solution
145	17	$s^2 + t^2 - r^2 = 26244$	$s = \ell + m$ $t = \ell - m$ $r^2 = 2\ell^2 + 2m^2 - 26244$
145	593	$s^2 + t^2 - r^2 = 544644$	$s = \ell + m$ $t = \ell - m$ $r^2 = 2\ell^2 + 2m^2 - 544644$
145	32993	$s^2 + t^2 - r^2 = 1098127044$	$s = \ell + m$ $t = \ell - m$ $r^2 = 2\ell^2 + 2m^2 - 1098127044$
40585	17	$s^2 + t^2 - r^2 = 1648522404$	$s = \ell + m$ $t = \ell - m$ $r^2 = 2\ell^2 + 2m^2 - 1648522404$
40585	593	$s^2 + t^2 - r^2 = 1695627684$	$s = \ell + m$ $t = \ell - m$ $r^2 = 2\ell^2 + 2m^2 - 1695627684$
40585	32993	$s^2 + t^2 - r^2 = 5413722084$	$s = \ell + m$ $t = \ell - m$ $r^2 = 2\ell^2 + 2m^2 - 5413722084$

III. COMPUTATION OF INTEGER SOLUTIONS VIA MATLAB

MATLAB programs were developed for Patterns I and II to search for integer solutions of the corresponding quadratic Diophantine equations. For each pattern, the code systematically varies the auxiliary parameters over prescribed ranges and evaluates s, t, r. The resulting integer solutions are displayed in Figure 1 to 4.

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Pattern I solutions:

EQUATION 1

n	a	s	t	r
0	1	820	820	1148
1	1	4756	4756	6724
2	1	27716	27716	39196

n	a	s	t	r
0	2	5576	2788	6232
1	2	100040	50020	111848
2	2	1795144	897572	2007032

n	a	s	t	r
0	3	18204	6068	19188
1	3	691260	230420	728652
2	3	26249676	8749892	27669588

EQUATION 2

n	a	s	t	r
0	1	3690	3690	5166
1	1	21402	21402	30258
2	1	124722	124722	176382

n	a	s	t	r
0	2	25092	12546	28044
1	2	450180	225090	503316
2	2	8078148	4039074	9031644

n	a	s	t	r
0	3	81918	27306	86346
1	3	3110670	1036890	3278934
2	3	118123542	39374514	124513146

EQUATION 3

n	a	s	t	r
0	1	165690	165690	231966
1	1	no solution	no solution	no solution
2	1	no solution	no solution	no solution

n	a	s	t	r
0	2	1126692	563346	1259244
1	2	20214180	10107090	22600116
2	2	362728548	181364274	405542844

n	a	s	t	r
0	3	3678318	1226106	3877146
1	3	139676670	46558890	147232134
2	3	5304035142	1768011714	5590943946

Fig. 1

EQUATION 4

n	a	s	t	r
0	1	203010	203010	284214
1	1	no solution	no solution	no solution
2	1	no solution	no solution	no solution

n	a	s	t	r
0	2	1380468	690234	1542876
1	2	24767220	12383610	27690564
2	2	444429492	222214746	496887276

n	a	s	t	r
0	3	4506822	1502274	4750434
1	3	171137430	57045810	180394686
2	3	6498715518	2166238506	6850247634

EQUATION 5

n	a	s	t	r
0	1	205890	205890	288246
1	1	no solution	no solution	no solution
2	1	no solution	no solution	no solution

n	a	s	t	r
0	2	1400052	700026	1564764
1	2	25118580	12559290	28083396
2	2	450734388	225367194	503936364

n	a	s	t	r
0	3	4570758	1523586	4817826
1	3	173565270	57855090	182953854
2	3	6590909502	2196969834	6947428626

EQUATION 6

n	a	s	t	r
0	1	367890	367890	515046
1	1	no solution	no solution	no solution
2	1	no solution	no solution	no solution

n	a	s	t	r
0	2	2501652	1250826	2795964
1	2	44882580	22441290	50180196
2	2	805384788	402692394	900447564

n	a	s	t	r
0	3	8167158	2722386	8608626
1	3	310131270	103377090	326907054
2	3	11776821102	3925607034	12413859426

Fig.2

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Pattern II solutions:

EQUATION 1

n	a	s	t	r
0	1	no solution	no solution	no solution
1	1	no solution	no solution	no solution
2	1	no solution	no solution	no solution

n	a	s	t	r
0	2	1312	656	1148
1	2	4920	2460	4264
2	2	18368	9184	15908

n	a	s	t	r
0	3	2952	984	2788
1	3	17220	5740	16236
2	3	100368	33456	94628

EQUATION 2

n	a	s	t	r
0	1	no solution	no solution	no solution
1	1	no solution	no solution	no solution
2	1	no solution	no solution	no solution

n	a	s	t	r
0	2	5904	2952	5166
1	2	22140	11070	19188
2	2	82656	41328	71586

n	a	s	t	r
0	3	13284	4428	12546
1	3	77490	25830	73062
2	3	451656	150552	425826

EQUATION 3

n	a	s	t	r
0	1	no solution	no solution	no solution
1	1	no solution	no solution	no solution
2	1	no solution	no solution	no solution

n	a	s	t	r
0	2	265104	132552	231966
1	2	994140	497070	861588
2	2	3711456	1855728	3214386

n	a	s	t	r
0	3	596484	198828	563346
1	3	no solution	no solution	no solution
2	3	no solution	no solution	no solution

Fig. 3

EQUATION 4

n	a	s	t	r
0	1	no solution	no solution	no solution
1	1	no solution	no solution	no solution
2	1	no solution	no solution	no solution

n	a	s	t	r
0	2	324816	162408	284214
1	2	1218060	609030	1055652
2	2	no solution	no solution	no solution

n	a	s	t	r
0	3	730836	243612	690234
1	3	no solution	no solution	no solution
2	3	no solution	no solution	no solution

EQUATION 5

n	a	s	t	r
0	1	no solution	no solution	no solution
1	1	no solution	no solution	no solution
2	1	no solution	no solution	no solution

n	a	s	t	r
0	2	329424	164712	288246
1	2	1235340	617670	1070628
2	2	no solution	no solution	no solution

n	a	s	t	r
0	3	741204	247068	700026
1	3	no solution	no solution	no solution
2	3	no solution	no solution	no solution

EQUATION 6

n	a	s	t	r
0	1	no solution	no solution	no solution
1	1	no solution	no solution	no solution
2	1	no solution	no solution	no solution

n	a	s	t	r
0	2	588624	294312	515046
1	2	2207340	1103670	1913028
2	2	no solution	no solution	no solution

n	a	s	t	r
0	3	1324404	441468	1250826
1	3	no solution	no solution	no solution
2	3	no solution	no solution	no solution

Fig. 4

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Likewise, the integer solutions ℓ, m, s, t, r for pattern III were obtained. For each equation, the parameters were varied over prescribed ranges, the conditions $\ell \neq m$ and $\ell, m \neq 0$ were enforced, and the corresponding values were computed. Only those tuples yielding integral values were retained, and the resulting solutions are displayed in Figure 5 to 10.

Equation 1

l	m	s	t	r
-100	-62	-162	-38	38
-100	-58	-158	-42	22
-100	58	-42	-158	22
-100	62	-38	-162	38
-99	-63	-162	-36	36
-99	63	-36	-162	36
-98	-64	-162	-34	34
-98	64	-34	-162	34
-97	-65	-162	-32	32
-97	-61	-158	-36	4
-97	61	-36	-158	4
-97	65	-32	-162	32
-96	-66	-162	-30	30
-96	66	-30	-162	30
-95	-67	-162	-28	28
-95	-65	-160	-30	16
-95	65	-30	-160	16
-95	67	-28	-162	28
-94	-68	-162	-26	26
-94	68	-26	-162	26
-93	-69	-162	-24	24
-93	69	-24	-162	24
-92	-70	-162	-22	22
-92	70	-22	-162	22
92	-70	22	162	22
92	70	162	22	22
93	-69	24	162	24

Fig. 5

Equation 2

l	m	s	t	r
-500	-190	-690	-310	166
-500	190	-310	-690	166
-499	-197	-696	-302	176
-499	197	-302	-696	176
-498	-156	-654	-342	6
-498	156	-342	-654	6
-497	-179	-676	-318	116
-497	179	-318	-676	116
-496	-166	-662	-330	50
-496	166	-330	-662	50
-493	-175	-668	-318	52
-493	175	-318	-668	52
-492	-174	-666	-318	6
-492	174	-318	-666	6
-491	-179	-670	-312	40
-491	179	-312	-670	40
-489	-183	-672	-306	24
-489	183	-306	-672	24
-488	-190	-678	-298	62
-488	190	-298	-678	62
-482	-200	-682	-282	2
-482	200	-282	-682	2
482	-200	282	682	2
482	200	682	282	2
488	-190	298	678	62
488	190	678	298	62
489	-183	306	672	24

Fig. 6

Equation 3

l	m	s	t	r
-16989	-16917	-33906	-72	7176
-16984	-16982	-33966	-2	7454
-16982	-16984	-33966	2	7454
-16975	-16657	-33632	-318	5752
-16965	-16713	-33678	-252	6012
-16954	-16828	-33782	-126	6566
-16947	-16875	-33822	-72	6768
-16941	-16779	-33720	-162	6240
-16936	-16642	-33578	-294	5426
-16933	-16919	-33852	-14	6916
-16926	-16728	-33654	-198	5874
-16922	-16684	-33606	-238	5594
-16919	-16933	-33852	14	6916
-16917	-16989	-33906	72	7176
-16913	-16835	-33748	-78	6388
-16900	-16730	-33630	-170	5734
-16891	-16621	-33512	-270	5000
-16877	-16799	-33676	-78	5996
-16877	-16651	-33528	-226	5104
-16875	-16947	-33822	72	6768
-16870	-16828	-33698	-42	6118
-16869	-16653	-33522	-216	5064

Fig. 7

Equation 4

l	m	s	t	r
-29789	23	-29766	-29812	11236
-29477	-5	-29482	-29472	9448
-29477	5	-29472	-29482	9448
-29107	29	-29078	-29136	6776
-29005	25	-28980	-29030	5836
-29004	18	-28986	-29022	5826
-29003	11	-28992	-29014	5816
-29002	-4	-29006	-28998	5806
-29002	4	-28998	-29006	5806
-29001	-3	-29004	-28998	5796
-29001	3	-28998	-29004	5796
-29000	-10	-29010	-28990	5786
-29000	10	-28990	-29010	5786
-28999	17	-28982	-29016	5776
-28998	24	-28974	-29022	5766
-28837	11	-28826	-28848	3824
-28802	16	-28786	-28818	3254
-28762	-4	-28766	-28758	2446
-28762	4	-28758	-28766	2446
-28719	21	-28698	-28740	1020
-28718	20	-28698	-28738	962
-28716	18	-28698	-28734	834
-28710	12	-28698	-28722	78
28710	12	28722	28698	78
28716	18	28734	28698	834
28718	20	28738	28698	962
28719	21	28740	28698	1020

Fig. 8

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Equation 5

l	m	s	t	r
-29940	18	-29922	-29958	9858
-29877	15	-29862	-29892	9468
-29702	16	-29686	-29718	8294
-29469	-9	-29478	-29460	6420
-29469	9	-29460	-29478	6420
-29415	15	-29400	-29430	5904
-29414	-8	-29422	-29406	5894
-29414	8	-29406	-29422	5894
-29413	-1	-29414	-29412	5884
-29413	1	-29412	-29414	5884
-29412	-6	-29418	-29406	5874
-29412	6	-29406	-29418	5874
-29411	13	-29398	-29424	5864
-29410	20	-29390	-29430	5854
-29323	11	-29312	-29334	4904
-29321	-1	-29322	-29320	4880
-29321	1	-29320	-29322	4880
-29184	-6	-29190	-29178	2790
-29184	6	-29178	-29190	2790
-29150	20	-29130	-29170	1954
-29126	-4	-29130	-29122	1010
-29126	4	-29122	-29130	1010
29126	-4	29122	29130	1010
29126	4	29130	29122	1010
29150	20	29170	29130	1954
29184	-6	29178	29190	2790
29184	6	29190	29178	2790
29321	-1	29320	29322	4880

Fig. 9

Equation 6

l	m	s	t	r
-40000	33578	-6422	-73578	6422
-39999	33579	-6420	-73578	6420
-39998	33580	-6418	-73578	6418
-39997	33299	-6698	-73296	1856
-39997	33581	-6416	-73578	6416
-39996	33582	-6414	-73578	6414
-39995	33583	-6412	-73578	6412
-39994	33416	-6578	-73410	4310
-39994	33584	-6410	-73578	6410
-39993	33585	-6408	-73578	6408
-39992	33586	-6406	-73578	6406
-39991	33587	-6404	-73578	6404
-39990	33588	-6402	-73578	6402
-39989	33311	-6678	-73300	1940
-39989	33589	-6400	-73578	6400
-39988	33290	-6698	-73278	898
-39988	33590	-6398	-73578	6398
-39987	33591	-6396	-73578	6396
-39986	33464	-6522	-73450	4870
-39986	33592	-6394	-73578	6394
-39985	33593	-6392	-73578	6392
-39984	33318	-6666	-73302	1974
-39984	33594	-6390	-73578	6390
-39983	33595	-6388	-73578	6388
-39982	33380	-6602	-73362	3442
-39982	33596	-6386	-73578	6386
-39981	33597	-6384	-73578	6384
-39980	33598	-6382	-73578	6382
-39979	33599	-6380	-73578	6380
-39978	33360	-6618	-73338	2922
-39978	33600	-6378	-73578	6378

Fig. 10

IV. CONCLUSION

The generalized quadratic Diophantine equation $s^2 + t^2 - r^2 = (\mathcal{K} + \mathcal{P})^2$ with three unknowns s, t, r, where $\mathcal{K}$ is a Krishnamurthy number and $\mathcal{P}$ corresponds to a Leyland prime ranging up to 5-digit, has been systematically investigated through three distinct patterns, each yielding explicit families of integer solutions. Across these patterns, appropriate substitutions and parameterizations were employed to reduce the original equation to more tractable forms, enabling the derivation of solution sets in a unified framework. Further, the corresponding integer solutions were computed meticulously using MATLAB, thereby illustrating both the theoretical results and their computational verification.

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