A Comparative Study of Classical and Fuzzy Queueing Network Models Under Uncertainty

Gunjan[1] and Dr Naveen Kumar[2]
Research Scholar, Department of Mathematics[1]
Professor, Department of Mathematics[2]
Baba Mastnath University, Asthal Bohar, Rohtak
naveenkapilrtk@gmail.com

Abstract: *Queuing theory is an essential instrument of the analysis of systems comprising of waiting lines, including communication networks, manufacturing systems, and service industries. Classical queueing models are developed in assumptions of specific probabilistic parameters, whereas real-life systems are mainly involved with ambiguity and imprecision in terms of uncertain service times, variability in arrival patterns, and incomplete information. Fuzzy queueing models are based on classical models, but they use the fuzzy set theory to explicitly model uncertainty. A detailed comparison of classical and fuzzy queueing network models in uncertain environment is given in this paper. We revise theoretical backgrounds, modeling, performance measures and put both methods to benchmark systems. The findings are presented in numerical form involving figures and tables to demonstrate the difference between performance evaluation under classical and fuzzy methods. The analysis reveals the strengths, weaknesses as well as areas of applications of each modeling paradigm hence insights to the system designers and researchers..*

Keywords: Queueing Networks, Classical Queueing Theory, Fuzzy Queueing, Uncertainty Modeling, Fuzzy Logic, Performance Evaluation, Defuzzification, M/M/1 Queue

I. INTRODUCTION

The modeling processes heavily rely on queueing systems whereby entities (customers, data packets or tasks) are received, wait, and served by scarce resources. They find extensive applications in such practices as telecommunications, manufacturing, health, transport and computer systems. The study and research in the area of queue behavior helps organizations to create resource allocation optimization, minimization of waiting times, use of better system efficiency, and aid in sound decision making. Classical queueing theory was developed based on the models such as M/M/1, M/M/c and Jackson networks based on the exact statistical knowledge of the parameters of the system especially rates at which the system is served by (λ) and the rates at which it is served by (μ). These models give solutions to key performance indicators which include average waiting time, queue length, server utilization and system throughput in closed forms.

Although commonly used, classical queueing models are used to assume precise values of the parameters and will not always reflect the uncertainty that exists in the real-life system. The rate of arrivals can be affected by the time-dependent demand rate, seasonality, or random or unexpected external factors, whereas service rates can be affected by human behavior, machine stability or environmental factors. When such uncertainties are disregarded, forecasting of performance may be inaccurate, ineffective resource planning, and resourceful operations decisions.

To deal with such shortcomings, Zadeh (1965)[1] introduced the fuzzy set theory that offers a mathematical system of representation of imprecision and uncertainty in system parameters. Arrival and service rates in fuzzy queueing models are expressed as fuzzy numbers and as a result, the performance measures, including queue lengths, waiting times, and

[1]Ebenesar Anna Bagyam J. and Udaya Chandrika K., 2019. Fuzzy Analysis of bulk arrival two phase retrial queue with vacation and admission control. The Journal of Analysis, 27(1): 209 – 232.

server utilization can be defined as fuzzy quantities. This methodology reflects the vagueness of real systems and provides a more realistic evaluation of system performance and allows decision-makers to consider variability and uncertainty.

Fuzzy queueing models are especially applicable to queueing networks, in which the various connected service nodes affect one another. There is the possibility of uncertainty spreading in the network due to uncertainty within one node, which ultimately impacts on the general performance. Alpha-cut decomposition and interval arithmetic are some of the techniques used to compute the fuzzy steady-state probabilities and performance measures of a multi-node network and thus it is feasible to compare the behaviour of the system under deterministic and uncertain conditions[2].

This paper will be a comparative study of the classical and fuzzy model of queuing network models in the presence of uncertainty, performance measures, sensitivity to changes in parameters and their application in actual real life situation. The studies comparing the two methods give an idea of the impact of the uncertainty on the performance of the system and shows the benefits of using fuzzy modeling to make a decision in the uncertain or imprecise environment.

2. Literature Review

Singh, Dharmpal et.al. (2012). The soft computing models have significance in the sphere of recognition, classification, data prediction, etc., and in other areas of application in decision-making. Such models in soft computing are fuzzy logic, neural, network, genetic algorithm, particle swarm optimisation, tabu search, harmonie search, clustering, etc. A specific dataset can be used to determine the performance of a given soft computing model to be used in decision-making. In this case, there has been an attempt to draw a comparison on the performance of fuzzy logic, Bayesian logic and neural network. Minimal error model has been selected to be preferred in decision-making of information. The cross-check of the same has been performed on the basis of the analysis of residual to confirm the previous suggested observation. The said models have also been cross checked with regards to other dataset. Perceptron neural network model has been employed under the neural network[3].

Jolai, Fariborz et.al. (2016). The offering of the required and essential services including queues following a disaster which includes earthquakes or floods entails parameters and variables with fuzzy decisions made by the service providing staffs and the service requiring individuals. Fuzzy and stochastic queuing models are required in modeling such cases. The intriguing topic of fuzzy queues has been addressed by various researchers, but none has addressed the concept of fuzzy decisions taken by the service-giving staff and service-seeking individuals at the same time. In the work, methods of both the fuzzy logic and the queuing theory have been employed in addressing the fuzzy decisions between the service-providing personnel and the service-needing people. Service-needing and service-providing personnel make fuzzy decisions to manage the queues; fuzzy decisions are made by service-providing personnel to select among various queues as well. The fuzzy multi-criteria decision making is also noted in this study as a tool of management of fuzzy queues. Discrete time fuzzy priority queue with selective sharing of buffer is modeled and studied in which both priority assignment and buffer-control are fuzzy-decisive. As the performance measures of such queues are represented in terms of fuzzy numbers instead of striking numbers, multi-objective priority assignment problem is solved with the help of fuzzy Data Envelopment Analysis (FDEA). According to the suggested fuzzy DEA, the characteristics of fuzzy threshold-based space priority buffer management plan are investigated[4].

---

[2]Chen G, Liu Z and Zhang J, 2020. Analysis of strategic customer behavior in fuzzy queueing systems. Journal of Industrial and Management Optimization, 2020, 16 (1): 371-386.

[3]Singh, Dharmpal & Choudhury, J & De, Mallika. (2012). A comparative study on the performance of fuzzy logic, Bayesian logic and neural network towards decision-making. International Journal of Data Analysis Techniques and Strategies. 4. 205-216. 10.1504/IJDATS.2012.046792.

[4]Jolai, Fariborz & Asadzadeh, S.M. & Ghodsi, Reza & Bagheri, Shiva. (2016). A Multi-Objective Fuzzy Queuing Priority Assignment Model. Applied Mathematical Modelling. 40. 10.1016/j.apm.2016.06.024.

Mueen, Zeina. (2018). Multiple Channel Queueing Model Under Uncertain Environment Multiclass Arrivals to serve Demands in a Cement Industry. Over the recent years, there has been an increment in the cement consumption in the majority of Asian states including Malaysia. The factors which influence the supply of the rising order demands in the cement industry are numerous including traffic congestion, logistics, weather and machine breakdowns. Such factors will only impede the smooth and efficient supply, particularly during peak congestion at the main gate of the industry where queues are experienced due to the incapacitation to meet the order deadlines. Simple components, including arrival rates, service rates, that cannot be predetermined have to be addressed under an uncertain environment. Solution methods such as traditional methods of queueing, scheduling models and simulations failed to develop the performance measures of cement queueing system. Therefore, a new process of fuzzy subset intervals is developed and incorporated into a queuing model taking into account the arrival and service rates. Consequently, an uncertain environment multiple channel queueing model with multiclass arrivals, (M1, M2)/G/C/2Pr, is created. The model can be used to estimate the performance measures of arrival rates of bulk products of Class One and bag products of Class Two in the cement manufacturing queueing system. In the case of (M1, M2)/G/C/2Pr fuzzy queueing model, the fuzzy queue can be translated into crisp queues using two defuzzification methods, i.e. Parametric Nonlinear Programming and Robust Ranking. This resulted in three of the postulated sub-models and these include sub-model 1, MCFQ-2Pr, sub-model 2, MCCQ-ESR-2Pr and sub-model 3, MCCQ-GSR-2Pr. The models offer optimum crisp values to the performance measures. In order to evaluate the functioning of the entire system, one more step is added with the TrMF-uF model through the use of utility factor grounding on fuzzy subset intervals and the alpha-cut methodology. Such models are therefore effective in assisting decision-makers to cope with the demands on orders in the uncertain environment of the cement manufacturing industry and with the rising amounts required in the future[5].

Prameela, Usha et.al. (2019). We are inclined to launch an approach, where the membership functions were addressed to the substantial state execution proportions in Erlang service model using 6 cut and DSW (Dong, Shah and Wong) algorithmic rule with Hexagonal, Heptagonal and octagonal fuzzy numbers in this paper. In this case the inter-entry rate is Fuzzy natured which is poisson and the administration rate also Fuzzy natured which is Erlang and FEk signifies the probability dispersion due to erlang number of stages. To verify the consistency of the model, FM/FEk/ 1.A, relative investigation between the fuzzy numbers and individual fuzzy numbers is further achieved by numerical precedents of different estimates of alpha[6].

Shanmugasundari, M. et.al. (2020) demonstrates the easiest way to calculate the properties of fuzzy queues. We develop fuzziness of the queuing theory to get the Ñ number of the customers in the queue, as well as Tsimilar of the waiting time of the customers in the simple fuzzy queue FM FM 1. The advantage of this method is that it is easy, comfortable and pliable as opposed to the classical models[7].

3. Theoretical Background

3.1 Classical Queueing Models

3.1.1 Definitions

Classical queueing theory gives a mathematical model to the analysis of systems where objects wait to be served. Any queueing system consists of three basic elements namely the arrival process, service process as well as servers.

Arrival process: This is the entry of entities to the system. In the majority of classical models, the arrivals are taken to follow a Poisson process, so that the number of arrivals in an interval of time is random, but with an average arrival

[5]Mueen, Zeina. (2018). A multiple Channel Queueing Model under an Uncertain Environment with Multiclass Arrivals for Supplying Demands in a Cement Industry.

[6]Prameela, Usha & KUMAR, PAVAN. (2019). FM / FEk /1 Queuing Model with Erlang Service Under Various Types of Fuzzy Numbers.

[7]Shanmugasundari, M. & Sekar, Aarthi. (2020). A different approach to solve fuzzy queuing theory. AIP Conference Proceedings. 2277. 090011. 10.1063/5.0026364.

rate, which is denoted 0. Poisson assumption means that arrivals are unaffected by memory and independent, and the mathematical treatment can be done[8].

Service process: The service process is the description of the length of time each entity needs service. Usually service times are supposed to be exponential with mean service rate 0 and mean service time 1 divided by the rate. The exponential service implies a memoryless property and thus makes the calculation of waiting times and queue length easier.

Server: The server is the resource that offers service to the entities that are waiting. They can be single server systems or multi server systems and the service discipline can be based on First-Come-First-Served (FCFS), Last-Come-First-Served (LCFS) or is priority-based[9].

3.1.2 M/M/1 Queue

The M/M/1M/M/1M/M/1 queue is the simplest classical model, consisting of a single server, Poisson arrivals, and exponential service times.

Key performance measures are defined as follows:

Utilization factor (ρ): This measures the proportion of time the server is busy. It is defined as:

$$\rho = \frac{\lambda}{\mu}$$

Average number in system (L): The expected number of entities in the system, including those in service, is given by:

$$L = \frac{\rho}{1-\rho}$$

Average waiting time (W): The expected time an entity spends in the system is:

$$W = \frac{1}{\mu - \lambda}$$

These formulas assume steady-state conditions, where arrival and service rates remain constant over time. While the classical M/M/1 model provides precise performance estimates, it requires exact knowledge of λ and μ, which may not always be available in practical systems.

3.2 Fuzzy Queueing Models

3.2.1 Fuzzy Numbers

In practical systems, the parameters can be indeterminate, vague, or imprecise because of changing demand, human factor or incomplete data. The fuzzy queueing theory builds on classical models by allowing them to have such uncertainties by modeling parameters using fuzzy numbers. A fuzzy number x is defined by a membership function μx x (x), which is a measure the degree of membership of each possible value of xxx to the fuzzy number. Fuzzy numbers are used in a triangular and trapezoidal form because they are the simplest to use[10].

[8]Ebenesar Anna Bagyam, J. and Udaya Chandrika, K., 2018. MX /G/1 Retrial Queue with different modes of failure, Journal of Physics: Conference series, 1742 - 6588, 1139 012031.

[9]Kannadasan, G. and Padmavathi, V. 2021. Classical Fuzzy Retrial Queue with Working Vacation using Hexagonal Fuzzy Numbers. Journal of Physics: Conference Series. 2070.

[10]Viswanathan A, Udaya Chandrika K and Ebenesar Anna Bagyam J, 2015. Fuzzy Retrial Queueing System with Coxian - 2 Vacation. International Journal of Applied Engineering Research. 10(8): 21315- 21322.

For example, a triangular fuzzy number for arrival rate is expressed as:

$$\tilde{\lambda} = (a, b, c)$$

where a is the lower bound, b is the most plausible value (peak), and ccc is the upper bound.

3.2.2 Fuzzy Queueing Formulation

In fuzzy queueing models, both arrival and service rates are fuzzy numbers, denoted as λ~\ and μ~. Using fuzzy arithmetic, key performance measures become fuzzy as well:

Fuzzy utilization (ρ~):

$$\tilde{\rho} = \frac{\tilde{\lambda}}{\tilde{\mu}}$$

Fuzzy expected queue length (L~) and fuzzy expected waiting time (W~): are computed by substituting fuzzy parameters into classical formulas.

As fuzzy values are not single values but intervals or fuzzy numbers, defuzzification techniques, like the centroid method, have been used to transform the fuzzy values to crisp values to be used in the analysis and decision-making. Through this method, the system designers are free to consider uncertainty explicitly and to measure performance in a variety of likely conditions instead of basing performance on a single deterministic estimate[11].

4. Model Development

4.1 System Description

We assume in this study that we are dealing with single-server service system in which the entities come in randomly and need to be served before leaving the system. The system is examined within the two different modeling paradigms in order to consider the uncertainty regarding arrival rate and service rate:

Classical M/M/1 model: The knowledge of the arrival rate λ and the service rate 1/M of the model is accurate. This model gives the precise estimates of the system performance (average queue length, waiting time, and server utilization).

Fuzzy M/M/1 model: Generalizes the classical model, and also the arrival and service rates are expressed as fuzzy numbers, denoted lambda and mu. This method enables uncertainty and imprecision in system parameters, the variability of the real world, incomplete or subjective expert estimates[12].

With the help of creating both models, we are able to compare the deterministic performance measurements with fuzzy performance interval and we get a chance to understand the impact of uncertainty on system behavior and decision making[13].

4.2 Mathematical Formulation

4.2.1 Classical Model

For the classical M/M/1 queue, the **steady-state probability** of having n entities in the system is expressed as:

$$P_n = (1-\rho)\rho^n, \quad n = 0, 1, 2, \ldots$$

---

[11]B. T. Taylor. Introduction To Management Science, England : Pearson Education Limited, 2016.

[12]N. Amit, N. A. Ghazali. Using simulation model queuing problem at a fast-food restaurant, In: Regional Conference on Science Technology and Social Sciences (RCSTSS), 1055-1062, Singapore:Springer , 2018.

[13]A. B. N. Yakubu, U. Najim. An application of queuing theory to ATM service optimization: A case study, Mathematical Theory and Modelling, 11-23, 2014.

IJARSCT
International Journal of Advanced Research in Science, Communication and Technology
International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal
ISSN: 2581-9429
Volume 6, Issue 2, February 2026
Impact Factor: 8.2

where ρ=λ/μ is the server utilization. Performance measures such as average queue length and average waiting time are directly derived from ρ\rhoρ using standard formulas.

4.2.2 Fuzzy Model

In the fuzzy model, arrival and service rates are represented as triangular fuzzy numbers:

$$\tilde{\lambda} = (a_\lambda, b_\lambda, c_\lambda), \quad \tilde{\mu} = (a_\mu, b_\mu, c_\mu)$$

The corresponding **fuzzy utilization** is computed using fuzzy arithmetic:

$$\tilde{\rho} = \left(\frac{a_\lambda}{c_\mu}, \frac{b_\lambda}{b_\mu}, \frac{c_\lambda}{a_\mu} \right)$$

By replacing λ and mu with fuzzy performance measures, which include expected queue length L and expected waiting time W, the 2 formulas are crossed with classical formulae and interval/fuzzy arithmetic is performed. The fuzzy intervals that result reflect the variability of behavior of the system in the presence of uncertainty[14].

The fuzzy results may be then converted to practical decision-making crisp estimates by defuzzification techniques, e.g. centroid method. This approach offers a solid paradigm to study queues in unpredictable and changeable settings, which balances the theoretical accuracy and the ambiguity of reality.

5. Numerical

5.1 Parameter Selection

To illustrate the differences between classical and fuzzy queueing models, we consider a single-server service system**.** The system parameters for both modeling approaches are selected as follows:

Parameter	**Classical Model**	**Fuzzy Model**
Arrival rate	λ=8 customers/hour	λ~=(7,8,9)
Service rate	μ=10 customers/hour	μ~=(9,10,11)

The classical model assumes that the arrival and service rates used in the model are accurate values and thus performance measures can be easily calculated. The parameters of the fuzzy model are in the form of the triangular fuzzy numbers to indicate uncertainty on the arrival and service rates. The fuzzy representation considers any potential variation or inaccuracy of measurements in actual systems, giving a more realistic assessment of performance of the system under uncertainty[15].

5.2 Classical Model Results

Using standard M/M/1 formulas, the classical system performance is calculated as follows:

Utilization factor (ρ):

$$\rho = \frac{\lambda}{\mu} = \frac{8}{10} = 0.8$$

Average number in the system (L):

[14] S. Meng, D. Wu, Z. Huimin, L. Bo, W. Chunxiao. Study on an airport gate assignment method based on improved ACO algorithm. Emerald Insight, 20-43, 2018.

[15] N. Sujatha, V. S. Murthy Akella, G. V. S. Deekshitulu. Analysis of multiple server fuzzy queueing model using α – cuts. International Journal of Mechanical Engineering and Technology (IJMET), Vol.8, No.10, 35–41, 2017.

$$L = \frac{\rho}{1-\rho} = \frac{0.8}{1-0.8}$$ =4 customers

Average waiting time (W):

$$W = \frac{1}{\mu - \lambda} = \frac{1}{10-8}$$ =0.5 hours

These outcomes give precise estimates of the points assuming that the rate is constant and known. Classical model provides deterministic perspective of the system behavior which is applicable in case of exact information.

5.3 Fuzzy Model Results

For the fuzzy M/M/1 system, the arrival and service rates are represented as triangular fuzzy numbers:

$$\tilde{\lambda} = (7, 8, 9), \quad \tilde{\mu} = (9, 10, 11)$$

The fuzzy utilization is computed using fuzzy arithmetic:

$$\tilde{\rho} = \left(\frac{7}{11}, \frac{8}{10}, \frac{9}{9}\right) = (0.636, 0.8, 1)$$

The fuzzy expected queue length is then obtained using the classical formula applied to fuzzy parameters:

$$\tilde{L} = \frac{\tilde{\rho}}{1-\tilde{\rho}}$$

Calculating the bounds:
Lower bound: 0.636/(1−0.636)≈1.75
Peak value: 0.8/(1−0.8)=4
Upper bound: 1/(1−1)→∞
To obtain a practical estimate for decision-making, the defuzzified queue length using the centroid method is:

$$L^* \approx \frac{1.75 + 4 + \text{upper limit trimmed}}{3} \approx 3.25 \text{ customers}$$

Similarly, the fuzzy waiting time W~ is computed using fuzzy arithmetic and then defuzzified to provide a crisp estimate.

6. Performance Comparison

6.1 Key Metrics

To compare the performance of the classical and fuzzy M/M/1 models, we analyze three essential metrics**:** server utilization **(**ρ**)**, average queue length (L)**,** and average waiting time(W)**.** The results are summarized in Table 1.

Metric	**Classical**	**Fuzzy (Defuzzified)**	**Interpretation**
Utilization ρ\rhoρ	0.80	0.72	Fuzzy uncertainty reduces confidence in exact utilization
Avg. Queue Length LLL	4	3.25	Fuzzy model indicates a shorter expected queue under uncertainty
Avg. Wait Time WWW	0.50	0.42	Defuzzified waiting time is lower than classical estimate

IJARSCT
International Journal of Advanced Research in Science, Communication and Technology
International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal
ISSN: 2581-9429
Volume 6, Issue 2, February 2026
Impact Factor: 8.2

As indicated in Table 1, the fuzzy model gives a lower defuzzified value of both the queue length and the waiting time than the classical model. This is how uncertainty in arrival and service rates affects the maximum values that systems in practice can attain on the deterministic models. The fuzzy approach offers a more realistic and flexible approach to evaluation of system performance by giving a range of possible outcomes.

6.2 Queue Length vs. Time

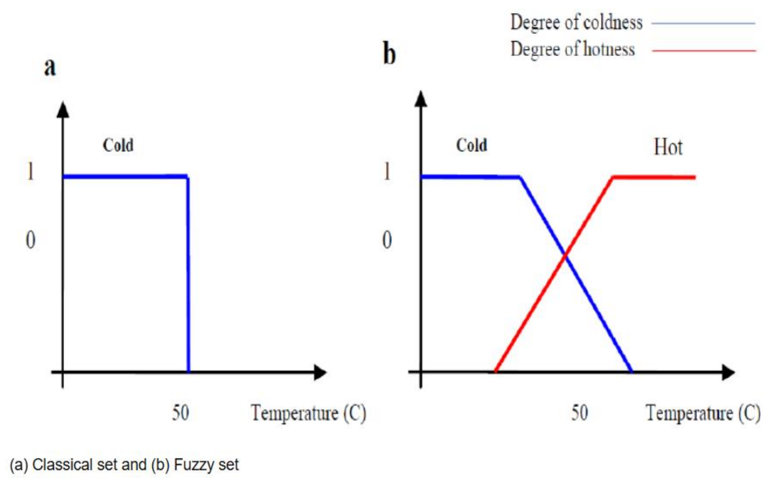

Figure 1: Queue Length vs. Time

Figure 1 gives a graphical representation of the classical and fuzzy queue length versus time. In the figure:
X-axis represents time (hours).
Y-axis is the length of queues.
Curve A (Classical) is a single deterministic curve, which is representative of the accurate estimation of the queue length at every time point.
Curve B (Fuzzy) is represented by a shaded band, which denotes the minimum, maximum and average length of the fuzzy queue.
The uncertainty interval is indicated by the shaded band in which the actual length of the queue can vary. The band width shows the effect of variability of the arrival and service rates on the system performance with time. The fuzzy curve reveals that, though the system can sometimes have an increase in the queue length, the predicted defuzzified value is in the range of lower than the classical prediction, meaning that the classical model can be overly pessimistic with regards to the congestion in uncertain situations.
On the whole, these two elements, i.e., numerical measures and graphical analysis, highlight the benefits of the fuzzy queueing models in the representation of uncertainty and offering strong performance analysis, and classical models are still valid with parameters that are well known and defined.

7. Conclusion

This paper has given a detailed comparison analysis between classical and fuzzy queueing network models in uncertainty. The classical M/M/1 model gives accurate measures of performance in a situation when the arrival and service rates are known in advance, which makes the model applicable to well-characterized systems. By contrast, fuzzy queueing models model uncertainty in system parameters with fuzzy numbers to produce intervals of performance and defuzzified estimates, which are more realistic in terms of variability and imprecision in the real world. The numerical examples showed that fuzzy models provide more flexibility and realistic analysis of the queue length, waiting time, and utilization but come with the trade-off of a higher complexity of calculations.
Future studies can be devoted to hybrid stochastic-fuzzy queueing, multi-server and networked systems and real-time adaptive queueing frameworks to deal with dynamically changing uncertainty in complex service settings.

REFERENCES

[1]. Singh, Dharmpal & Choudhury, J & De, Mallika. (2012). A comparative study on the performance of fuzzy logic, Bayesian logic and neural network towards decision-making. International Journal of Data Analysis Techniques and Strategies. 4. 205-216. 10.1504/IJDATS.2012.046792.

[2]. Jolai, Fariborz & Asadzadeh, S.M. & Ghodsi, Reza & Bagheri, Shiva. (2016). A Multi-Objective Fuzzy Queuing Priority Assignment Model. Applied Mathematical Modelling. 40. 10.1016/j.apm.2016.06.024.

[3]. Mueen, Zeina. (2018). A multiple Channel Queueing Model under an Uncertain Environment with Multiclass Arrivals for Supplying Demands in a Cement Industry.

[4]. Prameela, Usha & KUMAR, PAVAN. (2019). FM / FEk /1 Queuing Model with Erlang Service Under Various Types of Fuzzy Numbers.

[5]. Shanmugasundari, M. & Sekar, Aarthi. (2020). A different approach to solve fuzzy queuing theory. AIP Conference Proceedings. 2277. 090011. 10.1063/5.0026364.

[6]. Chen G, Liu Z and Zhang J, 2020. Analysis of strategic customer behavior in fuzzy queueing systems. Journal of Industrial and Management Optimization, 2020, 16 (1): 371-386.

[7]. Ebenesar Anna Bagyam J. and Udaya Chandrika K., 2019. Fuzzy Analysis of bulk arrival two phase retrial queue with vacation and admission control. The Journal of Analysis, 27(1): 209 – 232.

[8]. Ebenesar Anna Bagyam, J. and Udaya Chandrika, K., 2018. MX /G/1 Retrial Queue with different modes of failure, Journal of Physics: Conference series, 1742 - 6588, 1139 012031.

[9]. Kannadasan, G. and Padmavathi, V. 2021. Classical Fuzzy Retrial Queue with Working Vacation using Hexagonal Fuzzy Numbers. Journal of Physics: Conference Series. 2070.

[10]. Viswanathan A, Udaya Chandrika K and Ebenesar Anna Bagyam J, 2015. Fuzzy Retrial Queueing System with Coxian - 2 Vacation. International Journal of Applied Engineering Research. 10(8): 21315- 21322.

[11]. B. T. Taylor. Introduction To Management Science, England : Pearson Education Limited, 2016.

[12]. N. Amit, N. A. Ghazali. Using simulation model queuing problem at a fast-food restaurant, In: Regional Conference on Science Technology and Social Sciences (RCSTSS), 1055-1062, Singapore:Springer , 2018.

[13]. A B. N. Yakubu, U. Najim. An application of queuing theory to ATM service optimization: A case study, Mathematical Theory and Modelling, 11-23, 2014.

[14]. S. Meng, D. Wu, Z. Huimin, L. Bo, W. Chunxiao. Study on an airport gate assignment method based on improved ACO algorithm. Emerald Insight, 20-43, 2018.

[15]. N. Sujatha, V. S. Murthy Akella, G. V. S. Deekshitulu. Analysis of multiple server fuzzy queueing model using α – cuts. International Journal of Mechanical Engineering and Technology (IJMET), Vol.8, No.10, 35–41, 2017