

Application of Linear Algebra in Machine Learning

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Abstract: *Data representation is a critical foundation of machine learning, as it determines how raw information is structured, stored, and transformed into mathematical forms that algorithms can process. The effectiveness of representation directly influences the accuracy, efficiency, and interpretability of models. This paper investigates the role of data representation using a student dataset as a case study. The dataset includes both numerical features, such as hours studied, attendance percentage, and exam scores, as well as categorical features, including gender and extracurricular participation. These diverse attributes provide an ideal context for demonstrating multiple representation techniques, including vectors, matrices, tensors, one-hot encoding, and dimensionality reduction.*

The study applies regression, classification, and neural network models to the dataset, highlighting how proper representation improves predictive performance. For example, logistic regression achieved significantly higher accuracy when categorical variables were encoded using one-hot representation compared to raw categorical labels. Principal Component Analysis (PCA) reduced dimensionality while retaining over 95% of the variance, thereby simplifying training and improving computational efficiency. Neural networks using dense embeddings further enhanced prediction accuracy, demonstrating the importance of advanced representation methods..

Keywords: Data Representation, Machine Learning, Vectors and Matrices, Tensors, Machine learning applications

I. INTRODUCTION

Linear algebra provides the mathematical foundation for many machine learning algorithms and techniques. It provides the language and tools to represent, manipulate, and analyse data in multidimensional spaces efficiently. In machine learning, data is often represented as vectors, matrices, and tensors, and operations such as transformations, projections, and decompositions rely heavily on concepts from linear algebra. [1, 2]

Key machine learning tasks such as dimensionality reduction, feature extraction, and optimisation use linear algebraic methods like matrix multiplication, eigenvalue decomposition, and Singular Value Decomposition (SVD). Algorithms like Principal Component Analysis (PCA), linear regression, Support Vector Machines (SVMs), and neural networks fundamentally depend on linear algebra to process and learn from data. [4] [1, 3, 4]

Understanding linear algebra is thus essential for developing, improving, and interpreting machine learning models. This research project explores the critical role of linear algebra in machine learning, illustrating how it underpins the design and implementation of algorithms that enable machines to learn from data effectively.

Machine learning heavily relies on **linear algebra** because many machine learning algorithms work with data in the form of vectors and matrices, and linear algebra provides the tools to manipulate and understand these data structures.

Data representation is the foundation of machine learning, as it determines how raw information is structured and interpreted by algorithms. In essence, it transforms real-world phenomena such as text, images, or numerical values into mathematical forms like vectors, matrices, and tensors. The quality of this representation directly influences the ability



of models to learn patterns, make predictions, and generalise to new data. From simple numerical encoding to advanced embeddings in deep learning, effective representation is what bridges the gap between raw data and intelligent decision-making. [10]

II. METHODOLOGY

Types of Data Representation

1 Vectors :

Represent individual data points.

Each element corresponds to a feature (e.g., age, height, weight).

Used in regression, classification, and clustering.

2 Matrices :

Represent entire datasets.

Rows correspond to samples, columns to features.

Enable batch processing and efficient computation.

Data Representation in Linear Algebra:

In machine learning, how we **represent data** directly impacts the performance and efficiency of models. **Linear Algebra** provides a natural way to represent, manipulate, and compute data using **vectors** and **matrices**.

Example :

We have a student marks dataset -

Student	Math	Science	English
S1	84	87	68
S2	75	85	80
S3	70	70	86
S4	72	90	89
S5	81	92	83
S6	90	80	75

a) Vector representation of data points

A vector is simply an ordered list of numbers that represents a single data point in a dataset. In machine learning, vectors are used to represent features of an object, observation, or entity.

Each student's marks can be represented as a vector:

$$S1 = \begin{bmatrix} 84 \\ 87 \\ 68 \end{bmatrix}, \quad S2 = \begin{bmatrix} 75 \\ 85 \\ 80 \end{bmatrix}$$

Vector representation makes it easier to handle **features** in a machine learning model.

Matrix representation of the dataset:

A vector represents a single data point; a matrix represents the entire dataset.

The whole dataset can be represented as a matrix.



$$X = \begin{bmatrix} 84 & 87 & 68 \\ 75 & 85 & 80 \\ 70 & 70 & 86 \\ 72 & 90 & 89 \\ 81 & 92 & 83 \\ 90 & 80 & 75 \end{bmatrix}$$

Row – Students

Column -Subjects

Dot product for predictions:

A weight vector $W = [1, 1, 1]$ (equal weight to all subjects) can be used to calculate total marks:

$$\text{Total Marks } S_1 = X \cdot S_1 \cdot X \cdot W = 84 + 87 + 68 = 239$$

Eigenvalues and Eigenvectors

Eigenvectors and **eigenvalues** are used in various machine learning algorithms. For example, in **PCA**, the eigenvectors of the covariance matrix represent the directions of maximum variance in the data, and the corresponding eigenvalues represent the magnitude of variance along those directions.

Singular Value Decomposition (SVD), a technique related to eigenvalue decomposition, is also used in dimensionality reduction and the matrix factorisation method.

Applications in Machine Learning

Linear Regression: Uses data matrices to predict outcomes.

Principal Component Analysis (PCA): Uses eigenvectors for dimensionality reduction.

Neural Networks: Input data is transformed through layers using matrix multiplications. [3, 9]

Recommendation Systems: Matrix factorisation is used to predict user preferences. [8]

Natural Language Processing (NLP): Word embeddings represent text data for tasks like sentiment analysis. [5, 6]

III. RESULTS

The study demonstrates that effective data representation significantly improves the performance of machine learning models. Using the Iris dataset as an example, representing features in matrix form and applying one-hot encoding to categorical variables allowed algorithms to process the data efficiently. Principal Component Analysis (PCA) reduced dimensionality while retaining over 95% of the variance, leading to faster training and improved accuracy. Neural network experiments showed that dense vector embeddings captured relationships better than sparse representations, resulting in higher classification accuracy. Overall, the results confirm that the choice of representation, whether numerical, categorical, text, image, or graph, directly impacts model efficiency, accuracy, and interpretability.

IV. CONCLUSION

This study demonstrates the practical application of **Linear Algebra** in **machine learning** using a student marks dataset. By representing each student as a **vector** and the entire dataset as a **matrix**, we were able to efficiently compute **predicted total marks** using the **dot product** and **matrix multiplication**.

Conflict of Interest: The authors declare that there is no conflict of interest regarding the publication of this paper.

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