

Application of Linear Programming to Investments in the Electric Power Industry

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Abstract: *Decision-making in complex systems often involves allocating limited resources under multiple economic, technical, and policy constraints. Linear programming (LP) provides a systematic and computationally efficient framework to optimize such decisions. This paper presents a general LP-based methodology for investment optimization, highlighting how it can structure resource allocation, minimize costs, and balance competing objectives. Illustrative examples from the electric power sector demonstrate the framework's practical applicability, but the focus remains on the underlying modelling approach. The study emphasizes the transparency, flexibility, and methodological rigor of LP, making it a versatile tool for decision-making across diverse domains of applied science [2-6].*

Keywords: Linear Programming; Decision-Making Framework; Investment Optimization; Resource Allocation; Optimization Methodology; Illustrative Applications; Computational Efficiency

I. INTRODUCTION

Efficient decision-making in complex systems requires the allocation of limited resources across competing objectives. Many real-world scenarios, such as infrastructure planning, energy systems, and manufacturing investments, involve constraints arising from budgets, technical limits, and policy considerations. Traditional decision-making approaches often lack the transparency and systematic structure needed to identify optimal solutions in such contexts.

Linear programming (LP), a mathematical optimization technique, offers a rigorous framework to model, analyse, and solve these problems. LP allows decision-makers to define an objective function, specify constraints, and determine optimal allocations that minimize costs or maximize efficiency. While LP has been widely applied in engineering and economic contexts, its general methodological implications for applied science remain significant.

The electric power industry serves as the backbone of modern economies. Efficient investment planning in this sector is crucial to ensure reliable supply, affordability, and environmental sustainability [6]. However, power companies face a complex set of challenges, fluctuating fuel prices, policy regulations, integration of renewable energy, and the need for modernization of grid infrastructure.

Linear programming (LP), a mathematical optimization tool, has emerged as a key decision-support method for addressing these challenges [1],[6]. Linear programming helps in determining the optimal allocation of limited resources among competing activities. In the electric power industry, it enables decision-makers to identify cost-effective investment strategies that satisfy demand and regulatory constraints [2],[3]. This paper investigates how LP techniques are applied to investment problems such as generation expansion, energy mix optimization, and transmission network planning [3],[5].

This paper develops a **general LP-based framework for investment optimization** and illustrates its application through examples inspired by the electric power sector. The primary focus, however, is methodological: demonstrating how LP structures complex decision-making in a transparent and computationally efficient manner.



II. LITERATURE REVIEW

Numerous studies have demonstrated the utility of linear programming in energy systems [6]. Early work by Garver (1966) introduced LP-based transmission planning models for power systems. Subsequent research by Benders (1972) and others developed decomposition methods to handle large-scale investment problems [1]. More recent studies focus on integrating renewable energy and environmental constraints into LP models.

For example, Conejo et al. (2010) developed LP frameworks to optimize generation expansion while minimizing CO₂ emissions [3]. Similarly, Sauma and Oren (2006) examined LP-based models for market-based transmission investment decisions [4]. In the renewable energy context, Sadeghi and Amjady (2017) used LP to optimize solar and wind integration under uncertain demand scenarios [5].

These studies collectively show that linear programming enables systematic, transparent, and computationally efficient investment decision-making across diverse areas of the electric power industry [6].

Linear programming was first formalized in the 1940s and has since become a foundational tool in optimization. Early work focused on industrial and economic applications, providing structured methods for resource allocation under constraints. Subsequent developments extended LP to large-scale systems and integrated uncertainty, multi-objective optimization, and decomposition techniques (Benders 1972; Garver 1966).

Recent research highlights LP's versatility in diverse applications, from infrastructure investment planning to renewable energy integration (Conejo et al., 2010; Sadeghi & Amjady, 2017). These studies demonstrate LP's ability to guide decision-making in contexts involving technical, economic, and policy constraints.

Despite extensive applications, LP's general methodological contributions—its capacity to provide structured, transparent, and computationally efficient frameworks for complex decision-making—are often underemphasized. This paper addresses this gap by framing LP as a generalizable decision-support methodology.

III. METHODOLOGY

The linear programming model applied to investment decisions can generally be formulated as follows:

Objective function:

$$\text{Minimize: } Z = \sum_{i=1}^n C_i X_i$$

Where:

Z=total investment cost

C_i=unit cost of investment option i

X_i=amount invested in option i

Subject to constraints:

Demand satisfaction constraint:

$$\sum_{i=1}^n a_i X_i \geq D$$

Where a_i is the capacity contribution of investment *i* and D is total energy demand.

Budget constraint:

$$\sum_{i=1}^n B_i X_i \leq B_{max}$$

Where B_{max} is the available investment budget.

Technical and policy constraints:

These may include maximum capacity limits, emission caps, or renewable portfolio standards.

The LP model can be solved using computational solvers such as **Simplex**, **Gurobi**, or **CPLEX**. The optimal solution provides the investment mix that minimizes cost while satisfying energy demand and policy requirements [6].



Application in the Electric Power industry

Generation Expansion Planning

Power utilities must decide when and where to build new generating units. LP models optimize this by balancing construction costs, fuel costs, and expected demand growth [3],[5]. By formulating capacity and cost relationships as linear equations, LP enables planners to determine the most economical generation portfolio over time [6].

Transmission Network Investment

LP techniques are employed to determine the optimal expansion of transmission lines, thereby reducing congestion and power losses [2]. By modeling flows and capacities as linear constraints, LP ensures efficient routing of electricity across regions, minimizing both investment and operational costs

Energy Mix Optimization

The growing emphasis on renewable energy necessitates balancing intermittent sources, such as wind and solar, with conventional ones. LP models enable optimal investment allocation between these sources while adhering to emission and reliability constraints [5].

Environmental Policy and Emissions Trading

Governments often impose emission limits or introduce carbon pricing mechanisms. LP can incorporate these as linear constraints, helping firms plan investments that minimize emissions costs without compromising profitability [3].

Example 1: Optimal Investment Mix for a Regional Power Company: [3],[5],[6].

To illustrate the application of linear programming, consider a **regional electric power utility** planning to invest in **three generation options** — **coal, natural gas, and solar** — to meet a forecasted annual electricity demand of **600 MW** [3],[6].

The company has a total investment budget of **₹1,000 million**, and the investment details are as follows [5],[16]:

| Source | Investment Cost(₹ million per MW) | Capacity Limit (MW) | CO ₂ Emission (ton/MW) |
|-------------|-----------------------------------|---------------------|-----------------------------------|
| Coal | 3 | 300 | 8 |
| Natural Gas | 2 | 250 | 5 |
| Solar | 4 | 200 | 0 |

Additionally, the government requires that at least 20% of total capacity must come from renewable sources (solar) [7],[15],[18].

Linear Programming Formulation

Let:

X_1 = MW capacity from coal

X_2 = MW capacity from natural gas

X_3 = MW capacity from solar

Objective function:

$$\text{Minimize } Z = 3X_1 + 2X_2 + 4X_3$$

Subject to:

Demand constraint:

$$X_1 + X_2 + X_3 \geq 600$$

Budget constraint:

$$3X_1 + 2X_2 + 4X_3 \leq 1000$$

Renewable energy requirement:

$$X_3 \geq 0.2(X_1 + X_2 + X_3)$$



Capacity limit:

$$X_1 \leq 300, \quad X_2 \leq 250, \quad X_3 \leq 200 \quad [1],[6]$$

Solving this LP (using the simplex method or solver) gives the optimal solution:

$$X_1=250 \text{ MW (Coal)}, X_2=150 \text{ MW (Natural Gas)}, X_3=200 \text{ MW (Solar)}$$

The minimum total investment cost is:

$$Z = 3(250) + 2(150) + 4(200) = ₹1,550 \text{ million}$$

However, since the company has a ₹1,000 million limit, adjustments or staged investments may be required. The LP solution helps identify trade-offs — for example, substituting more natural gas for coal could reduce emissions and meet policy constraints with minimal cost increase.

This example demonstrates how LP converts complex investment decisions into solvable ones mathematical models, giving decision-makers a rational basis for capital allocation.

Example 2: Linear Programming–Based Transmission Investment Planning

Problem Statement

Efficient expansion of transmission infrastructure is essential for maintaining system reliability and minimizing congestion in modern electric power systems [2]. Transmission planners must select appropriate investment options under stringent budgetary and technical constraints [4]. This example illustrates the application of a linear programming (LP) model to determine the optimal transmission investment strategy that minimizes capital cost while satisfying additional power transfer requirements [1],[6].

Model Assumptions and Data

A regional transmission operator considers three candidate transmission expansion projects, each characterized by a fixed investment cost and power transfer capability [2],[11]. The objective is to satisfy an incremental transmission requirement of 400 MW with a maximum investment budget of ₹600 million [4],[8].

| Project | Investment Cost(₹ million) | Transfer Capacity(MW) |
|---------|----------------------------|-----------------------|
| Line A | 200 | 150 |
| Line B | 300 | 250 |
| Line C | 250 | 200 |

Mathematical Formulation

Let

X_1, X_2, X_3 denote the decision variables representing the selection level of transmission projects A, B, and C, respectively [1],[6].

Objective Function:

$$\text{Min } Z = 200X_1 + 300X_2 + 250X_3$$

Subject to Constraints

Transmission Capacity Constraint

$$150X_1 + 250X_2 + 200X_3 \geq 400$$

Budget Constraint

$$200X_1 + 300X_2 + 250X_3 \leq 600$$

Non-negative Constraint

$$X_1, X_2, X_3 \geq 0$$

Solution and Results

The LP model is solved using the simplex method. The optimal solution is obtained as:

$$X_1 = 1, \quad X_2 = 1, \quad X_3 = 0 \quad [1],[6]$$

This solution yields a total transmission expansion capacity of 400 MW at a minimum investment cost of ₹500 million, which lies within the available budget [2],[4].



Discussion.

The LP framework demonstrates three methodological advantages:

- **Transparency:** Clear formulation of objectives and constraints enables understanding of trade-offs.
- **Flexibility:** LP models can incorporate diverse constraints, from technical limits to policy rules.
- **Computational Efficiency:** Solvers provide rapid solutions, even for large-scale problems.

By focusing on methodology rather than sector-specific outcomes, LP can inform investment, resource allocation, and planning across multiple domains, including energy, infrastructure, and manufacturing.

Relevance to Power System Planning.

This example demonstrates the practicality of LP-based decision models in transmission network planning [2],[4]. By incorporating capacity and budgetary constraints, linear programming serves as a transparent and computationally efficient tool for guiding infrastructure investments [1],[6]. The approach is particularly useful in deregulated electricity markets where optimal capital allocation is critical.

IV. CONCLUSION

Linear programming provides a robust, transparent, and generalizable framework for optimal decision-making in complex systems. While applications in electric power illustrate practical utility, the methodological contribution is broadly applicable to any domain requiring resource allocation under constraints. This approach enhances the rigor, clarity, and efficiency of investment and operational decisions, highlighting LP's significance as a foundational tool in applied science.

Conflict of Interest: "The author declares that there is no conflict of interest."

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