

Application of Bootstrap Exponentially Weighted Moving Average-Mann-Whitney (Bootstrap-EWMA-MW) Control Chart in Monitoring HAFED Polytechnic Bakery Production Process

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Abstract: *In every successful production settings, there should be some means by which the production process is monitored in real time. This study seeks to monitor the Bread production process of Hussaini Adamu Federal Polytechnic Bakery industry using the Bootstrap-EWMA-MW control chart. To achieve this specific aim, five loaves of bread were randomly chosen and their weight read every day for forty days, beginning from 1st November to 10th December, 2025. Applying the Bootstrap-EWMA-MW control chart on Phase II sample, the software output showed that a shift is detected at monitoring period 3. This implies that the chart is reliable and very effective in monitoring processes with same distributional properties as the Bread weight data from Bakery production process of the HAFED Polytechnic Bakery industry. The study thus recommends that in addition to applying Bootstrap-EWMA-MW control chart in general bakery settings, the production process of the HAFED Polytechnic Bakery factory be closely monitored as it operates under out-of-control condition, contrary to the belief of the management of the Bakery plant.*

Keywords: Bootstrap-EWMA-MW Control Chart, In-Control, Out-of-Control, Bakery Plant, Bread and Weight

I. INTRODUCTION

Quality, a measure that gives a sense of how the properties of a monitored process deviate from that of an in-control process, is the objective of any industries and their customers (Woodcock et al., 2025). The methods by which the quality can be achieved has been contentious because different problem requires different approach. The approaches to any quality monitoring problem differ based on the distribution the monitored process follow (Abidet al., 2024). However, local industries barely know anything about the distributional properties of their processes, something that mar their effort towards utilization of control charts in monitoring and enhancement of quality (Kazeemet al., 2025). More adversely, production processes of such local industries are mostly unchecked using any Statistical Process Control (SPC) technique and are therefore very likely to market poor quality product and consequently de-market the business as a whole (Kazeemet al., 2025). In Hussaini Adamu Federal (HAFED) Polytechnic Bread industry for example, the quality characteristic of their process as measured by weight, has never been monitored using Bootstrap Exponentially Weighted Moving Average-Mann-Whitney (Bootstrap-EWMA-MW) or any SPC method. This highlights the need to introduce a suitable method into the production process of the industry. As earlier hinted, suitability of an SPC method to the HAFED Polytechnic Bakery production process hinges on whether the process quality characteristic is compatible with the properties of the method in question. As weight data is more likely to be skewed and/or heavy-tailed (Hermanussen et al., 2001) as a result of being bound on one side and unbounded on the other, hence the suitability of the Bootstrap-EWMA-MW control chart for monitoring such a process. In a nutshell, this



study seeks to apply Bootstrap-EWMA-MW control chart to monitor and guide the production process of HAFED Polytechnic bread production process.

II. MATERIALS AND METHODS

2.1 MATERIALS

The study was conducted to apply Bootstrap-EWMA-MW control chart to investigate the performance of Bread production process of HussainiAdamu Federal Polytechnic Bakery industry. The polytechnic is located in Kazaure, Jigawa state, Nigeria. In collecting the Bread weight data, a weighing balance is used.

2.2 METHODS

The sampling technique used in selecting the bread sample was simple random sampling without replacement (SRSWOR). This technique was chosen because the sampling units are identical and equally likely in terms of the study variable. Moreover, the method enabled different loaf of bread to be tested as a result of non-repetition of sample. The bootstrap samples were drawn on the basis of sampling with replacement. The bootstrap samples were drawn 100,000 times in order to have better idea of the distribution of the charting statistic of the Bootstrap-EWMA-MW control chart. Finally, the data for the study was analyzed using R Package.

2.2.1 THE Bootstrap-EWMA-MW CONTROL CHART

1. Collect m in-control subgroups, each of size n :

$$Y_{jk}, j = 1, \dots, n; k = 1, \dots, m; \quad (2.1)$$

2. Pool the Phase I data into a single reference sample:

$$Y = \{Y_{11}, \dots, Y_{mn}\} = \{Y_k : k = 1, \dots, mn\} \quad (2.2)$$

3. Compute the in-control mean and variance of the Mann-Whitney statistic by estimating μ and σ^2 using B bootstrap pseudo-Phase II subgroups samples drawn from Y :

$$\hat{\mu} = \frac{1}{B} \sum_{b=1}^B U^{*(b)} \quad (2.3)$$

and

$$\hat{\sigma}^2 = \frac{1}{B-1} \sum_{b=1}^B \left(U^{*(b)} - \hat{\mu} \right)^2 \quad (2.4)$$

where $\hat{\mu}$ and $\hat{\sigma}^2$ are procedurally estimated as follows: Generate B bootstrap pseudo-subgroups from Phase I. Draw a sample of size n from each of the b bootstrap sample. Compute the Mann-Whitney statistic $U^{*(b)}$ comparing Phase I sample with the pseudo-subgroups. Compute the mean and variance of $U^{*(b)}$ to estimate $\hat{\mu}$ and $\hat{\sigma}^2$ respectively.

4. Choose EWMA smoothing parameter θ .

5. Observe a Phase II subgroup of size n at period i :

$$X_i = \{X_{i1}, \dots, X_{in}\} \quad (2.5)$$

6. Compute the Mann-Whitney Statistic comparing the pooled Phase I sample with the Phase II subgroup:

$$U_i = \sum_{k=1}^{mn} \sum_{j=1}^n I(X_{ij}, Y_k) \quad (2.6)$$

where



$$I(X_{ij}, Y_k) = \begin{cases} 1, & X_{ij} > Y_k \\ \frac{1}{2}, & X_{ij} = Y_k \\ 0, & X_{ij} < Y_k \end{cases} \quad (2.7)$$

7. Standardize the Mann-Whitney Statistic:

$$W_i = \frac{U_i - \hat{\mu}_U}{\hat{\sigma}_U} \quad (2.8)$$

using bootstrap-based estimated $\hat{\mu}_U$ and $\hat{\sigma}_U$.

8. Update the EWMA statistic recursively:

$$EMW_i = \theta W_i + (1 - \theta) EMW_{i-1}, \quad EMW_0 = 0 \quad (2.9)$$

The Control Limits

For each fixed monitoring period i :

9. Generate B bootstrap samples from the Phase I pooled sample:

9.1 For each bootstrap replication $b = 1, \dots, B$:

9.1.1. A bootstrap reference sample $Y^{*(b)}$ is generated by sampling with replacement from the Phase I pooled sample:

$$Y^{*(b)} = \{Y_k^{*(b)} : k = 1, \dots, mn\} \quad (2.10)$$

9.1.2. Create a pseudo-Phase II subgroup $X_i^{*(b)}$ of size n by sampling with replacement from bootstrap reference sample:

$$X_i^{*(b)} = \{X_{i1}^{*(b)}, \dots, X_{in}^{*(b)}\} \quad (2.11)$$

9.1.3. Compute Mann-Whitney Statistic $U_i^{*(b)}$ comparing bootstrap reference sample $Y_k^{*(b)}$ with pseudo-Phase II subgroup $X_i^{*(b)}$ corresponding to i^{th} period:

$$U_i^{*(b)} = \sum_{k=1}^{mn} \sum_{j=1}^n I(X_{ij}^{*(b)}, Y_k^{*(b)}) \quad (2.12)$$

where

$$I(X_{ij}^{*(b)}, Y_k^{*(b)}) = \begin{cases} 1, & \text{if } X_{ij}^{*(b)} > Y_k^{*(b)} \\ \frac{1}{2}, & \text{if } X_{ij}^{*(b)} = Y_k^{*(b)} \\ 0, & \text{if } X_{ij}^{*(b)} < Y_k^{*(b)} \end{cases} \quad (2.13)$$

9.1.4. Standardize the Mann-Whitney Statistic to get $W_i^{*(b)}$:

$$W_i^{*(b)} = \frac{U_i^{*(b)} - \hat{\mu}_{U^{*(b)}}}{\hat{\sigma}_{U^{*(b)}}} \quad (2.14)$$

9.1.5. Compute bootstrap EWMA Statistic:

$$EMW_i^{*(b)} = \theta W_i^{*(b)} + (1 - \theta) EMW_{i-1}^{*(b)}, \quad EMW_0^{*(b)} = 0 \quad (2.15)$$

10. Collect the bootstrap EWMA Statistics to obtain the distribution of the proposed chart charting statistic:



$$\{EMW_i^{*(1)}, \dots, EMW_i^{*(B)}\} \quad (2.16)$$

11. Determine control limits at period i as the empirical Quantiles of the bootstrap EWMA Statistic distribution:

$$LCL_i = Q_{\alpha/2}, \quad UCL_i = Q_{1-\alpha/2} \quad (2.17)$$

12. The proposed Bootstrap-EWMA-MW chart signals out-of-control if:

$$EMW_i < LCL_i \text{ or } EMW_i > UCL_i, \text{ other wise the process is in - control} \quad (2.18)$$

2.2.2 The Phase I Analysis and Process Property Assessment Methods

The Phase I analysis was employed to ensure that the Phase I sample meet the criteria for control chart implementation. A data set qualifies to be a Phase I sample for a process if in addition to the sample being part of the process, it is also generated when the process operates under in-control condition (Coelho *et al.*, 2015). Phase I analysis was conducted using the traditional \bar{X} and R chart as recommended by Oakland & Oakland, (2007). According to the same authors, \bar{X} and R chart is given as:

$$LCL = \bar{\bar{X}} - A_2 \bar{R}, \quad (2.19)$$

$$CL = \bar{\bar{X}} \quad (2.20)$$

and

$$UCL = \bar{\bar{X}} + A_2 \bar{R} \quad (2.21)$$

The study utilized normal Quantile-Quantile (Q-Q) plots and Shapiro Wilk test check if the data is normal. Öztuna *et al.*, (2006) identified normal Q-Q plots as one of the most important visual tools for diagnosing departures from normality. Q-Q plot is a graph of the Quantile of the variable being tested against that for a normally distributed variable. To construct a Q-Q plot for $x_i, i = 1, 2, 3, \dots, n$, the following procedure is adopted (Thode, 2002). Obtain the sample ordered statistic $x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$ and the sample Quantiles probabilities as $p_i = \frac{i-0.375}{n-0.25}$, where p_i represents cumulative probabilities (Blom, 1958). Calculate the theoretical normal Quantiles $q_i = \Phi^{-1}(p_i)$, where Φ^{-1} is the inverse cumulative distribution of the standard normal distribution. Plot the sample Quantiles $x_{(i)}$ on the y-axis against the theoretical normal Quantiles q_i . If $x_i, i = 1, 2, 3, \dots, n$ is sampled from normal distribution, all the points in the Q-Q plot will lie on or near a straight line drawn through the middle half of the points (Ghasemi & Zahediasl, 2012).

Shapiro Wilk test was due to Shapiro and Wilk who introduced the concept purposely to facilitate a powerful assessment of possible departures of some processes from normality (Shapiro & Wilk, 1965). Given $x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$, Shapiro & Wilk, (1965) provides a representation for the test as:

$$W = \frac{b^2}{S^2} \quad (2.22)$$

where $S^2 = \sum_{i=1}^n (x_i - \bar{x})^2$, $b = \sum_{i=1}^k a_{n-i+1} (x_{n-i+1} - x_i)$ if n is even ($n = 2k$),

$b = a_n (x_n - x_{n-(n-1)}) + a_{n-1} (x_{n-1} - x_{n-(n-2)}) + \dots + a_{k+2} (x_{k+2} - x_k)$ if n is odd ($n = 2k + 1$), a_{n-i+1} are



constants tabulated in Shapiro &Wilk, (1965) and k is a given positive integer. SW test tests the following Null (H_0) and Alternative (H_1) hypotheses:

$$\begin{aligned} H_0 &: \text{the sample comes from a distribution } F(x) \\ H_1 &: \text{the sample does not come from a distribution } F(x) \end{aligned} \quad (2.23)$$

According to the test, H_0 is rejected if W exceeds the significance percentage point available in Shapiro &Wilk, (1965) or the P-value is less than the set significance level α .

The study assessed the independence assumption using Runs test, following the approach of Herdianiet *al.*, (2025). The Runs test, originally introduced by Wald and Wolfowitz, (1940), evaluates the following hypotheses:

$$\begin{aligned} H_0 &: \text{data is random} \\ H_1 &: \text{data is not random} \end{aligned} \quad (2.24)$$

The test statistic for the test is given as:

$$Z = \frac{r - \mu_r}{\sigma_r} \quad (2.25)$$

where μ_r and σ_r are respectively given as:

$$\mu_r = \frac{2n_1n_2}{n_1n_2} \quad (2.26)$$

$$\sigma_r = \sqrt{\frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1 + n_2)^2(n_1 + n_2 - 1)}} \quad (2.27)$$

where r stands for sequence of identical symbols “+” or “-“, the symbol “+” is used if an observation is above the median, the symbol “-“ is used if an observation is below the median. n_1 and n_2 stand for values above and the median observation respectively. The test rejects H_0 if $|Z| > Z_{\frac{\alpha}{2}}$.

2.2.2 Data Sources and Collection

The study used a primary data collected directly from HussainiAdamu Federal (HAFED) Polytechnic Bakery plant production process. The data was collected over forty days, from 1stNovember to 10thDecember, 2025. The data was partitioned into two, firstpartition (1st to 14thNovember, 2025) and the second partition (15th November to 10thDecember, 2025). The first partition was used as Phase I sample and the second as monitored sample. On each day, five loaves of bread were selected based on Simple Random Sampling without Replacement (SRSWOR) and their Wight reading taken in Kilogram (Kg). Some intervals of timewere observed between each sample so that the sample can be a good representative of each day’s production. The Bootstrap-EWMA-MW control chart was implemented on the collected data using an open access R Package version “R version 4.5.1 (2025-06-13 ucrt)”.

III. RESULTS AND DISCUSSION

3.1 RESULTS

3.1.1 Phase I Analysis and Process Properties Assessment

Table 3.1 represents the Shapiro Wilk and Run tests results, while Figure 3.1 represents the Normal Quantile-Quantile plot.



Table 3.1. Assumptions Checks on Phase I Data.

Values	Tests	
	Shapiro Wilk Test	Run Test
Test Statistics	0.4632	0.5684
P-Values	0.0000	0.5698

From Table 4.12 and Figure 4.8, it is seen that the Phase I data does not meet the normality assumption as the test rejects the null hypothesis of normality, p-value = 0.0000. On the contrary, by the P-value = 0.5698 of the Run test in Table 3.9, the Phase I data is seen to satisfy the assumption of independence.

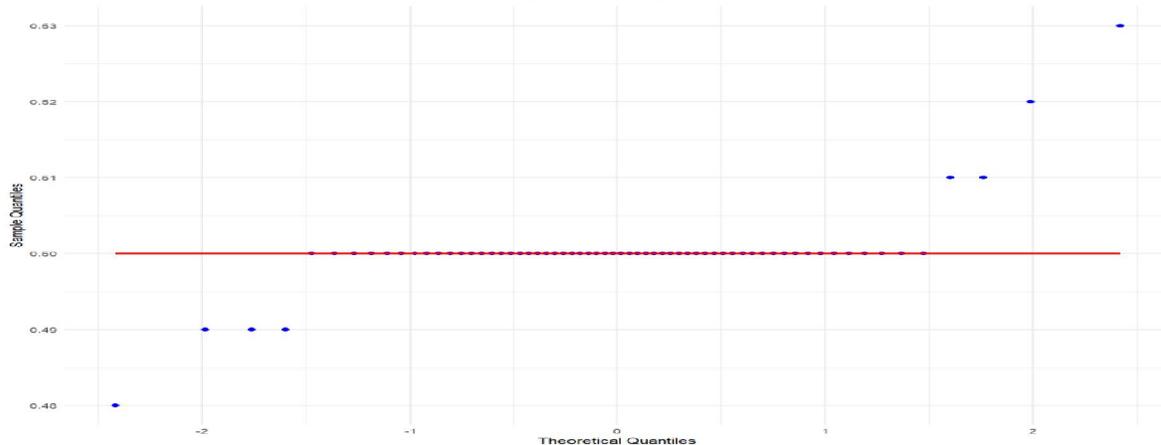


Figure 3.1. Normal Q-Q Plot of Phase I Data

In light of the results of these tests and the properties of the Bootstrap-EWMA-MW control chart in question, it is observed that the data set is perfectly suitable to be tracked, applying the model. This is because, Bootstrap-EWMA-MW control chart assumes independence but not normality.

Table 3.1 represents the preliminary chart for Phase I analysis. The analysis was conducted using the \bar{X} and R Chart.

Table 3.1. Preliminary Chart for Phase I Analysis of the Bread-Weight Process Data using \bar{X} and R Chart.

Period	Bread-Weight Reading (kg)					Average	Range	LCL	UCL	Decision
1	0.50	0.50	0.50	0.50	0.50	0.5000	0.0000	0.4959	0.5047	IC
2	0.50	0.50	0.50	0.50	0.50	0.5000	0.0000	0.4959	0.5047	IC
3	0.50	0.50	0.50	0.50	0.50	0.5000	0.0000	0.4959	0.5047	IC
4	0.53	0.50	0.50	0.5	0.48	0.5040	0.0500	0.4959	0.5047	IC
5	0.50	0.50	0.50	0.50	0.50	0.5000	0.0000	0.4959	0.5047	IC
6	0.50	0.50	0.5	0.50	0.50	0.4980	0.0100	0.4959	0.5047	IC
7	0.50	0.50	0.50	0.50	0.50	0.5000	0.0000	0.4959	0.5047	IC
8	0.50	0.50	0.50	0.50	0.50	0.5000	0.0000	0.4959	0.5047	IC
10	0.50	0.5	0.5	0.50	0.52	0.5040	0.0300	0.4959	0.5047	IC
11	0.50	0.50	0.50	0.50	0.50	0.5000	0.0000	0.4959	0.5047	IC
12	0.50	0.50	0.50	0.49	0.50	0.4980	0.0100	0.4959	0.5047	IC
13	0.50	0.50	0.50	0.50	0.50	0.5000	0.0000	0.4959	0.5047	IC
14	0.50	0.50	0.50	0.50	0.50	0.5000	0.0000	0.4959	0.5047	IC

From Table 3.1, it is seen that the data point for period removed to ensure an in-control Phase I process.



3.1.2 Ongoing Process Monitoring

Table 3.2 shows the application of the Bootstrap-EWMA-MW control chart. The bootstrapping was conducted using 100,000 bootstrap samples. The samples were used for the estimation of the Quantile-based control limits of the chart.

Table 3.2. Application of the Optimal Bootstrap-EWMA-MW ($\theta = 0.11$, $\alpha = 0.0045$) Control Chart to Bread Weight Monitoring.

Period	Weight Readings (kg)					U_i	W_i	EMW_i	LCL_i	UCL_i	Decision
1	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	-0.0263	-0.2743	0.2745	IC
2	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	-0.0497	-0.2967	0.2503	IC
3	0.49	0.5	0.49	0.5	0.5	100.5	-2.7470	-0.3464	-0.2855	0.2896	OOO
4	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	-0.3345	-0.5837	-0.0327	IC
5	0.5	0.5	0.52	0.51	0.5	224.0	2.2491	-0.0503	-0.6341	-0.0582	OOO
6	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	-0.0711	-0.3195	0.2308	IC
7	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	-0.0895	-0.3382	0.2121	IC
8	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	-0.1059	-0.3554	0.1952	IC
9	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	-0.1206	-0.3692	0.1806	IC
10	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	-0.1336	-0.3833	0.1678	IC
11	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	-0.1452	-0.3935	0.1570	IC
12	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	-0.1555	-0.4044	0.1461	IC
13	0.5	0.5	0.52	0.5	0.52	224.5	2.2693	0.1113	-0.4736	0.1035	OOO
14	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	0.0728	-0.1758	0.3745	IC
15	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	0.0385	-0.2097	0.3396	IC
16	0.52	0.5	0.5	0.51	0.5	224.0	2.2491	0.2816	-0.2941	0.2743	OOO
17	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	0.2244	-0.0236	0.5249	IC
18	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	0.1734	-0.0764	0.4753	IC
19	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	0.1281	-0.1199	0.4289	IC
20	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	0.0877	-0.1603	0.3884	IC
21	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	0.0518	-0.1973	0.3547	IC
22	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	0.0198	-0.2304	0.3222	IC
23	0.5	0.5	0.5	0.5	0.5	162.5	-0.2388	-0.0086	-0.2574	0.2923	IC
24	0.5	0.5	0.5	0.5	0.5	193.0	0.9950	0.1018	-0.3098	0.2520	IC
25	0.9	0.7	0.9	0.6	0.5	162.5	-0.2388	0.0643	-0.1845	0.3650	IC
26	0.5	0.5	0.5	0.51	0.5	162.5	-0.2388	0.0310	-0.2172	0.3314	IC

Table 3.2 and Figure 3.2 show that the Bootstrap-EWMA-MW chart detected the process shift at periods 3, 5, 13 and 16, illustrating its high sensitivity to changes in the process location. By this result, it can be seen that the chart helps the management of the industry to not only detect an early signal in the process, but also to prevent the management and the industry's reputation by not allowing defective products into the market.



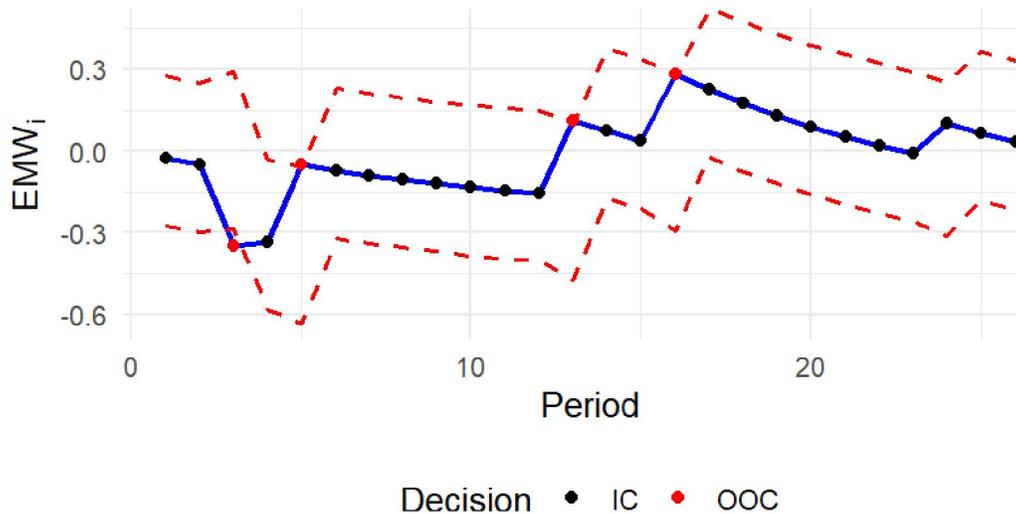


Figure 3.2. The Plot of the Bootstrap-EWMA-MW ($\theta = 0.11$, $\alpha = 0.0045$) as Applied to HAFED Polytechnic Bakery Bread Weight Data

3.2 DISCUSSIONS

Statistical process control is of course indispensable in every quest for more profitable production business such as the Bakery business (Bertie & Hartiati, 2023). The observed relevance of Bootstrap-EWMA-MW control chart in tracking less quality product in the HAFED Polytechnic Bakery production business is a confirmation to the claim of Bertie & Hartiati, (2023). In essence, the industry serving as the case study for this study is bound to make more profit if their product evaluation and monitoring unit can utilize SPC for decision making.

IV. CONCLUSION

Statistical quality control was utilized in many industrial settings and for different purposes. One common aim to all is quality improvement through real-time monitoring of processes. This study aimed at same quality improvement in the output of HAFED Polytechnic Bakery production process through the application of Bootstrap-EWMA-MW control chart. The present study therefore concludes that Bootstrap-EWMA-MW control chart is a good tool to track processes and ensure that they operate within the set target. Thus, the study recommends that the production process in question be continuously monitored, particularly using a suitable control chart such as the one utilized herein.

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