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Analysis for Cartan's Second Curvature Tensor in Finsler Space

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Abstract: *The decomposition of curvature tensors have been studied by the Finslerian geometrics. The aim of the present paper is to three decomposable of Cartan's second curvature tensor* P^i_{jkh} *to prove that Cartan's second curvature tensor* P^i_{jkh} *in affinely connected space is symmetric in first and second indices of their decomposable.*

Keywords: Decomposition of Cartan's Second Curvature Tensor P^i_{jkh}, Symmetric Property

I. Introduction

The analysis for Cartan's fourth curvature tensor in Finsler space discussed by Qasem and Nasr [6]. The decomposition of Berwald curvature tensor H^i_{jkh} and Cartan's fourth curvature tensor K^i_{jkh} for some spaces in sense of Berwald and Cartan were studied by Pandey [11]. The decomposition of Cartan's third curvature tensor R^i_{jkh} and Cartan's fourth curvature tensor K^i_{jkh} equipped with non − symmetric connection in Finsler space were discussed by Mishra et al. [10] and Al−Qashbari [3], respectively. The decomposition of normal projective curvature tensor in Finsler space was discussed by Qasem and Saleem [5]. Hit [12] introduced Berwald curvature tensor which be decomposable and obtained several results. Al_Qufail [8] studied decomposability of curvature tensors in non-symmetric recurrent Finsler space, Nor [2] introduced the decomposability of Cartan's forth curvature tensor K^i_{jkh} in Finsler space.Also, Bisht and Neg [9] studied decomposition of normal projective curvature tensor fields in Finsler manifolds.In this paper, we find the condition for Cartan's second curvature tensor P^i_{jkh} to be symmetric of their decomposable.

II. Preliminaries

In this section, we introduce some conditions and definitions which are needed in this paper.

Let F_n be an $n-$dimensional Finsler space equipped with the metric function $F(x,y)$ satisfying the request conditions [4, 7]. The vector y_i defined by

$$y_i = g_{ij}(x,y)\, y^j \tag{2.1}$$

The two sets of quantities g_{ij} and g^{ij} which are components of the metric tensor and associate metric tensor are related by

$$g_{ij} g^{jk} = \delta^k_i = \begin{cases} 1 & if \quad i = k \ , \\ 0 & if \quad i \neq k \ . \end{cases} \tag{2.2}$$

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We know that P^i_{jkh} is called $hv-$ *curvature tensor* (*Cartan's second curvature tensor*) and defined by [7]

$$(2.3)\qquad P^i_{jkh} = C^i_{kh|j} - g^{ir} C_{jkh|r} + C^r_{jk} P^i_{rh} - P^r_{jh} C^i_{rk} \quad ,$$

where

$$(2.4)\qquad P^i_{jkh}\, y^j = \Gamma^{*i}_{jkh} y^j = P^i_{kh} = C^i_{kh|r}\, y^r$$

and

$$(2.5)\qquad P^i_{kh} y_i = 0 \,,$$

where P^i_{kh} is $v(hv)-$torsion tensor of the curvature tensor P^i_{jkh}.

In view of (2.3), the $hv-$ curvature tensor P^i_{jkh} satisfies the following:

$$(2.6)\qquad P^i_{jkh} - P^i_{kjh} = C^i_{kh|j} + C^i_{sj} P^s_{kh} - j/k \,.$$

However, in affinely connected space or Berwald's space, Alaa et al. [1] were obtained

$$(2.7)\qquad C^r_{jk|h} = 0 \,.$$

Let us consider a Finsler space whose Cartan's second curvature tensor P^i_{jkh} is decomposition. Since the curvature tensor is a mixed tensor of the type (1,3), *i.e.* of rank 4, it may be written as product of contravariant (or covariant) vector and a tensor of rank 3 , *i.e.* covariant tensor of the type (0,3) {or mixed tensor of the type (1,2)} as following [5 , 6]

$$(2.8)\qquad \text{a)}\ P^i_{jkh} = X^i Y_{jkh} \quad , \qquad \text{b)}\ P^i_{jkh} = X_j Y^i_{kh} \quad ,$$

$$\text{c)}\ P^i_{jkh} = X_k Y^i_{jh} \qquad \text{and} \qquad \text{d)}\ P^i_{jkh} = X_h Y^i_{jk}$$

as first case.

Or in the second case as product of two tensors each them of rank 2, *i.e.* mixed tensors of the type (1,1) and covariant tensor of the type (0,2) as following [5, 6]

$$(2.9)\qquad \text{a)}\ P^i_{jkh} = T^i_j \psi_{kh} \quad , \qquad \text{b)}\ P^i_{jkh} = T^i_k \psi_{jh}$$

and

$$\text{c)}\ P^i_{jkh} = T^i_h \psi_{jk} \quad .$$

Here, in this paper, we discuss the possible forms in three decomposable of the tensor, two decompositions for the first case (the other are similar) and one decomposition for the second case (the other are similar). Clearly, from all several possibilities, we study the possibilities which are given by (2.8a), (2.8b) and (2.9a).

III. MAIN RESULTS

In this section, several theorems have been established and proved. Let Cartan's second curvature tensor P^i_{jkh} is decomposable in the form (2.8a). Transvecting (2.8a) by y^j and using (2.4), we get

$$(3.1)\qquad P^i_{kh} = X^i Y_{jkh}\, y^j \ .$$

Transvecting eq. (3.1) by y_i and using (2.5), we have at least one of the following condition

$$(3.2)\qquad \text{a)}\ y_i X^i = 0 \qquad \text{and} \qquad \text{b)}\ Y_{jkh} y^j = 0 \,.$$

Transvecting (2.8a) by y_i and in view of eq. (3.2a), we obtain

$$(3.3)\qquad y_i P^i_{jkh} = 0 \,.$$

In view of (2.1) and (2.4), then eq. (3.3) can be written

$g_{ij}P^i_{kh}=0$.

Transvecting above equation by g^{jn} and using (2.2), we get

$P^n_{kh}=0$.

If eq. (3.2b) holds, then eq. (3.1) reduces to

$P^i_{kh}=0$.

According to the previous , the equations (3.2a) and (3.2b) lead to the $v(hv)-$torsion tensor P^i_{kh} is vanishing. Then, by using this fact and (2.7) in (2.6), we deduce

$P^i_{jkh}=P^i_{kjh}$.

Thus, we conclude the following theorem

Theorem 3.1. *If Cartan's second curvature tensor P^i_{jkh} is decomposable in the form (2.8a), where X^i and Y_{jkh} are non-zero contravariant vector and tensor field, respectively, then in affinely connected space, the curvature tensor P^i_{jkh} is symmetric in first and second indices.*

Let Cartan's second curvature tensor P^i_{jkh} is decomposable in the form (2.8b).

Transvecting (2.8b) by y^j and using (2.4), we get

(3.4) $P^i_{kh}=X_j y^j Y^i_{kh}$.

Transvecting eq. (3.4) by y_i and using (2.5), we get at least of the following:

(3.5) a) $X_j y^j=0$ and b) $y_i Y^i_{kh}=0$.

Transvecting of (2.8b) by y_i , using (2.1) and (2.4),then in view of eq. (3.5b), we get

$g_{ij}P^i_{kh}=0$.

Transvecting above equation by g^{jn} and using (2.2), we get

$P^n_{kh}=0$.

If eq. (3.5b) holds, then eq. (3.4) reduces to

$P^i_{kh}=0$.

According to the previous, the equations (3.5a) and (3.5b) lead to the $v(hv)-$torsion tensor P^i_{kh} is vanishing. Then, by using this fact and (2.7) in (2.6), we deduce

$P^i_{jkh}=P^i_{kjh}$.

Thus, we conclude the following theorem

Theoram 3.2. *If Cartan's second curvature tensor P^i_{jkh} is decomposable in the form (2.8b) where X_j and Y^i_{kh} are non-zero covariant vector and tensor field, respectively, then in affinely connected space, the curvature tensor P^i_{jkh} is symmetric in first and second indices.*

Let Cartan's second curvature tensor P^i_{jkh} is decomposable in the form (2.9a).

Transvecting (2.9a) by y^j and using (2.4), we get

(3.6) $P^i_{kh} = T^i_j \psi_{kh} y^j$.

Transvecting eq. (3.6) by y_i and using (2.5), we obtain at least of the following:

(3.7) a) $T^i_j y_i = 0$ and b) $\psi_{kh} y^j = 0$.

Transvecting of (2.9b) by y_i , using (2.1) and (2.4), then in view of eq. (3.7b), we get

$g_{ij} P^i_{kh} = 0$.

Transvecting above equation by g^{jn} and using (2.2), we get

$P^n_{kh} = 0$.

If eq. (3.7b) holds, then eq. (3.6) reduces to

$P^i_{kh} = 0$.

According to the previous, the equations (3.7a) and (3.7b) lead to the $v(hv)-$torsion tensor P^i_{kh} is vanishing. Then, by using this fact and (2.7) in (2.6), we deduce

$P^i_{jkh} = P^i_{kjh}$.

Thus, we conclude the following theorem

Theoram 3.3. *If Cartan's second curvature tensor P^i_{jkh} is decomposable in the form (2.9a) where T^i_j and ψ_{kh} are the decomposition tensors field, then in affinely connected space, the curvature tensor P^i_{jkh} is symmetric in first and second indices.*

IV. Conclusion

The possibilities of decomposition for Cartan's second curvature tensor P^i_{jkh} have been studied. We obtained that Cartan's second curvature tensor P^i_{jkh} in affinely connected space is symmetric in first and second indices of their decomposable.

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