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On Some Applications of New Integral Transform "IMAN Transform"

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Abstract: *In this study a new integral transform, namely Iman transform was applied to solve linear ordinary differential equations with constant coefficients. In particular we apply this new transform technique to solve linear dynamic systems and signals-delay differential equations and the renewal equation in statistics.*

Keywords: Iman Transform- Differential Equations--Applications

I. INTRODUCTION

Many problems of physical interest are described by differential and integral equations with appropriate initial or boundary conditions. These problems are usually formulated as initial value problem, boundary value problems, or initial – boundary value problem that seem to be mathematically more vigorous and physically realistic in applied and engineering sciences. Iman transform method is very effective for solution of differential and integral equations.

The technique that we used is Iman transform method which is based on Fourier transform, it introduced by Iman Ahmed see [1, 2, 3, 4, 5, 6].

In this study, Iman transform is applied to solve linear dynamic systems and signals-delay differential equations and the renewal equation in statistics, which the solution of these equations have a major role in the fields of science and engineering.

When a physical system is modeled under the differential sense, if finally gives a differential equation.

Recently . Iman Ahmed introduced a new transform and named as Iman transform [1] which is defined by:

$$\mathrm{I}[f(t),v] = L(v) = \frac{1}{v^2}\int_0^\infty f(t)e^{-tv^2}dt = L(v) \quad , \ k1 \le v \le k2 \, , \ t \ge 0$$

Or for a function $f(t)$ which is of exponential order,

$$|f(t)| < \begin{cases} Me^{-\frac{t}{k1}} & , \quad t \le 0 \\ Me^{-\frac{t}{k2}} & , \quad t \ge 0 \end{cases}$$

Theorem (1)

Let $L(v)$ be Iman transform of $f(t)$, $I(f(t)) = L(v)$, then

$$\mathrm{I}\left[\frac{\partial f(x,t)}{\partial t}\right] = v^2 L(x,v) - \frac{1}{v^2} f(x,0)$$

$$\mathrm{I}\left[\frac{\partial^2 f(x,t)}{\partial t^2}\right] = v^4 L(x,v) - f(x,0) - \frac{1}{v^2}\frac{\partial f(x,0)}{\partial t}$$

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Proof: (i)

By the definition we have:

$$\mathrm{I}[f'(t)] = \frac{1}{v^2}\int_0^\infty f'(t)e^{-tv^2}dt$$

$$\mathrm{I}\left[\frac{\partial f(x,t)}{\partial t}\right] = v^2 L(x,v) - \frac{1}{v^2} f(x,0)$$

(ii)

$let\ g(x,t) = f'(x,t)\ then$

$\mathrm{I}\left[\frac{\partial g(x,t)}{\partial t}\right] = v^2 \mathrm{I}[g(x,t)] - \frac{1}{v^2} g(x,0)$ using (i) we find that:

$$\mathrm{I}\left[\frac{\partial^2 f(x,t)}{\partial t^2}\right] = v^4\, L(x,v) - f(x,0) - \frac{1}{v^2}\,\frac{\partial f(x,0)}{\partial t}$$

Theorem (2)

$$A=\{f(t): \exists M, k1, k2>0, |f(t)|<Me^{\frac{|t|}{k_j}};\ t\in(-1)^j \times [0,\infty[\ \}$$

With Laplace transform $F(s)$, Then Iman transform $L(v)$ of $f(t)$ is given by:

$$L(v) = \frac{1}{v^2}F(v^2)$$

Proof

Let $f(t) \in A$, Then for , $k1\leq v\leq k2$, $L(v) = \frac{1}{v^2}\int_0^\infty f\left(\frac{t}{v^2}\right)e^{-t}\,dt$

Let $w = \frac{t}{v^2}$ then we have:

$L(v) = \frac{1}{v^2}\int_0^\infty f(w)e^{-wv^2}v^2\,dw = \frac{1}{v^2}\int_0^\infty f(w)e^{-wv^2}dw = \frac{1}{v^2}\,F(\ v^2)$

Also we have that $L(1) = F(1)$ so that both the Iman and Laplace transforms

must coincide at $v = s = 1$.

Theorem (3)

Let $f(t)$ *and* $g(t)$ be defined in A having Laplace transforms $F(s)$ and $G(s)$ and Iman transforms $M(v)$ and $N(v)$. Iman transform of the convolution of f and g

$$(f * g)(t) = \int_0^\infty f(t)g(t-\tau)d\tau \qquad \text{Is given by: } I[(f*g)(t)] = v^2 M(v)N(v)$$

Proof

The Laplace transform of $(f * g)$ is given by: $\ell(f * g)(t) = F(s)\,G(s)$

By the duality relation (Theorem (2)), we have: $[(f*g)(t)] = \frac{1}{v^2}\ell(f\ *\ g)(t),$

and since

$$M(v) = \frac{1}{v^2}F(v^2), \quad N(v) = \frac{1}{v^2}G(v^2)$$

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Then

$$I[(f*g)(t)] = \frac{1}{v^2}F(v^2)G(v^2)$$
$$= \frac{1}{v^2}[v^2M(v)v^2N(v)]$$
$$= v^2[M(v)N(v)]$$

Theorem (4)

$I(f(t)) = L(v)$, then

$I[f(t-a)H(t-a)] = e^{-av^2}L(v)$

Where $H(t-a)$ is Heaviside unit step function.

Proof

It follows from the definition that:

$I[f(t-a)H(t-a)] = \frac{1}{v^2}\int_0^\infty f(t-a)H(t-a)e^{-av^2}dt =$

$$\frac{1}{v^2}\int_0^\infty f(t-a)e^{-av^2}dt$$

Let $t-a=\tau$, then we have:

$$e^{-av^2}\ \frac{1}{v^2}\int_0^\infty f(\tau)e^{-\frac{t}{\tau}}\,d\tau = e^{-av^2}L(v)$$

In particular if $f(t) = 1$, then:

$$I[H(t-a)] = \frac{1}{v^2}e^{-av^2}$$

Also we can prove by mathematical induction that:

$$I\left[\frac{(t-a)^{n-1}}{\Gamma(n)}H(t-a)\right] = \left(\frac{1}{v^2}\right)^{n+1} e^{-av^2}$$

Example (1) (Linear dynamical systems and signals)

In physical and engineering sciences, a large number of linear dynamical systems with a time dependent input signal $f(t)$ that generates an output signal $x(t)$ can be described by the ordinary differential equation with constant coefficients.

$$(D^n + a_{n-1}D^{n-1} + \cdots + a_0)x(t) = (D^m + b_{m-1}D^{m-1} + \cdots + b_0)f(t) \qquad (1)$$

Where

$D = \frac{d}{dx}$, $\quad a_0, a_1, \ldots, a_{n-1},\ b_0, b_1, \ldots, b_{m-1}$ are constants

We apply Iman transform to find the output $x(t)$ so that (1) becomes.

$\overline{p_n}(v)\,\bar{x}(v) - \overline{R_{n-1}}(v) = \overline{q_m}(v)\,\bar{f}(v) - \overline{S_{m-1}}(v) \qquad (2)$

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Where,

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It is convenient to express (2) in the form

$$\bar{x}(v) = \bar{h}(v)\,\bar{f}(v) + \bar{g}(v) \quad (3)$$

Where

$$\bar{h}(v) = \frac{\overline{q_m}(v)}{\overline{p_n}(v)} \quad , \bar{g}(v) = \frac{\overline{R_{n-1}}(v) - \overline{S_{m-1}}(v)}{\overline{p_n}(v)} \quad (4)$$

And $\bar{h}(v)$ is usually called the transfer function. The inverse Iman transform combined with the convolution theorem leads to the formal solution,

$$x(t) = \int_0^t f(t-\tau)h(\tau)d\tau + g(t) \quad (5)$$

With zero initial data $\bar{g}(v) = 0$. the transfer function takes the simple form.

$$\bar{x}(v) = \bar{h}(v)\,\bar{f}(v)$$

If $f(t) = \delta(t)$ and $h(t) = e^t$ then, the output function is

$$x(t) = I^{-1}\left[\frac{1}{v^4(v^2+1)}\right] = e^t - 1$$

$h(t)$ is known as the impulse response.

Example (2) (Delay Differential Equations)

In many problems the derivatives of the unknown function $x(t)$ are related to its value at different times $t-\tau$ .this leads us to consider differential equations of the form:

$$\frac{dx}{dt} + ax(t-\tau) = f(t) \quad (7)$$

Where a is a constant and $f(t)$ is a given function. Equations of this type are called delay differential equations. In general , initial value problems for these equations involve the specification of $x(t)$ in the interval $t_0 - \tau \le t \le t_0$and this information combined with the equation it self, is sufficient to determine $x(t)$ for $t_0 > t$. We show how equation (7) can be solved by Iman transform when $t_0 = 0$ And$x(t) = x_0$for $t\ \delta\ 0$. In view of the initial condition , we can write. $x(t-\tau) = x(t-\tau)H(t-\tau)$ So equation (7) is equivalent to,

$$\frac{dx}{dt} + ax(t-\tau)H(t-\tau) = f(t) \quad (8)$$

Applying Iman transform to (8) gives,

$$v^2\bar{x}(v) - \frac{1}{v^2}x(0) + ae^{-\tau v^2}\,\bar{x}(v) = \bar{f}(v)$$

And

$$\bar{x}(v) = \left[\frac{1}{v^2}\bar{f}(v) + \frac{1}{v^4}x(0)\right]\sum_{n=0}^{\infty}(-1)^n\left(\frac{a}{v^2}\right)^n e^{-\tau n v^2} \quad (9)$$

The inverse Iman transform gives the formal solution:

$$x(t) = I^{-1}\left[\frac{1}{v^2}\bar{f}(v) + \frac{1}{v^4}x(0)\right]\sum_{n=0}^{\infty}(-1)^n\left(\frac{a}{v^2}\right)^n e^{-\tau n v^2} \quad (10)$$

In order to write an explicit solution, we choose $x_0 = 0$ and $f(t) = t$ and hence (10) be comes.

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$$x(t) = I^{-1}\left[\frac{1}{v^8}\sum_{n=0}^{\infty}(-1)^n\left(\frac{a}{v^2}\right)^n e^{-\tau n v^2}\right] = \sum_{n=0}^{\infty}(-1)^n (a)^n \frac{(t-n\tau)^{n+2}}{(n+2)!}H(t-n\tau)\text{ , t >0}$$

Example (3) (Renewal Equation in statistics)

The random function $x(t)$ of time t represents the number of times some event has occurred between time 0 and time t, and usually referred to as a counting Process. A random variable X_n that recodes the time it assumes for X to get the value n from the $n-1$ is referred to as an inter-arrival time .If the random variables X_1 ,X_2 .... are independent and identically distributed, then the counting process $X(t)$ is called a renewal process. We denote their common Probability distribution function by $F(t)$ and the density function by $f(t)$ so that $F'(t) = f(t)$.Then renewal function is defined by the expected number of time the event being counted occurs by time t and is denoted by $r(t)$ so that .

$$r(t) = I[X(t)] = \int_0^{\infty} I\{[x(t): X_1 = x]\} f(x)dx \qquad (11)$$

Where $I\{[x(t): X_1 = x]\}$ is the conditional expected value of $X(t)$ under the condition that $X_1 = x$ and has the value.

$$I\{[x(t): X_1 = x]\} = [1 + r(t-x)]H(t-x) \qquad (12)$$

Thus

$$r(t) = \int_0^{t} [1 + r(t-x)] f(x)dx \qquad (13)$$

This is called the renewal equation in mathematical statistics .We solve the equation by taking Iman transform with respect to t, and Iman transformed equation is,

$$\bar{r}(v) = \bar{F}(v) + v^2\bar{r}(v)\bar{f}(v)$$

The inverse transform gives the formal solution $r(t)$ of renewal function.

If $F(t) = e^t$ and $f(t) = e^t$, then:

$$r(t) = \frac{1}{\sqrt{2}} sinh\sqrt{2}\, t$$

(ii) If $F(t) = t$ and $f(t) = 1$, then:

$$r(t) = e^t - 1$$

II. CONCLUSION

In this study, we introduced new integral transform called Iman transform to solve linear dynamical systems and signals – delay differential equations and the renewal equation in statistics. It has been shown that Iman transform is a very efficient tool for solving these equations

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Table of Functions and their Iman Transform

$f(t)$	$I[f(t)] = F(v)$
1	$\frac{1}{v^4}$
t	$\frac{1}{v^6}$
t^2	$\frac{2!}{v^8}$
$t^n \quad n \in N$	$\frac{n!}{v^{2n+4}}$
e^{at}	$\frac{1}{v^2(v^2-a)}$
$\sin(at)$	$\frac{a}{v^2(v^4+a^2)}$
$\cos(at)$	$\frac{1}{v^4+a^2}$
$H(t-a)$	$\frac{1}{v^4}e^{-av^2}$
$\delta(t-a)$	$\frac{1}{v^2}e^{-av^2}$
$\sinh(at)$	$\frac{a}{v^2(v^4-a^2)}$
$\cosh(at)$	$\frac{a}{v^4-a^2}$
$t^{a-1}/\Gamma(a)$, a>0	$\left(\frac{1}{v^2}\right)^{a+1}$