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A Review of Important Double Integral Transformations with Some Elementary Functions

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Abstract: *Through their applications in many domains, integral transformations have demonstrated their importance in mathematics for decades. Mathematicians have found that double integral transforms may provide unexpected characteristics and enable new applications after extensive research on integral transforms. It analyzes 29 double integral transformations and their applicability to fundamental functions. These transformations help solve partial differential equations in physics, engineering, medicine, and mechanics. Our evaluation uses 29 references to illustrate their real-world relevance*

Keywords*:* Basic Functions, Elementary Functions, Double Integral Transformations.

I. INTRODUCTION

Since Laplace integral transformation, integral transformations have been extensively studied because they can solve differential equations in their ordinary and partial forms. However, not all integral transformations can handle all partial differential equation applications. Due to the importance of partial differential equations in various applications, further integral transforms were investigated to find a variation on the existence of transformations that can handle partial integral transform applications. Additional integral transformations were needed to study partial differential equations in various applications. This project sought to provide a transformation framework for all partial integral transform applications. This theory suggested double integral transformation. The double integral transformations combine current integral transforms to create new integral transformations with additional characteristics and treat partial differential equations differently. The inverse of double integral transforms requires less algebraic operations, proving its suitability for certain applications.

This study will provide current double integral transformations and their applications to fundamental and often used functions as a reference for these deliberate integral transformations.

DOUBLE SUMUDU INTEGRAL TRANSFORMATION

Jean M. and Nyimvua S. proposed double Sumudu integral transformation (2007). Double integral transformation for two-variable function (x, y) as:

$$S_2\{f(x,y)\} = F(s,r) = \frac{1}{sr}\int_0^\infty \int_0^\infty e^{-\left(\frac{y}{r}+\frac{x}{s}\right)} f(x,y)dxdy.$$

$x, y > 0.$

The double Sumudu integral transformation for some basic functions are:

$S_2\{k\} = k, k$ is an aribtrary constant.

$S_2\{x^m y^n\} = m!\,n!\,s^m r^n$, where $n, m \in \mathbb{N}$.

$S_2\{e^{ax+by}\} = \dfrac{1}{(1-as)(1-br)}$, where $a, b \in \mathbb{R}$

$S_2\{\sin(ax+by)\} = \dfrac{as+br}{(1+a^2s^2)(1+b^2r^2)}, a, b \in \mathbb{R}$

$S_2\{\cos(ax+by)\} = \dfrac{1-absr}{(1+a^2s^2)(1+b^2r^2)}, a, b \in \mathbb{R}$

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DOUBLE LAPLACE INTEGRAL TRANSFORMATION

Debnath L. developed double Laplace integral transformation (2016). The double integral transformation for the two-variable function (x, y) is defined as:

$$L_2\{f(x,y)\} = F(s,r) = \int_0^\infty \int_0^\infty e^{-(sx+ry)} f(x,y)dxdy,$$
$$x, y > 0.$$

The double Laplace integral transformation for some fundamental functions are:

$L_2\{x^m y^n\} = \frac{m!n!}{s^{m+1}r^{n+1}}$, where $m, n \in N$.

$L_2\{e^{ax+by}\} = \frac{1}{(s-a)(r-b)}$ where $a, b \in \mathbb{R}$.

$L_2\{e^{t(ax+by)}\} = \frac{(rs-ab)+l(as+br)}{(s^2+a^2)(r^2+b^2)}$ where $a, b \in \mathbb{R}$.

$L_2\{\sin(ax+by)\} = \dfrac{as+br}{(s^2+a^2)(r^2+b^2)^2}$ where $\mathrm{a,b} \in \mathbb{R}$.

$L_2\{\cos(ax+by)\} = \frac{rs-ab}{(s^2+a^2)(r^2+b^2)}$ where $a, b \in \mathbb{R}$.

$L_2\{\sinh(ax+by)\} = \frac{as+br}{(r^2-b^2)(s^2-a^2)}$ where $a, b \in \mathbb{R}$.

$L_2\{\cosh(ax+by)\} = \frac{rs+ab}{(r^2-b^2)(s^2-a^2)}$ where $a, b \in \mathbb{R}$.

DOUBLE NATURAL INTEGRAL TRANSFORMATION

In (2017) Kilicman A. and Omran M. introduced the double natural integral transformation. This double integral transformation is defined as:

$$N_2\{f(x,y)\} = F(r,s,u,v) = \int_0^\infty \int_0^\infty e^{-(sx+ry)} f(ux,vy)dxdy,$$
$$x, y > 0$$

The double natural integral transformation for some basic functions are:

$N_2\{k\} = \frac{k}{sT} \cdot k$ is an arbitrary constant.

$N_2\{x^m y^n\} = \frac{u^m v^n m!n!}{s^{m+1}r^{n+1}}$ where $n, m \in \mathbb{N}$.

$N_2\{e^{ax+by}\} = \frac{1}{(s-au)(r-bv)^2}$ where $a, b \in \mathbb{R}$.

$N_2\{e^{l(ax+by)}\} = \frac{(sr-abuv)+i(sbv+aru)}{(s^2+a^2u^2)(r^2+b^2v^2)}$ $a, b \in \mathbb{R}$.

$N_2\{\sin(ax+by)\} = \frac{sbv+aru}{(s^2+a^2u^2)(r^2+b^2v^2)}$ $a, b \in \mathbb{R}$.

$N_2\{\cos(ax+by)\} = \frac{(sr-abuv)}{(s^2+a^2u^2)(r^2+b^2v^2)}$ $a, b \in R$.

$N_2\{\sinh(ax+by)\} = \frac{-(sbv+aru)}{(s^2+a^2u^2)(r^2+b^2v^2)}$ $a, b \in \mathbb{R}$.

$N_2\{\cosh(ax+by)\} = \dfrac{(sr+abuv)}{(s^2+a^2u^2)(r^2+b^2v^2)}, a, b \in \mathbb{R}$.

DOUBLE ABOODH INTEGRAL TRANSFORMATION

In (2017) K.S. Aboodh et al. introduced the double Aloodh integral transformation. This double integral transformation is defined as:

$$A_2\{f(x \mid y)\} = \frac{1}{sr} \int_0^\infty e^{-(sx+ry)} f(x,y)dxdy = F(s,r),$$
$$\text{where } x, y > 0.$$

The double Aboodh integral transformation for some basic functions are:

$A_2\{k\} = \frac{k}{s^2r^2}$, where k is an arbitrary constant.

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$A_2\{x^m y^r\} = \dfrac{m!\,n!}{s^{m+2}r^{n+2}}, m, n \in \mathbb{N}.$

$A_2\{e^{ax+by}\} = \dfrac{1}{(s^2 - as)(r^2 - br)}, a, b \in \mathbb{R}.$

$A_2\{\sin(ax+by)\} = \dfrac{ab}{\mathrm{sr}\,(s^2 + a^2)(r^2 + b^2)}, a, b \in \mathbb{R}.$

$A_2\{\cos(ax+by)\} = \dfrac{1}{(s^2 + a^2)(r^2 + b^2)}, a, b \in \mathbb{R}.$

$A_2\{\sinh(ax+by)\} = \dfrac{ab}{sr(s^2 - a^2)(r^2 - b^2)}, a, b \in \mathbb{R}$

$A_2\{\cosh(ax+by)\} = \dfrac{s^2 r^2(1 + absr)}{(s^2 - a^2)(r^2 - b^2)}, a, b \in \mathbb{R}.$

DOUBLE KAMAL INTEGRAL TRANSFORMATION

In (2019) Sonawane S.M. and Kiwne S.B. introduced the double Kamal integral transformation. This double integral transformation is defined as:

$$K_2\{f(x,y)\} = F(s,r) = sr\int_0^\infty \int_0^\infty e^{-(x+y)} f(sx, ry)\,dx\,dy,$$
$$x, y > 0 \text{ and } r, s \in \mathbb{R}$$

The double Kamal integral transformation for some fundamental functions are:

$K_2\{x^m y^n\} = m!\,n!\,s^{m+1}r^{n+1}, m, n \in \mathbb{N}.$

$K_2\{e^{ax+by}\} = \dfrac{sr}{(1 - as)(1 - br)^2}$ where $a, b \in \mathbb{R}.$

$K_2\{\sin(ax+by)\} = \dfrac{sr(as + br)}{(1 + a^2s^2)(1 + b^2r^2)}, a, b \in \mathbb{R}.$

$K_2\{\cos(ax+by)\} = \dfrac{sr(1 - absr)}{(1 + a^2s^2)(1 + b^2r^2)}, a, b \in \mathbb{R}.$

$K_2\{\sinh(ax+by)\} = \dfrac{sras + br)}{(1 - a^2s^2)(1 - b^2r^2)}, a, b \in \mathbb{R}.$

$K_2\{\cosh(ax+by)\} = \dfrac{sr(1 - abs)}{(1 - a^2s^2)(1 - b^2r^2)}, a, b \in \mathbb{R}.$

DOUBLE ELZAKI INTEGRAL TRANSFORMATION

In (2020) Jadhav S. et al. introduced the double Elzaki integral transformation. This double integral transformation is defined as:

$$E_2\{f(x,y)\} = F(s,r) = sr\int_0^\infty \int_0^\infty e^{-\left(\frac{x}{s}+\frac{y}{r}\right)} f(x, y)\,dx\,dy,$$
$$x, y > 0.$$

The double Elzaki transformation for some basic functions are:

$E_2\{k\} = ks^2r^2$ where k is an arbitrary constant.

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$E_2\{x^m y^n\} = m!\,n!\,s^{m+2}r^{n+2}, n,m \in \mathbb{N}.$

$E_2\{e^{ax+by}\} = \dfrac{(rs)^2}{(1-as)(1-br)^2}, a,b \in \mathbb{R}.$

$E_2\{\sin(ax+by)\} = \dfrac{(rs)^2(as+by)}{(1+a^2s^2)(1+b^2r^2)}, a,b \in \mathbb{R}.$

$E_2\{\cos(ax+by)\} = \dfrac{(rs)^2(1-absr)}{(1+a^2s^2)(1+b^2r^2)}, a,b \in \mathbb{R}.$

$E_2\{\sinh(ax+by)\} = \dfrac{-r^2s^2(as+br)}{(1-a^2s^2)(1-b^2r^2)}, a,b \in \mathbb{R}.$

$E_2\{\cosh(ax+by)\} = \dfrac{r^2s^2(1+ars)}{(1-2)}, a,b \in \mathbb{R}.$

DOUBLE SHEHU INTEGRAL TRANSFORMATION

In (2020) Alfaqeih S. and Misirli E. generalized the concept of Shehu integral transformation to the double Shehu integral transformation. This double integral transformation is defined as:

$$H_2\{f(x,y)\} = F((s,r),(u,v)) = \int_0^\infty \int_0^\infty e^{-\left(\frac{s}{u}x+\frac{r}{v}y\right)} f(x,y)\,dx\,dy,$$
$$x,y > 0.$$

The double Shehu integral transformation for some basic functions are:

$H_2\{k\} = \frac{kuv}{s\pi}$, where k is a constant.

$H_2\{x^m y^n\} = m!\,n!\left(\frac{u}{s}\right)^{m+1}\cdot\left(\frac{v}{r}\right)^{n+1}, n,m \in \mathbb{N}.$

$H_2\{e^{ax+by}\} = \frac{uv}{(s-au)(r-bv)}, a,b \in \mathbb{R}.$

$H_2\{e^{i(ax+by)}\} = \frac{uv(rs+abvv)+iuv(asu+brv)}{(s^2+a^2u^2)(r^2+b^2v^2)}, a,b \in \mathbb{R}, i^2=-1.$

$H_2\{\sin(ax+by)\} = \frac{uv(bsv+aru)}{(s^2+a^2u^2)}, a,b \in \mathbb{R}.$

$H_2\{\cos(ax+by)\} = \frac{uv(rs+abuv)}{(r^2+b^2v^2)}, a,b \in \mathbb{R}$

$H_2\{\sinh(ax+by)\} = \frac{1}{2}\left[\frac{uv}{(s-au)(r-bv)} - \frac{uv}{(s+au)(r+bv)}\right], a,b \in \mathbb{R}$

$H_2\{\cosh(ax+by)\} = \frac{1}{2}\left[\frac{uv}{(v-au)(r-h)} + \frac{uv}{(s+av)(r+bw)}\right], a,b \in v$

DOUBLE MAHGOUB TRANSFORMATION

In (2020) Patil D.P. introduced the double Mahgoub integral transformation. This double integral transformation is defined by:

$$M_2\{f(x,y)\} = F(s,r) = sr\int_0^\infty \int_0^\infty e^{-(sx+ry)} f(x,y)\,dx\,dy$$

where $x, y > 0$.

The double Mahgoub integral transformation for some elementary functions are:

$M_2\{k\} = k$, where k is a constant.

$M_2\{x^m y^r\} = \frac{m!n!}{s^m r^n}, n,m \in \mathbb{N}$

$M_2\{e^{ax+by}\} = \frac{sr}{(s-a)(r-b)}, a,b \in \mathbb{R}$

$M_2\{\sin(ax+by)\} = \frac{abrs}{(a^2+s^2)(b^2+r^2)}, a,b \in \mathbb{R}$

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$M_2\{\cos(ax+by)\} = \frac{s^2r^2}{(a^2+s^2)(b^2+r^2)}, a, b \in \mathbb{R}$

$M_2\{\sinh(ax+by)\} = \frac{abrs}{(s^2-a^2)((s^2-b^2)}, a, b \in \mathbb{R}$

$M_2\{\cosh(ax+by)\} = \frac{(rs)^2}{(s^2-a^2)(s^2-b^2)}, a, b \in \mathbb{R}$

NEW GENERAL DOUBLE INTEGRAL TRANSFORMATION

In (2021) Hossein Jafari et al. introduced the new general double integral transformation. This double integral transformation is defined as:

$$T_2\{f(x,y)\} = F(s,r) = p(s)q(r)\int_0^\infty \int_0^\infty e^{-(\varphi(s)x+\psi(r)y)} f(x,y)dxdy$$
$$x, y > 0, p(s), q(r) \neq 0$$

The new general double integral transformation for some fundamental functions are:

(1) $T_2\{k\} = \frac{kp(s)q(r)}{\varphi(s)\psi(r)}$, k is a constant number.

$T_2\{x^m y^n\} = \frac{m!\,n!\,p(s)q(r)}{[\varphi(s)]^{m+1}[\psi(r)]^{n+1}}, n, m \in \mathbb{N}$

$T_2\{e^{ax+by}\} = \frac{p(s)q(r)}{(\varphi(s)-a)(\psi(r)-b)}, a, b \in \mathbb{R}$

$T_2\{\sin(ax+by)\} = \frac{[a\psi(r)+b\varphi(s)]p(s)q(r)}{([\varphi(s)]^2+a^2)([\psi(r)]^2+b^2)}, a, b \in \mathbb{R}$

$T_2\{\cos(ax+by)\} = \frac{[\varphi(s)\psi(r)-ab]p(s)q(r)}{([\varphi(s)]^2+a^2)([\psi(r)]^2+b^2)}, a, b \in \mathbb{R}$

$T_2\{\sinh(ax+by)\} = \frac{[a\psi(r)+b\varphi(s)]p(s)q(r)}{([\varphi(s)]^2-a^2)([\psi(r)]^2-b^2)}, a, b \in \mathbb{R}$

$T_2\{\cosh(ax+by)\} = \frac{[\varphi(s)\psi(r)+ab]p(s)q(r)}{([\varphi(s)]^2-a^2)[[\psi(r)]^2-b^2)}, a, b \in \sqrt{R}$

DOUBLE ARA INTEGRAL TRANSFORMATION

In (2022) Rania S. introduced the double ARA integral transformation. This double integral transformation is defined as:

$$G_2\{f(x,y)\} = F(s,r) = sr\int_0^\infty \int_0^\infty e^{-(sx+ry)} f(x,y)dxdy,$$
$$x, y > 0 \text{ also } s, r > 0.$$

The double ARA integral transformation for some fundamental functions are:

$G_2\{x^m y^n\} = \frac{m^m n!}{s^m r^n}, n, m \in \mathbb{N}.$

$G_2\{e^{ax+by}\} = \frac{rs}{(s-a)(r-b)}, a, b \in \mathbb{R}.$

$G_2\{e^{i(ax+by)}\} = \frac{s}{(s-ia)(r-ib)}, a, b \in \mathbb{R}, i^2 = -1$

$G_2\{\sin(ax+by)\} = \frac{rs(sb+ra)}{(s^2+a^2)(r^2+b^2)}, a, b \in \mathbb{R}.$

$G_2\{\cos(ax+by)\} = \frac{rs(sr-ab)}{(s^2+a^2)(r^2+b^2)}, a, b \in \mathbb{R}.$

$G_2\{\sinh(ax+by)\} = \frac{rs(sb+ra)}{(s^2-a^2)(r^2-b^2)}, a, b \in \mathbb{R}$

$G_2\{\cosh(ax+by)\} = \frac{rs(s\pi+ab)}{(s^2-a^2)(r^2-b^2)}, a, b \in \mathbb{R}$

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DOUBLE EMAD-FALIH INTEGRAL TRANSFORMATION

In (2022) Emad A. Kuffi and Saed M. Turq introduced the double Emad-Falih integral transformation. This double integral transformation is defined by:

$$E_2\{f(x,y)\} = f(s,r) = \frac{1}{sr}\int_0^\infty \int_0^\infty e^{-(s^2x+r^2y)} f(x,y)dxdy$$
$$x,y,s,r > 0$$

The double Emad-Falih integral transform for some elementary important functions are:

$E_2\{x^m y^n\} = \dfrac{m!\,n!}{s^{2m+3} r^{2n+3}}$, where $n, m \in \mathbb{N}$.

$E_2\{e^{ax+by}\} = \dfrac{1}{sr(s^2-a)(r^2-b)}$, where $a, b \in \mathbb{R}$

$E_2\{e^{i(ax+by)}\} = \dfrac{1}{sr(s^2-ia)(r^2-tb)}$, $a, b \in \mathbb{R}$ and $i^2 = -1$.

$E_2\{\sin(ax+by)\} = \dfrac{bs^2+ar^2}{sr(s^4+a^2)(r^4+b^2)}$, $a, b \in \mathbb{R}$

$E_2\{\cos(ax+by)\} = \dfrac{s^2r^2-ab}{\mathrm{sr}\,(s^4+a^2)(r^4+b^2)}$, $a, b \in \mathbb{R}$.

$E_2\{\sinh(ax+by)\} = \dfrac{bs^2+ar^2}{s\pi(s^4-a^2)(r^4-b^2)}$, $a, b \in \mathbb{R}$

$E_2\{\cosh(ax+by)\} = \dfrac{s^2r^2+ab}{\pi(s^4-a^2)(r^4-b^2)}$, $a, b \in \mathbb{R}$

DOUBLE SEE INTEGRAL TRANSFORMATION

In (2022) Emad A. Kuffi and Eman A. Mansour introduced the double SEE integral transformation. This double integral transformation is defined as:

$$\mathrm{S}_2\{f(x,y)\} = \frac{1}{(sr)^n}\int_0^\infty \int_0^\infty e^{-(sx+ry)} f(x,y)dxdy = F(s,y),$$
$$n \in \mathbf{Z}, x, y > 0.$$

The double SEE integral transformation for some elementary functions are:

$S_2\{k\} = \frac{k}{(sr)^{n+1}}$ where k is a constant.

$S_2\{x^m y^m\} = \dfrac{m!\,m!}{(sr)^{n+n+1}}$, where $n, m \in \mathbb{N}$.

$S_2\{e^{ax+by}\} = \frac{1}{(sr)^n(s-a)(r-b)^2}$ where a and b are constants.

$S_2\{\sin(ax+by)\} = \dfrac{as-br}{(sr)^n(s^2+a^2)(r^2+b^2)}$, $a, b \in \mathbb{R}$.

$S_2\{\cos(ax+by)\} = \dfrac{s-ab}{(rs)^n(s^2+a^2)(r^2+b^2)}$, $a, b \in \mathbb{R}$.

$S_2\{\sinh(ax+by)\} = \dfrac{ar+bs}{(rs)^n(s^2+a^2)(r^2-b^2)}$, $a, b \in \mathbb{R}$.

$S_2\{\cosh(ax+by)\} = \dfrac{srab}{(rs)^n(s^2-a^2)(r^2-b^2)}$, $a, b \in \mathbb{R}$.

DOUBLE GENERAL INTEGRAL TRANSFORMATION

In (2022) Patil D. P. et al. introduced the double general integral transformation. This double integral transformation is defined as:

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$$T_2\{f(x,y)\} = F(s) = p_1(s)p_2(s)\int_0^\infty e^{-(q_1(s)x+q_2(s)y)} f(x,y)dxdy,$$
$$p_1(s), p_2(s) \neq 0, q_1(s), q_2(s) > 0, \text{ and } x, y > 0.$$

The double general transformation for some basic functions are:

$T_2\{x^m y^n\} = \frac{m!n!p_1(s)p_2(s)}{[q_1(s)]^{m+1}[q_2(s)]^{n+1}}$, where $n, m \in \mathbb{N}$.

$T_2\{e^{ax+by}\} = \frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)}, a, b \in \mathbb{R}$.

$T_2\{e^{i(ax+by)}\} = \frac{p1(s)p_2(s)}{(q_1(s)-ta)(q_2(s)-ib)}, a, b \in \mathbb{R}$, and $i^2 = -1$.

$T_2\{\sin{(ax+by)}\} = \frac{1}{2i}\left[\frac{p_1(s)p_2(s)}{(q_1(s)-ia)(q_2(s)-ib)} - \frac{p_1(s)p_2(s)}{(q_1(s)+ia)(q_2(s)+ib)}\right]$, where $a, b \in \mathbb{R}$ and $i^2 = -1$. **GENERALIZED**

$T_2\{\cos{(ax+by)}\} = \frac{1}{2}\left[\frac{p_1(s)p_2(s)}{(q_1(s)-ia)(q_2(s)-ib)} + \frac{p_1(s)p_2(s)}{(q_1(s)+ia)(q_2(s)+b)}\right]$, where $a, b \in \mathbb{R}, i^2 = -1$.

$T_2\{\sinh{(ax+by)}\} = \frac{1}{2}\left[\frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)} - \frac{p_1(s)p_2(s)}{(q_1(s)+a)(q_2(s)+b)}\right]$, where $a, b \in \mathbb{R}, i^2 = -1$

$T_2\{\cosh{(ax+by)}\} = \frac{1}{2}\left[\frac{p_1(s)p_2(s)}{(q_1(s)-a)(q_2(s)-b)} + \frac{p_1(s)p_2(s)}{(q_1(s)+a)(q_2(s)+b)}\right]$, where $a, b \in \mathbb{R}$

DOUBLE INTEGRAL TRANSFORMATION

In (2022) Darle M.S. et al. introduced a generalized double Rangaig integral transformation. This double integral transformation is defined as:

$$\eta_2\{f(x,y)\} = F(s,y) = \frac{1}{s^m r^n}\int_{-\infty}^0\int_{-\infty}^0 e^{p(s)x+q(r)y} f(x,y)dxdy$$
$$\text{where } m, n \in \mathbb{N}\cdot\frac{1}{\lambda_1} < s < \frac{1}{\lambda_2}, \frac{1}{\psi_1} < r < \frac{1}{\psi_2}, \text{and } \lambda_1, \lambda_2, \psi_1, \psi_2 \in \mathbb{R}$$

The double generalized Rangaig integral transformation for some elementary functions are:

$$\eta_2\{x^m y^n\} = \frac{1}{s^m r^n}\left(\frac{(-1)^{m+n}m!\,n!}{[p(s)]^{m+1}[q(r)]^{n+1}}\right), m, n \in \mathbb{N}.$$
$$\eta_2\{e^{ax+by}\} = \frac{1}{s^m r^n}\left(\frac{1}{(p(s)+a)(q(r)+b)}\right), a, b \in \mathbb{R}, n, m \in \mathbb{N}.$$
$$\eta_2\{\cos{(ax+by)}\} = \frac{1}{s^m r^n}\left(\frac{-aq(r)-bp(s)}{[p(s)]^2+a^2)([q(s)]^2+b^2)}\right), a, b \in \mathbb{R}, n, m \in \mathbb{N}.$$
$$\eta_2\{\cosh{(ax+by)}\} = \frac{1}{s^m r^n}\left(\frac{p(s)q(r)+ab}{([p(s)]^2-a^2)([q(r)]^2-b^2)}\right), a, b \in \mathbb{R}, n, m \in \mathbb{N}.$$

DOUBLE KUSHARE INTEGRAL TRANSFORMATION

In (2022) Patil D.P. et al. introduce the double Kushare transformation. This double integral transformation is defined as:

$$k_2\{f(x,y)\} = F(s,r)\} = sr\int_0^\infty\int_0^\infty e^{-(s^\delta x+r^\delta y)} f(x,y)dxdy,$$
$$x, y > 0 \text{ and } \delta \in Z$$

The double Kushare transformation for some functions are:

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Impact Factor: 7.53

$$K_2\{x^m y^n\} = \frac{m!\,n!}{s^{\delta((m+1)-1)} r^{\delta((n+1)-1)}}, m, n \in \mathbb{N}.$$
$$K_2\{e^{ax+by}\} = \frac{sr}{(s^\delta - a)(r^\delta - b)}, a, b \in \mathbb{R}.$$
$$K_2\{e^{i(ax+by)}\} = \frac{sr}{(s^\delta - ia)(r^5 - ib)}, a, b \in \mathbb{R}, i^2 = -1.$$
$$K_2\{\sin(ax+by)\} = \frac{1}{2i}\left[\frac{sr}{(s^\delta - ia)(r^5 - ib)} - \frac{sr}{(s^\delta + ia)(r^\delta + ib)}\right], a, b \in \mathbb{R}, i^2 = -1.$$
$$K_2\{\cos(ax+by)\} = \frac{1}{2}\left[\frac{sr}{(s^\delta - ia)(r^\delta - ib)} + \frac{sr}{(s^\delta + ia)(r^\delta - ib)}\right], a, b \in \mathbb{R} i^2 = -1.$$
$$K_2\{\sinh(ax+by)\} = \frac{1}{2}\left[\frac{s}{(s^5 - a)(r^\delta - b)} - \frac{sr}{(s^\delta + a)(r^s + b)}\right], a, b \in \mathbb{R}.$$
$$K_2\{\cosh(ax+by)\} = \frac{1}{2}\left[\frac{sr}{(s^\delta - a)(r^\delta - b)} + \frac{sr}{(s^\delta + a)(r^\delta + b)}\right], a, b \in \mathbb{R}.$$

DOUBLE LAPLACE -ARA INTEGRAL TRANSFORMATION

In (2022) Sedeeg A.K. et al. introduced the double Laplace-ARA integral transformation. This double integral transformation is defined for the function (x, y) by:

$$\mathrm{LG}_2\{f(x,y)\} = F(s,r) = r\int_0^\infty \int_0^\infty e_e^{-(sx+ry)} f(x,y)\,dxdy,$$

The double Laplace-ARA integral transformation for some functions are:

$LG_2\{x^m y^n\} = \frac{m!\,n!}{s^{m+1} r^n}$, where $n, m \in \mathbb{N}$.

$$LG_2\{e^{ax+by}\}\frac{r}{(s-a)(r-b)}, a, b \in \mathbb{R}.$$
$$LG_2\{\sin(ax+by)\} = \frac{r(sb+ra)}{(s^2+a^2)(r^2+b^2)}, a, b \in \mathbb{R}.$$
$$LG_2\{\mathrm{Cos}(ax+by)\} = \frac{r(rs-ab)}{(s^2+a^2)(r^2+b^2)}, a, b \in \mathbb{R}.$$
$$LG_2\{\sinh(ax+by)\} = \frac{r(sb+ra)}{(s^2-a^2)(r^2-b^2)}, a, b \in \mathbb{R}.$$
$$LG_2\{\cosh(ax+by)\} = \frac{r(ss+ab)}{(s^2-a^2)(r^2-b^2)}, a, b \in R.$$

DOUBLE ARA-SUMUDU INTEGRAL TRANSFORMATION

In (2022) Qazza A. et al. introduced the double ARA-Sumudu integral transformation. This double integral transformation is defined as:

$$GS_2\{f(x,y)\} = F(s,r) = \frac{s}{r}\int_0^\infty \int_0^\infty e^{-\left(sx+\frac{y}{r}\right)} f(x,y)\,dxdy$$
$$s, r, x, y > 0.$$

The double ARA-Sumudu transformation for some functions are:

$GS_2\{x^m y^n\} = \frac{m!\,r^n n!}{s^m}, m, n \in \mathbb{N}$

$GR_2\{e^{ax+by}\} = \frac{s}{(s-a)(1-br)}, a, b \in \mathbb{R}.$

$GR_2\{e^{t(ax+by)}\} = \frac{is}{(s-ia)(rb+i)}, a, b \in \mathbb{R}$ and $i^2 = -1.$

$GR_2\{\sin(ax+by)\} = \frac{s(a+bsr)}{(a^2+s^2)(b^2r^2+1)}, a, b \in \mathbb{R}$

$GR_2\{\cos(ax+by)\} = \frac{s(s-abr)}{(a^2+s^2)(b^2r^2+1)}, a, b \in \mathbb{R}$

$GR_2\{\sinh(ax+by)\} = \frac{s(a+bsr)}{(a^2-s^2)(b^2r^2-1)}, a, b \in \mathbb{R}.$

$GR_2\{\cosh(ax+by)\} = \frac{s(s+abr)}{(a^2-s^2)(b^2r^2-1)}, a, b \in \mathbb{R}.$

DOUBLE LAPLACE-SUMUDU INTEGRAL TRANSFORMATION

In (2022) Ahmed S. Aet al. introduced the Laplace-Sumudu integral transformation. This double integral transformation is defined as:

$$LS_2\{f(x,y)\} = F(s,r) = \frac{1}{r}\int_0^{\infty} e^{-\left(sx+\frac{y}{r}\right)} dxdy, \text{ where } x, y > 0.$$

The double Laplace-Sumudu integral transformation for some functions are:

$LS_2\{x^m y^n\} = \frac{m!n!r^n}{s^{m+1}}, n, m \in \mathbb{N}.$

$LS_2\{e^{ax+by}\} = \frac{s}{(s-a)(1-br)}, a, b \in \mathbb{R}.$

$LS_2\{e^{i(ax+by)}\} = \frac{1}{(s-ia)(1-ibr)}, a, b \in \mathbb{R}, i^2 = -1$

$LS_2\{\sin(ax+by)\} = \frac{a+bsr}{(s^2+a^2)(1+b^2r^2)}, a, b \in \mathbb{R}.$

$LS_2\{\cos(ax+by)\} = \frac{s-abr}{(s^2+a^2)(1+b^2r^2)}, a, b \in \mathbb{R}.$

$LS_2\{\sinh(ax+by)\} = \frac{a+bsr}{(s^2-a^2)(1-b^2r^2)}, a, b \in \mathbb{R}.$

$LS_2\{\mathrm{Cosh}(ax+by)\} = \frac{s-abr}{(s^2-a^2)(1-b^2r^2)}, a, b \in \mathbb{R}.$

DOUBLE FARMABLE INTEGRAL TRANSFORMATION

In (2022) Ghazal B. et al. introduced the double farmable integral transformation. This double integral transformation is defined as:

$$R_2\{f(x,y)\} = F(r,s,v,u) = \frac{s}{u}\frac{v}{r}\int_0^{\infty}\int_0^{\infty} e^{-\left(\frac{y}{r}x+\frac{s}{u}y\right)} f(x,y)dxdy.$$

$$x, y > 0$$

The double farmable transformation for some functions are:

$R_2\{k\} = k$, where k is an arbitrary constant.

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$R_2\{x^m y^n\} = m!\,n!\left(\frac{u}{s}\right)^m \left(\frac{r}{v}\right)^n, m, n \in \mathbb{N}.$

$R_2\{e^{ax+by}\} = \frac{sr}{(s-au)(v-br)}, a, b \in \mathbb{R}.$

$R_2\{e^{l(ax+by)}\} = \frac{sv(sv-abur)+Lsv(srb+uva)}{(s^2+a^2u^2)(v^2+b^2r^2)}, a, b \in \mathbb{R}$, and $i^2 = -1.$

$R_2\{\sin(ax+by)\} = \frac{sv(srb+uva)}{(s^2+a^2u^2)(v^2+b^2r^2)}, a, b \in \mathbb{R}$

$R_2\{\cos(ax+by)\} = \frac{sv(sv-abur)}{(s^2+a^2u^2)(v^2+b^2r^2)}, a, b \in \mathbb{R}.$

$R_2\{\sinh(ax+by)\} = \frac{sv(srb+uva)}{(s^2-a^2u^2)(v^2-b^2r^2)}, a, b \in \mathbb{R}.$

$R_2\{\cosh(ax+by)\} = \frac{sv(sv+abur)}{(s^2-a^2u^2)(v^2-b^2r^2)}, a, b \in \mathbb{R}.$

DOUBLE LAPLACE-ABOODH INTEGRAL TRANSFORMATION

In (2022) Hunaiber M. and AL-Aati A. introduced the double Laplace-Aboodh integral transformation. This double integral transformation defined as:

$$\mathrm{LA}_2\{f(x,y)\} = F(s,r) = \frac{1}{r}\int_0^\infty \int_0^\infty e^{(sx+ry)} f(x,y)\,dxdy,$$
$$x, y > 0.$$

The double Laplace-Aboodh (LA_2) integral transformation for some basic functions are:

$LA_2\{k\} = \frac{k}{sr^2}$, where k is a constant.

$LA_2\{x^m y^n\} = \frac{m!\,n!}{s^{m+1} r^{n+2}}, n, m \in N.$

$LA_2\{e^{ax+by}\} = \frac{1}{(s-a)(r^2-bb)}, a, b \in \mathbb{R}.$

$LA_2\{e^{t(ax+by)}\} = \frac{(sr-ab)+(bs+ar)}{(s^2+a^2)(r^3+b^2r)}, a, b \in \mathbb{R}.$

$LA_2\{\sin(ax+by)\} = \frac{bs+ar}{(s^2+a^2)(r^3+b^2r)}, a, b \in \mathbb{R}$

$LA_2\{\cos(ax+by)\} = \frac{sr-ab}{(s^2+a^2)(r^3+b^2r)}, a, b \in \mathbb{R}.$

$LA_2\{\sinh(ax+by)\} = \frac{bs+ar}{(s^2-a^2)(r^3-b^2r)}, a, b \in \mathbb{R}.$

$LA_2\{\cosh(ax+by)\} = \frac{sr+ab}{(s^2-a^2)(r^3-b^2r)}, a, b \in \mathbb{R}.$

DOUBLE SUMUDU-KAMAL TRANSFORMATION

In (2022) Hunaiber M. and AL-Aati A. introduced the double Sumudu-Kamal integral transformation. This double integral transformation is defined as:

$$SK_2\{f(x,y)\} = F(s,r) = \frac{1}{s}\int_0^\infty \int_0^\infty e^{-\left(\frac{xy}{sr}\right)} f(x,y)\,dxdy,$$
$$x, y > 0.$$

The double Sumudu-Kamal integral transformation for some basic functions are:

$\mathrm{SK}_2\{c\} = cs$, where c is a constant.

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$SK_2\{x^m y^n\} = m!\, n!\, s^m r^{n+1}$, where $n, m \in \mathbb{N}$.

$Sk_2\{e^{ax+by}\} = \dfrac{r}{(1-as)(1-br)}, a, b \in \mathbb{R}$.

$SK_2\{\sin(ax+by)\} = \dfrac{r(as+br)}{(1+a^2s^2)(1+b^2r^2)}, a, b \in \mathbb{R}$.

$SK_2\{\cos(ax+by)\} = \dfrac{r-absr^2}{(1+a^2s^2)(1+b^2r^2)}, a, b \in \mathbb{R}$.

$SK_2\{\sinh(ax+by)\} = \dfrac{r(asbr)}{(1-a^2s^2)(1-b^2r^2)}, a, b \in \mathbb{R}$.

$SK_2\{\cosh(ax+by)\} = \dfrac{r+absr^2}{(1-a^2s^2)(1-b^2s^2)}, a, b \in \mathbb{R}$.

DOUBLE COMPLEX SEE INTEGRAL TRANSFORMATION

In (2023) Eman A. Mansour et al. introduced the double complex SEE integral transformation. This complex double integral transformation is defined as:

$$S_2^c\{f(x,y)\} = T(s,r) = \frac{1}{(sr)^n}\int_0^\infty \int_0^\infty e^{-i(sx+ry)} f(x,y)dxdy$$

where s and r are complex parameters, $i^2 = -1, n \in \mathbf{Z}$, and $x, y > 0, \mathrm{Im}\,(s) < 0$, and $\mathrm{Im}\,(r) < 0$. The double complex SEE transformation for some basic functions are:

$S_2^c\{k\} = \frac{-k}{(Sr)^{n+1}}$, k is a constant.

$S_2^c\{xy\} = \frac{1}{(sr)^{n+2}}$

$S_2^c\{x^2y^2\} = \frac{-4}{(sr)^{n+3}}$.

$S_2^c\{e^{ax+by}\} = \frac{1}{(sr)^n}\left[\frac{ab-sr}{(ab-rs)^2(ar+bs)^2} + \frac{i(ar+bs)}{(ab-rs)^2+(ar+bs)^2}\right], a, b \in \mathbb{R}$ and $i \in \mathbb{R}$

$S_2^c\{\sin(ax+by)\} = \dfrac{i(ar+bs)}{(sr)^n(r^2-b^2)(s^2-a^2)^2}$

$S_2^c\{\cos(ax+by)\} = \dfrac{-(ar+bs)}{(sr)^n(s^2-a^2)(r^2-b^2)}$

$S_2^c\{\sinh(ax+by)\} = \dfrac{i(ar+bs)}{(sr)^n(r^2+b^2)(s^2+a^2)^2}$

$S_2^c\{\cosh(ax+by)\} = \dfrac{(ab-sr)}{(sr)^n(r^2+b^2)(s^2+a^2)^2}$

DOUBLE COMPLEX SADIK INTEGRAL TRANSFORMATION

In (2023) Turq Saed M. and Emad A. Kuffi introduced the complex double Sadik integral transformation. This double complex integral transformation is defined as:

$$D^c\{f(x,y)\} = F(s,r) = \frac{1}{(sr)^\beta}\int_0^\infty \int_0^\infty e^{-t(s^\alpha x + r^\alpha y)} f(x,y)dxdy.$$

Where s, r are complex parameters, α is any nonzero number and β is any real number and $\mathrm{Im}\,(s^\alpha) < 0, \mathrm{Im}\,(r^\alpha) < 0$. Complex double Sadik integral transformation for some fundamental functions are:

$D^c\{k\} = \frac{-k}{(s\pi)^{\alpha+\beta}}$, k is a constant.

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$D^c\{x^m y^n\} = \frac{(-l)^{m+n+2} m!\, n!}{s^{m\alpha+(\alpha+\beta)} r^{n\alpha+(\alpha+\beta)}}, n, m \in \mathbb{N}.$

$D^c\{e^{ax+by}\} = \frac{1}{(sr)^\beta}\left[\frac{(a+is^\alpha)\cdot(b+ir^\alpha)}{(s^{2\alpha}+a^2)(r^{2\alpha}+b^2)}, a, b \in \mathbb{R}.\right.$

$D^c\{e^{i(ax+by)}\} = \frac{-1}{(sr)^\beta (s^\alpha - a)(r^\alpha - b)}.$

$D^c\{\sin(ax+by)\} = i\left[\frac{ar^\alpha + bs^\alpha}{(sr)^\beta (s^{2\alpha}-a^2)(r^{2\alpha}-b^2)}\right].$

$D^c\{\cos(ax+by)\} = \frac{-(s^\alpha r^\alpha + ab)}{(sT)^\beta (s^{2\alpha}-a^2)(r^{2\alpha}-b^2)^2}$

$D^c\{\sinh(ax+by)\} = i\left[\frac{ar^\alpha + bs^\alpha}{(sr)^\beta (r^{2\alpha}+b^2)(s^{2\alpha}+a^2)}\right] +$

$D^c\{\cosh(ax+by)\} = \frac{ab - s^\alpha r^\alpha}{(sr)^\beta (r^{2\alpha}+b^2)(s^{2\alpha}+a^2)}$

DOUBLE GUPTA INTEGRAL TRANSFORMATION

In (2022) Gupta R. introduced the double Gupta integral transformation. This double integral transformation is defined as:

$$R_2\{f(x,y)\} = G(s,r) = \frac{1}{(sr)^3}\int_0^\infty \int_0^\infty e^{-(sx+ry)} f(x,y)\,dxdy$$

where $x, y > 0$

DOUBLE ARA-FORMABLE TRANSFORMATION

In (2023) Rania S. and Mustafa M.M. introduced the double ARA-Formable integral transformation. This double integral transformation is defined as:

$$GR_2\{f(x,y)\} = \frac{rs}{u}\int_0^\infty \int_0^\infty x^{n-1} e^{-\left(sx+\frac{ry}{u}\right)} f(x,y)\,dxdy = F(s,r,u).$$

$x, y > 0.$

The double ARA-Farmable transformation for some fundamental functions are:

$GR_2\{k\} = \frac{k\Gamma(n)}{s^{n-1}}, R_e(s) > 0, k$ is a constant, $n > -1$.

$GR_2\{x^a y^b\} = \frac{u^s\Gamma(b+n)\Gamma(a+m)}{r^b s^{a+n-1}},, n > -1, b > -1$

$GR_2\{e^{ax+by}\} = \frac{rs\Gamma(n)}{(s-a)^n(r-ub)}, s > a, \frac{r}{u} > b.$

$GR_2\{\sin(ax+by)\} = \frac{rs(n)}{2i(s-ia)^n(r-tub)} - \frac{rs\Gamma(n)}{2t(s+ia)^n(r+iub)^n}$, where $i \in \mathbb{C}$

$GR_2\{\cos(ax+by)\} = \frac{rs\Gamma(n)}{2i(s-ia)^n(r-tub)} + \frac{rs\Gamma(n)}{2i(s+ia)^n(r+iub)^2}$

$GR_2\{\sinh(ax+by)\} = \frac{rs\Gamma(n)}{2(s-a)^n(r-ub)} - \frac{rs\Gamma(n)}{2(s+a)^n(r+ub)^n}$

$GR_2\{\cosh(ax+by)\} = \frac{rs\Gamma(n)}{2(s-a)^n(r-ub)} + \frac{rs\Gamma(n)}{2(s+a)^n(r+ub)}.$

DOUBLE SEJI INTEGRAL TRANSFORMATION

In (2023) Jinan A. Jassim et al. introduced the double SEJI integral transformation. This double integral transformation is defined as:

$$T_2^c\{f(x,y)\} = F(s,r) = p_1(s)p_2(r)\int_0^\infty \int_0^\infty e^{-[[\phi_1(s)x+\phi_2(r)y]} f(x,y)\,dxdy$$

Provided that the integral exists for some $\Phi_1(s)$, $\Phi_2(r)$ and $x, y > 0$.

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$I(\Phi_1(s)) < 0$, $Im(\Phi_2(s)) < 0$.

The double SEJI integral transformation for some elementary functions are:

$T_2^c\{k\} = \frac{-kp_1(s)p_2(r)}{\phi_1(s)\phi_2(r)}$, where k is a constant.

$$T_2^c\{e^{ax+by}\} = \frac{p_1(s)p_2(r)}{(a^2+[\phi_1(s))^2)(b^2+[\phi_2(r)]^2)}(a \mid +i\phi_1(s)(b+i\phi_2(r)), \text{WheI}$$

$$T_2^c\{e^{i(ax+by)}\} = \frac{-p_1(s)p_2(r)}{(\phi_1(s)-a)(\phi_2(r)-b)}, a, b \in \mathbb{R}.$$

$$T_2^c\{\sin(ax+by)\} = \frac{-ip_1(s)p_2(r)[b\phi_1(s)) + a\phi_2(r)]}{[\phi_1(s)]^2-a^2)[[\phi_2(r)]^2-b^2)}, a, b \in \mathbb{R} \text{ and } i \in \mathbb{R}.$$

$$T_2^c\{\cos(ax+by)\} = \frac{-p_1(s)p_2(r)[\phi_1(s)\phi_2(r)+ab]}{([\phi_1(s)]^2-a^2)([\phi_2(r)]^2-b^2)}, a, b \in \mathbb{R}.$$

$$T_2^c\{\sinh(ax+by)\} = \frac{-ip_1(s)p_2(r)[b\phi_1(s)+a\phi_2(r)]}{([\phi_1(s)]^2+a^2)([\phi_2(r)]^2+b^2)}, a, b \in \mathbb{R} \text{ and } i \in \mathbb{R}.$$

$$T_2^c\{\cosh(ax+by)\} = \frac{-p_1(s)p_2(r)[\phi_1(s)\phi_2(r)+ab]}{[[\phi_1(s)]^2+a^2)([\phi_2(r)]^2+b^2)}, a, b \in \mathbb{R}.$$

$$T_2^c\{(xy)^n\} = \frac{(-i)^{2(n+1)}[r(n+1)]^2 p_1(s)p_2(r)}{[\phi_1(s)\phi_2(r)]^{n+1}}, n > 0.$$

$$T_2^c\{x^m y^n\} = \frac{(-i)^{m+1}(-i)^{n+1}\Gamma(n+1)\Gamma(m+1)p_1(s)p_2(r)}{[\phi_1(s)]^{m+1}[\phi_2(r)]^{n+1}}, m, n > 0.$$

DOUBLE COMPLEX EE INTEGRAL TRANSFORMATION

In (2023) Ahmad Issa and Emad A. Kuffi introduced the double EE integral transformation. This double complex transformation is defined by:

$$D_{EE}^c\{f(x,y)\} = F(s,r) = \int_0^\infty \int_0^\infty e^{l(s^n x + r^n y)} f(x,y)\,dxdy,$$

where $n \in \mathbb{Z}$, $x, y > 0$, $i \in \mathbb{C}$. $\mathrm{Im}(s^n) < 0$, $\mathrm{Im}(rn) < 0$.

The double complex EE transformation for some basic functions are:

$D_{EE}^c\{k\} = \frac{-k}{(uv)^n}$, where k a constant $n \in \mathbb{Z}$.

$D_{EE}^c\{x^p y^m\} = \frac{p!m!}{(is)(p+1)n(ir)^{(m+1)^n}}, n \in \mathbf{Z}, m, p > 0.$

$D_{EE}^c\{e^{ax+by}\} = \frac{(a+is^n)(b+lr^n)}{(a^2+s^{2n})(b^2+r^{2n})}.$

$D_{EE}^c\{e^{i(ax+by)}\} = \frac{-1}{(s^n+a)((\ \ ^n-b)^n}$

$D_{EE}^c\{\sin(ax+by)\} = \frac{i(bs^n+ar^n)}{(s^{2n}-a^2)(r^{2n}-b^2)}.$

$D_{EE}^c\{\cos(ax+by)\} = \frac{-(sr)^n+ab)}{(s^{2n}-a^2)(b^{2n}-b^2)}.$

$D_{EE}^c\{\sinh(ax+by)\} = \frac{t(ar^n+bs^n)}{(a^2+s^{2n})(b^2+r^{2n})^4}$

$D_{EE}^c\{\cosh(ax+by)\} = \frac{ab-(s)^n}{(a^2+s^{2n})(b^2+r^{2n})}.$

DOUBLE ZZ INTEGRAL TRANSFORMATION

In (2022), Rehab Ali Shaban et al. introduced the double ZZ integral transformation. This double integral transform in defined as:

$$\beta_2\{\varphi(x,y)\} = \frac{\theta}{\omega}\cdot\frac{\rho}{\sigma}\int_0^\infty \int_0^\infty \varphi(x,y)e^{-\left(\frac{\theta}{\omega}x+\frac{\rho}{y}y\right)}dxdy$$

Where θ, ω, ρ and γ are complex parameters.

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The double ZZ Transform for some basic functions are:

$\beta_2\{k\} = k, k$ is a constant.

$$\beta_2\{e^{ax+by}\} = \frac{\theta\rho}{(\theta - aw)(\rho - b\gamma)^*}$$

$$\beta_2\{x^a y^b\} = a!\,b!\left(\frac{w}{\theta}\right)^a \left(\frac{\gamma}{\rho}\right)^b, a, b = 0,1,2,3, \dots$$

$$\beta_2\{\text{Cos}\,(ax + by)\} = \frac{\theta\rho(\theta\rho - ab\gamma w)}{(\theta^2 + b^2w^2)(\rho^2 + b^2\gamma^2)^2}$$

$$\beta_2\{\sin\,(ax + by)\} = \frac{\theta\rho(a\theta\gamma + a)^2)}{(\theta^2 + a^2w^2)(\rho^2 + b^2\gamma^2)^2}$$

$$\beta_2\{\sinh\,(ax + by)\} = \frac{\theta\rho(a\gamma\theta + a\rho\omega)}{(\theta^2 - a^2\omega^2)(\rho^2 - b^2\gamma^2)}.$$

$$\beta_2\{\cosh\,(ax + by)\} = \frac{\theta\rho(\theta\rho + ab\gamma\omega)}{(\theta^2 - a^2\omega^2)(\rho^2 - b^2\gamma^2)}.$$

DOUBLE COMPLEX ZZ INTEGRAL TRANSFORMATION

In (2024), Rehab Ali et al. introduced the complex double ZZ integral transformation. This complex double integral transform is defined as:

$$\beta_2^c\{\varphi(x,y)\} = \frac{\theta}{\omega} \cdot \frac{\rho}{\gamma} \int_0^\infty \int_0^\infty \varphi(x,y) e^{-i\left(\frac{\theta}{\omega}x + \frac{\rho}{\gamma}y\right)} dxdy.$$

Where θ, ω, ρ and γ are complex parameters, and every Im $\left(\frac{\theta}{\omega}\right) < 0$ and Im $\left(\frac{\rho}{\gamma}\right) < 0$

The complex double ZZ transform for some elementary functions are:

$\beta_2^c\{k\} = -ik, k$ is a constant.

$$\beta_2^c\{e^{ax+by}\} = \frac{\theta\rho(awb\gamma - \theta\rho)}{(ab\gamma\omega - \theta\rho)^2 + (\theta b\gamma - a\omega\rho)^2} + i\frac{\theta\rho(\theta b\gamma - aw\rho)}{(ab\gamma\omega - \theta\rho)^2 + (\theta b\gamma - aw\rho)^2}.$$

$$\beta_2^c\{x^a y^b\} = a!\,b!\left(\frac{\omega}{\theta}i\right)^a \left(\frac{\gamma}{\rho}i\right)^b, a, b = 0,1,2, \dots.$$

$$\beta_2^c\{\cos\,(ax + by)\} = \frac{-\theta\rho(\theta\rho + a\omega b\gamma)}{(\theta^2 - a^2\omega^2)(\rho^2 - b^2\gamma^2)}.$$

$$\beta_2^c\{\sin\,(ax + by)\} = \frac{i\theta\rho(2b\theta\gamma + 2aw\rho)}{(\theta^2 - a^2\omega^2)(\rho^2 - b^2\gamma^2)}$$

$$\beta_2^c\{\cosh\,(ax + by)\} = \frac{\theta\rho(ab\omega\gamma - \theta\rho)}{(\theta^2 + a^2w^2)(\rho^2 + b^2\gamma^2)^2}.$$

$$\beta_2^c\{\sinh\,(ax + by)\} = \frac{i\theta\rho(b\gamma\theta + a\rho\omega)}{(\theta^2 + a^2\omega^2)(\rho^2 + b^2\gamma^2)^2}.$$

II. CONCLUSION

This study found that double integral transforms cannot be constructed without a single integral transform, either the same transform repeated to generate its own double or two separated integral transforms combined to generate a new integral transform. There are no visible distinctions between single and double integral transforms, hence doubles are not preferable than singles. The double integral transformation may handle particular applications with fewer algebraic operations than the single integral transformation.

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