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Anisotropic Plane Symmetric Inflationary Universe with Massless Scalar Field

G. U. Khapekar and Pallavi Dakare
Department of Mathematics
Jagadamba Mahavidyalaya, Achalpur, Amravati, Maharashtra, India
ganeshkhapekar777@gmail.com

Abstract: *A Plane symmetric homogeneous space-time in the presence of mass less scalar field with a flat potential is investigated. To get an inflationary universe, we have considered a flat region in which potential V is constant. It is assumed that scale factor is* $a(t) = e^{Ht}$ *where H is the Hubble constant. Some physical and kinematical properties of the model are also discussed.*

Keywords: Plane symmetry, Inflationary cosmological model, scalar field etc.

I. Introduction

In recent years, there has been a lot of interest in cosmological models of the universe which are important in understanding the mysteries of the early stages of its evolution. In particular, inflationary models of the universe it play a vital role in solving a number of outstanding problems in cosmology like the homogeneity, the isotropy and the flatness of the observed universe. The standard explanation for the flatness of the universe is that it has undergone at an early stage a period of exponential expansion known as inflation.

Guth [1], Linde [2] and La and Steinhardt [3] are some of the authors who have investigated different aspects of inflationary universes in general relativity. Scalar fields are the simplest classical fields and there exist an extensive literature containing numerous solutions of the Einstein equations where the scalar field is minimally coupled to the gravitational field. In particular, self –interacting scalar fields play a central role in the study of inflationary cosmology. Burd and Barrow [4], Wald [5], Barrow [6], Stein-Schabes [7], Ellis and Madsen [8] and Heusler [9] studied different aspects of scalar field in the evolution of the universe and FRW models. Bhattacharjee and Baruah [10], Bali and Jain [11] and Rahaman et al. [12] have studied the role of self interacting scalar fields in inflationary cosmology.

Reddy et al.[13], Reddy and Naidu [14] have discussed inflationary universe in general relativity in four and five dimensions. Reddy et al. [15] have investigated a plane symmetric Bianchi type-I inflationary universe in general relativity. Reddy [16] has discussed Bianchi type -V inflationary universe in general relativity. Also Reddy et al. [17], Katore and Rane [18] have studied the Kantowski-Sachs inflationary universe in general relativity. Inflationary scenario in locally rotationally symmetric Bianchi type II space-time with massless scalar field with flat potential, have been discussed by Bali and Laxmi [19]. Recently, Bali and Swati [20] investigated LRS Bianchi type-II inflationary scenario for a massless scalar field with flat potential and time varying Λ. Kaluza-Klein inflationary universe in general relativity has been studied by Adhav [21]. A five-dimensional Bianchi type-I inflationary universe is investigated in the presence of massless scalar field with a flat potential by Katore et al.[22]. Katore et al. [23] have investigated a plane symmetric inflationary universe with massless scalar field and time varying Λ.

Motivated by the above discussion, we have investigated inflationary scenario in anisotropic plane symmetric Bianchi Type-I space-time with flat potential, and assuming the condition scale factor $a \sim e^{Ht}$, H is Hubble constant as introduced by Kirzhnitz and Linde [24]. **In section 2**, we give a brief discussion of metric and field equations. **In section 3**, we find the solution of field equation i.e anisotropic the plane symmetric inflationary model. This model represents an anisotropic universe which isptropizes for large value of t under special condition as shown by Rothman and Ellis [25]. The model represents uniform expansion but accelerating universe. The model leads to de-sitter space-time. Lastly, **section 4** contains concluding remarks.

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II. Metric and Field Equations

We consider the homogeneous and anisotropic Bianchi type -I space - time described by the line element

$$ds^2 = dt^2 - A^2(t)\,dx^2 - B^2(t)\,(dy^2 + dz^2) \tag{1}$$

Where A, B are functions of t only.

Using Stein -Schabes [7] approach, the lagrangian of gravity minimally coupled with Higg's scalar field ϕ having effective potential $V(\phi)$ is given by ,

$$L = \int \sqrt{-g}\ (R - \frac{1}{2} g^{ij}\, \partial_i \phi \partial_j \phi - V(\phi))\, d^4 x \tag{2}$$

Where, notations have their usual meaning.

The variation of this action s with respect to the dynamical field leads to the Einstein field equations (Here geometrized unit, $8\pi G = c = 1$)

$$R_{ij} - \frac{1}{2} R\, g_{ij} = -T_{ij} \tag{3}$$

Where

$$T_{ij} = \partial_i \phi \partial_j \phi - \left[\frac{1}{2} \partial_\rho \phi\, \partial^\rho \phi + V(\phi)\right] g_{ij} \tag{4}$$

With

$$\frac{1}{\sqrt{-g}} \partial_i \left[\sqrt{-g}\, \partial^i \phi\right] = -\frac{\partial V}{\partial \phi} \tag{5}$$

Here, ϕ is the Higg's field, V the potential and g_{ij} the metric tensor

Now, Einstein's field equations (3) for metric in Eqn.(1) with the help of Eqn.(4) give a set of equations

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \tag{7}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = \left[\frac{1}{2}\dot{\phi}^2 - V(\phi)\right] \tag{8}$$

Eqn.(5) for scalar field ϕ leads to

$$\ddot{\phi} + \left(2\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\right)\dot{\phi} = -\frac{dV}{d\phi} \tag{9}$$

Where (.) indicates the derivative with respect to 't

III. Solution of the Field Equations

We assume that region is flat, so for flat region the effective potential is constant.

$$V(\phi) = \text{constant} = \text{k (say)} \tag{10}$$

Solving Eqn. (9), we get

$$\dot{\phi} = \frac{l}{A B^2} \tag{11}$$

Where, l is constant of integration

To get a deterministic model, we consider the condition [Zel'dovich et al. [26]; Kirzhnitz [27]] that scale factor can be expressed as

$$a = e^{Ht}$$

This leads to $a^3 = AB^2 = e^{3Ht}$ (12)

Using Eqns. (11) and (12), we have

$$\dot{\phi}=l\,e^{-3Ht} \tag{13}$$

Subtracting Eqns. (6) and (7), we get

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}+\frac{\dot{B}^2}{B^2}-\frac{\dot{A}\dot{B}}{AB}=0 \tag{14}$$

This gives

$$\frac{d}{dt}\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)+\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)\left[2\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\right]=0 \tag{15}$$

Integrating the above equation, we get

$$\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)=\frac{L}{AB^2}=\frac{L}{e^{3Ht}} \tag{16}$$

Where, L is constant of integration

Using Eqns. (12) and (16), we get

$$A=P\,e^{Ht}\exp\left\{\frac{2L\,e^{-3Ht}}{9H}\right\} \tag{17}$$

$$B=Q\,e^{Ht}\exp\left\{\frac{-L\,e^{-3Ht}}{9H}\right\} \tag{18}$$

Where P and Q are constants of integration

Using Eqns. (17) and (18) in Eqn. (1) and after suitable transformation of co-ordinates with choice of constants, the plane symmetric inflationary model is given by

$$ds^2=dt^2-P^2\,e^{2Ht}\exp\left\{\frac{4L\,e^{-3Ht}}{9H}\right\}dx^2-Q^2\,e^{2Ht}\exp\left\{\frac{-2L\,e^{-3Ht}}{9H}\right\}(dy^2+dz^2) \tag{19}$$

The model in Eqn. (19) has no singularity at t=0.

Integrating Eqn. (13), we get the scalar Higgs field

$$\phi=S-\frac{1}{3H}l\,e^{-3Ht} \tag{20}$$

Where, S is constant of integration.

3.1 Physical Properties

The spatial volume for the model (19) is given by

$$V=AB^2=e^{3Ht} \tag{21}$$

The average expansion anisotropy parameter is defined as $\bar{A}=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2$

Where, $\Delta H_i=H_i-H,\ H_1=\frac{\dot{A}}{A},\ H_2=H_3=\frac{\dot{B}}{B}$

This gives

$$\bar{A}=\frac{2L^2\,e^{-6Ht}}{9H^2} \tag{22}$$

For isotropy, we need $\bar{A}=0$. Therefore, the plane symmetric universe isotropizes when $L=0$

The shear scalar σ^2 is given by

$$\sigma^2=\frac{3}{2}\bar{A}H^2$$

This gives

$$\sigma^2 = \frac{1}{3} L^2 e^{-6Ht} \quad (23)$$

The condition $L = 0$ implies that the shear scalar vanishes i.e. equal to zero for isotropy.

The deceleration parameter q is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1 \quad (24)$$

IV. Conclusion

In this paper, we have presented the anisotropic plane symmetric inflationary cosmological model in the presence of massless scalar field with flat region of constant potential. The model (19) in general represents an anisotropic universe. However the model isotropized under the special condition as pointed out by Rothman and Ellis [25]. The spatial volume increases with time. Hence inflationary scenario exists in anisotropic Bianchi Type-I space-time. The expansion scalar is constant and the shear scalar $\sigma \neq 0$. There is uniform expansion and deceleration parameter, $q = -1$. Hence the model leads to de-sitter space time, and the model represents accelerating universe.

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