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A Review of Fractional Transforms and their Practical Applications

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Abstract: *The fundamental concepts and practical applications of fractional transforms, a potent mathematical instrument that has found pervasive use in various disciplines, are the focus of this review paper. Fractional transforms are generalizations of classical integral transforms that extend their applicability to signals and functions without integer orders. The article provides a comprehensive examination of fractional calculus, including a discussion of various types of fractional transforms and an examination of their utility in practical applications. Furthermore, the paper emphasizes the practical applications of fractional transformations in a variety of relevant fields, including finance, physics, image analysis, and signal processing*

Keywords: Fractional Calculus, Signal Processing, Differential Equations

I. INTRODUCTION

Fractional calculus is a branch of mathematics that applies the principles of differentiation and integration to non-integer orders, typically involving real or fractional values. It deals with fractional derivatives and integrals, which enable us to analyze and manipulate functions with non-integer degrees of differentiability. The importance of fractional calculus is its ability to be applied to a variety of intricate systems and phenomena that are not completely elucidated by conventional integer-order calculus. It is employed in a variety of disciplines, including finance, signal processing, engineering, and physics. Fractional calculus offers valuable instruments for modeling and analyzing complex real-world systems, facilitating a more profound comprehension of time-dependent processes with memory effects, long-range interactions, and anomalous behavior.

HISTORICAL BACKGROUND AND DEVELOPMENT OF FRACTIONAL TRANSFORMS

Fractional transforms originated in the late 19th and early 20th eras. Liouville, Riemann, and Grunwald invented fractional calculus, which deals with non-integer derivatives and integrals. Finally, fractional transforms advanced in the 1970s. Mathematicians and engineers investigated fractional transformations for signal processing, image analysis, and complicated differential equations. Due of its capacity to examine signals with time-varying spectra, Namias' 1980 fractional Fourier transform garnered attention. Since then, academics have developed fractional transform theory and application, integrating them into many scientific and engineering domains. Fractional transforms are fundamental in contemporary mathematics and engineering for signal processing, communications, and control systems.

Fractional Transform Basics A significant mathematical technique utilized in signal processing, image analysis, and quantum physics is fractional transforms. Fractional transforms may capture finer data details and are more adaptable than integral and discrete transforms because they use non-integer orders. The Fractional Fourier Transform (FRFT), which generalizes the traditional Fourier transform, is well-known. It analyzes signals with smooth or abrupt time-frequency variations. The Fractional Laplacian models anomalous diffusion and long-range interactions in complex systems. Researchers and practitioners studying signals and data with non-integer scaling must understand these transformations' features and applicability.

Derivatives and fractional integrals Fractional integrals and derivatives apply integer-order calculus to non-integer orders. Orders in fractional calculus might be real or complex. Antiderivatives may be extended to fractional integrals to integrate

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functions with non-integer differentiation orders. However, fractional derivatives represent the rate of change of a function with non-integer orders, enabling us to study fractal-like phenomena or long-range interactions. These operators are used in physics, engineering, signal processing, and finance, where complex systems and anomalies occur. Understanding fractional calculus helps us model and understand such systems, making it useful in current science and technology.

RIEMANN-LIOUVILLE, CAPUTO, AND OTHER DEFINITIONS

Riemann-Liouville and Caputo are two different definitions related to fractional calculus, a branch of mathematics that generalizes differentiation and integration to non-integer orders.

In fractional calculus, the Riemann- Liouville fractional integral and derivative are defined based on a real number order, typically denoted by α. The Riemann- Liouville fractional integral Iα(f)(x) of a function f(x) is given by the integration of f(x) over a certain interval to the power of α. On the other hand, the Riemann- Liouville fractional derivative Dα(f)(x) of a function f(x) represents the differentiation of f(x) to the power of α, followed by dividing it by the appropriate constant.

Fractional calculus also uses the Caputo fractional derivative, which is more useful for real-world issues. To calculate the Caputo fractional derivative, first take the standard derivative of a function and then apply the Riemann-Liouville fractional integral with the same order α.

Fractional calculus is used in signal processing, viscoelasticity, and fractional-order control systems. These concepts enable the modeling and study of complicated systems with memory and hereditary features, revealing phenomena that traditional calculus cannot completely represent.

FRACTIONAL LAPLACIAN AND FRACTIONAL FOURIER TRANSFORM

Fractional Transform Types Fractional transformations generalize integer-order transforms like Fourier transforms. They are utilized in signal and image processing, science, and engineering. The Fractional Fourier Transform (FRFT) is a popular fractional transform that combines the spatial and frequency domains to rotate data fractionally. The Fractional Laplacian, which generalizes the ordinary Laplacian operator to non-integer orders, is used in diffusion processes and random walk models. Fractional calculus also uses Caputo and Riemann-Liouville transformations to expand differentiation and integration to non-integer orders. These fractional transformations help analyze complicated data and handle non-linear and non-stationary events. Fractional Mellin transform

Signal processing, image analysis, and pattern identification employ the Fractional Mellin Transform. The traditional Mellin transform is extended to accommodate non-integer transformation parameter values. Fractional Mellin Transform is the fractional power integral of a function weighted by a complex exponential factor. This transform captures local and global data properties, making it suitable for fractal-like signals. Feature extraction, picture denoising, and scale-invariant pattern identification are its uses. Researchers and practitioners are using the Fractional Mellin Transform to solve complicated issues in several fields, improving signal processing and analysis.

FRACTIONAL HANKEL TRANSFORM

A mathematical integral transform utilized in signal and image processing, science, and engineering is the Fractional Hankel Transform (FHT). The conventional Hankel transform is extended to accept non-integer transform parameter values. The FHT effectively extracts radial characteristics from two-dimensional functions, especially in circular symmetry situations. It is used in optics, medical imaging, and pattern recognition. The FHT may be calculated numerically or using specialized algorithms to reveal radial information in input data.

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OTHER FRACTIONAL TRANSFORMS AND THEIR PROPERTIES

Fractional transforms, also known as fractional linear transforms or Möbius transforms, are important mathematical tools with diverse properties. Besides the widely known linear fractional transform $f(z) = (az + b)/(cz + d)$ (where a, b, c, and d are complex numbers with ad - bc ≠ 0), there exist other intriguing fractional transforms with distinct characteristics. For instance, the Schwarzian derivative, defined as $S(f)(z) = (f'(z)f(z)'')(1/2)(f(z))^2$, important in complex analysis and geometric function theory. It occurs in univalent function and differential equation studies. Another example is the Erdős-Baker map, a torus-based irrational rotation with unique dynamical features. These transformations have complex mathematical structures and are used in physics, cryptography, and computer graphics. Understanding these additional fractional transformations improves complicated analysis and introduces new mathematical avenues.

NUMERICAL TECHNIQUES FOR FRACTIONAL TRANSFORMS

Signal and image processing, partial differential equations with fractional derivatives, and other scientific and engineering applications depend on fractional transform numerical methods. FrFT and FLT address non-local and non-linear characteristics that integer-order transformations cannot. Fractional transformations are complicated and lack analytical answers for many issues. To effectively calculate fractional transforms, academics have created several new numerical algorithms, including fast Fourier-based techniques, quadrature rules, and iterative methods. These methods allow us to use fractional calculus in many domains to improve data analysis and problem-solving.

CHALLENGES IN COMPUTING FRACTIONAL TRANSFORMS

Fractional calculus and Fourier transformations are difficult to compute in mathematics and signal processing. Fractional orders, which are non-integer quantities, need sophisticated mathematical formulations. Accurate numerical approximations and methods for non-integer ordering are needed. Fractional transformations are computationally costly and numerically unstable due to singularities and non-smooth functions. Conventional computer techniques may not be enough to handle fractional transformations, thus specialized hardware and software are needed. Addressing these issues is vital to unleashing these transforms' full potential in scientific study and real-world applications in signal processing, physics, and engineering.

NUMERICAL ALGORITHMS AND APPROXIMATIONS

Numerous sciences and engineering sectors depend on numerical methods and approximations. These algorithms employ arithmetic to tackle complicated issues that are difficult to analyze. Numerical algorithms are needed to simulate physical processes in physics and engineering and optimize financial models in economics and finance. They include root-finding, interpolation, numerical integration, and linear algebra solvers. Complex mathematical functions or series are simplified using approximations to make them computationally tractable and precise. Despite their limits and inaccuracies owing to discrete approximations, numerical techniques are essential for solving real-world problems without closed-form answers. Continuous numerical algorithm research improves accuracy, efficiency, and resilience, helping scientists and engineers make informed judgments and acquire deeper insights into their fields.

FRACTIONAL TRANSFORMS IN SIGNAL PROCESSING

Fractional transformations are important in signal processing because they allow non-integer frequency analysis and manipulation. The Fractional Fourier Transform (FrFT) and Fractional Laplacian add a fractional exponent to Fourier and Laplace transformations. This fractional exponent lets the transform handle time-varying or non-stationary signals, making it helpful in communication, picture processing, and audio analysis. FrFTs can easily evaluate signals with chirp-like components, whereas Fractional Laplacians can examine fractional Brownian motion and other signals with long-range dependencies. As signal processing applications expand and need more diverse tools, fractional transformations help investigate and grasp complicated and non-standard data.

APPLICATION OF FRACTIONAL TRANSFORMS IN TIME-FREQUENCY ANALYSIS

The strong mathematical tool fractional transformations is used in time-frequency analysis. Combining fractional calculus with Fourier and wavelet transformations improves their ability to capture non-stationary and non-linear signal characteristics. A common example is the fractional Fourier transform (FRFT), which permits unrestricted time-frequency signal rotation. This feature is beneficial in signal processing applications with chirp-like or time-varying spectral characteristics. Fractional wavelet transformations (FWTs) can also analyze complicated time-frequency signals due to their improved localization and adaptive resolution. Fractional transforms in time-frequency analysis increase signal processing, communication, and pattern detection in telecommunications, image processing, and biological signal analysis.

Denoising and fractional filtering Signal and image processing uses fractional filtering and denoising to remove noise and artifacts to improve data quality. Fractional operators like fractional derivatives or integrals are used to the signal to control the filtering process more precisely than integer-order filters. This is helpful when the signal has non-local and long-range dependencies. However, denoising removes noise while keeping signal properties. In medical imaging, telecommunications, and audio processing, wavelet transform, statistical modeling, and machine learning techniques are essential. Researchers and engineers may enhance data quality and extract relevant information from noisy and complicated signals and pictures by combining fractional filtering and denoising.

FRACTIONAL TRANSFORMS IN IMAGE ANALYSIS

Fractal transformations are powerful picture manipulation and analysis tools. These transformations extend integer-order transforms like Fourier and Laplace transforms for more accurate and versatile image processing. In this field, FRFT and FLT are popular. FRFT handles complicated picture structures better by selectively filtering and rotating image features at any angle. FLT improves non-local image feature analysis, making it suitable for texture analysis and edge identification. Fractional transforms help image analysts improve medical imaging, remote sensing, and computer vision outcomes.

SOFTWARE PACKAGES FOR COMPUTING FRACTIONAL TRANSFORMS

Software for computers Fractional transformations are crucial in signal processing, image analysis, and numerical mathematics. These packages enable complicated fractional calculus procedures, which are essential for modeling non-local and memory-dependent systems. These software programs calculate fractional derivatives, integrals, and transformations efficiently and accurately using strong numerical algorithms and libraries. These technologies help researchers, engineers, and scientists understand complex systems, advancing control theory, biology, and finance. As demand for complex phenomenon analysis rises, these software packages must be continuously developed and improved to meet real-world application requirements.

HARDWARE IMPLEMENTATIONS AND OPTIMIZATIONS

Modern computer systems' efficiency and performance depend on hardware implementations and optimizations. Engineers may boost speed, power consumption, and functionality by customizing hardware components. GPUs and FPGAs, which surpass general-purpose CPUs in parallel computing, are used for graphics rendering, artificial intelligence, and encryption. Hardware improvements like pipelining, caching, and branch prediction reduce data bottlenecks and boost system performance. As technology improves, hardware implementations become more energy-efficient, quicker, and smarter, powering the growing digital world.

Challenges and Prospects Society faces interconnected challenges and future views in a continuously changing environment. Environmental concerns and technological upheavals need new answers as we grow. Action is needed to limit climate change's negative consequences on our planet and its people. The ethical implications of developing technologies like artificial intelligence and biotechnology need rigorous analysis and regulation to guarantee their responsible development and use. Economic and social inequality remain, needing a deliberate effort to promote inclusion and social justice. Despite obstacles, the future is bright. Renewable energy, sustainable habits, and green technology give promise for a sustainable society. Additionally, medical and space exploration advances provide new paths for human advancement. Embracing these possibilities and facing problems will help future generations have a better and more egalitarian future.

LIMITATIONS AND OPEN QUESTIONS IN THE FIELD

Scientific research is continually changing, and the area at hand has limits and unresolved questions. Although advances have been made, some restrictions still plague researchers and practitioners. Data availability and quality might impede generalizability in certain areas due to a lack of comprehensive datasets or biased information. The topic matter's intricacy frequently leads to simplistic models and assumptions, which may overlook key aspects affecting results. Interdisciplinary partnerships are essential for deeper insights, but they also present obstacles in harmonizing approaches and terminology. The exact amount of long-term impacts and the processes behind some events are yet unknown. These constraints must be accepted and open questions addressed with rigor and openness to advance the discipline and solve complicated challenges.

Possible multidisciplinary uses Combining knowledge and talents from other sectors may help solve complicated problems and advance innovation. The combination of AI, biology, and medicine might transform healthcare. AI-powered algorithms can evaluate genetic databases for customized treatment and medication development. Sustainable innovations like better solar panels and energy storage solutions are enabled by materials science, engineering, and renewable energy research. Psychology, design, and technology may also create user-centric products and services that improve human experiences. Using multidisciplinary techniques promotes cooperation, creativity, and complex problem-solving, leading to a more connected and inventive society.

EMERGING TRENDS AND RESEARCH DIRECTIONS

New trends and research paths will affect our world in many fields as we go forward. As technology advances, explainable AI and ethical issues for transparency and justice in its use are becoming more important. Quantum computing may solve complicated problems that traditional computers cannot, enabling new paths for scientific discovery and optimization. Precision medicine and gene editing are transforming healthcare, enabling focused therapeutics and disease prevention. Sustainable energy options including enhanced solar, energy storage, and carbon capture are being researched to mitigate climate change. Critical research on digital infrastructure security is needed as the Internet of Things (IoT) devices become increasingly linked, stretching data security and privacy limitations. Researchers in social sciences are exploring innovative mental health therapies and support systems to meet society's changing mental health needs. These trends and research lines have the potential to alter businesses and communities, but they also raise ethical and regulatory issues that must be addressed. Academics, business, and governments must work together to maximize these breakthroughs' advantages and minimize their hazards.

II. CONCLUSION

Finally, technology's fast growth has changed many elements of our existence. Technology has transformed our lives, work, and relationships in communication, healthcare, education, and more. This achievement raises ethical issues and the necessity for responsible AI development. To fully use technology to improve society, people, organizations, and governments must work to encourage ethical, inclusive, and sustainable innovation. We can build a future where technology improves lives and solves global problems by doing so. Accepting problems and seizing chances will lead to a better and more egalitarian future.

REFERENCES

[1]. E. Capelas de Oliveira, S. Jarosz, J. Vaz, (2019) “Fractional calculus via Laplace transform and its application in relaxation processes” Volume 69, 2019, ISSN 1007-5704, https://doi.org/10.1016/j.cnsns.2018.09.013.

[2]. David Mendlovic and Haldun M. Ozaktas, "Fractional Fourier transforms and their optical implementation: I," J. Opt. Soc. Am. A 10, 1875- 1881 (1993)

[3]. Viera-Martin, E., Gómez-Aguilar, J.F., Solís-Pérez, J.E. et al. Artificial neural networks: a practical review of applications involving fractional calculus. Eur. Phys. J. Spec. Top. 231, 2059–2095 (2022). https://doi.org/10.1140/epjs/s11734-022-00455-3.

[4]. Z. Wang, X. Wang, Y. Li, X. Huang, Stability and Hopf bifurcation of fractional-order complex- valued single neuron model with time delay. Int. J. Bifurc. Chaos 27(13), 1750209 (2018)

[5]. R. Rakkiyappan, G. Velmurugan, F.A. Rihan, S. Lakshmanan, Stability analysis of memristor-based complex-valued recurrent neural networks with time delays. Complexity 21(4), 14– 39 (2016)

[6]. C.J. Zúñiga-Aguilar, H.M. Romero-Ugalde, J.F. Gómez-Aguilar, R.F. Escobar-Jiménez, M. Valtierra-Rodríguez, Solving fractional differential equations of variable-order involving operators with Mittag–Leffler kernel using artificial neural networks. Chaos Solitons Fract. 103, 382–403 (2017)

[7]. S. Qureshi, A. Yusuf, A. Shaikh, M. Inc, Transmission dynamics of varicella zoster virus modeled by classical and novel fractional operators using real statistical data. Phys. A 534, 1–12 (2019)

[8]. Jajarmi, A. Yusuf, D. Baleanu, M. Inc, A new fractional HRSV model and its optimal control: a non-singular operator approach. Phys. A 547, 1– 12 (2020)

[9]. S. Qureshi, A. Yusuf, A. Ali Shaikh, M. Inc, D. Baleanu, Mathematical modeling for adsorption process of dye removal nonlinear equation using power law and exponentially decaying kernels. Chaos Interdiscip. J. Nonlinear Sci. 30, 1–10 (2020)

[10]. Gorenflo, R., & Mainardi, F. (1997). Fractional calculus. In Fractals and fractional calculus in continuum mechanics, 223-276.

[11]. Hilfer, R. (Ed.). (2000). Applications of fractional calculus in physics. World scientific.

[12]. Sun, H., Zhang, Y., Baleanu, D., Chen, W., & Chen, Y. (2018). A new collection of real world applications of fractional calculus in science and engineering. Communications in Nonlinear Science and Numerical Simulation, 64, 213-231.

[13]. Ray, S.S., Atangana, A., Noutchie, S.C., Kurulay, M., Bildik, N., & Kilicman, A. (2014). Fractional calculus and its applications in applied mathematics and other sciences. Mathematical Problems in Engineering.

[14]. Mansour, E.A., Mehdi, S., & Kuffi, E.A. (2021). The new integral transform and its applications. International Journal of Nonlinear Analysis and Applications, 12(2), 849-856.