IJARSCT
International Journal of Advanced Research in Science, Communication and Technology
International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal
ISSN: 2581-9429
Volume 5, Issue 6, June 2025
Impact Factor: 7.67

Design of PID Control Strategies for Spherical and Triangular Vessel Storage System

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Abstract: *Liquid level control in storage vessels with complex geometries, such as spherical and triangular tanks, presents significant challenges due to their nonlinear and time-varying dynamics. Accurate control is essential to maintain process efficiency, safety, and product quality in industrial applications. This paper investigates the design and performance of classical Proportional-Integral-Derivative (PID) controllers applied to spherical and triangular vessel storage systems using three widely adopted tuning methods: Ziegler-Nichols, Lambda tuning, and Kappa-Tau (τ-κ) tuning. Mathematical models of the vessels are developed based on first principles and fluid dynamics, capturing the nonlinear behavior inherent to each geometry. PID controllers are then designed and tuned using each method, and their performance is evaluated through simulation using standard control performance metrics such as rise time, settling time, overshoot, and integral error criteria. The comparative analysis reveals that while Ziegler-Nichols provides faster response, it often leads to higher overshoot. Lambda tuning offers smoother and more robust control, whereas the Kappa-Tau method achieves a balance between responsiveness and stability. The study concludes that the choice of tuning method significantly influences control performance and should be selected based on the specific dynamics of the vessel geometry.*

Keywords: *Liquid level.*

I. INTRODUCTION

In process control industries, maintaining the liquid level in storage vessels is crucial for operational safety, quality control, and system efficiency. While conventional cylindrical tanks are common, spherical and triangular vessels are also used in specialized applications where geometric and spatial constraints or specific fluid dynamics are key considerations. The nonlinear behavior arising from the changing cross-sectional area with height in these vessels makes accurate level control more challenging.

Traditional Proportional–Integral–Derivative (PID) controllers remain the industry standard due to their simplicity, cost-effectiveness, and well-established tuning methodologies. However, their effectiveness depends heavily on accurate tuning, especially in nonlinear and variable-area systems such as spherical and triangular tanks. The nonlinear dynamics result in varying time constants and gains at different liquid levels, complicating controller design.

This paper presents a comprehensive investigation into the design and performance of PID control strategies for nonlinear storage systems with spherical and triangular vessel geometries. These two tank shapes, due to their varying cross-sectional areas with respect to liquid height, introduce nonlinearities that challenge traditional control approaches. To address this, nonlinear mathematical models for both tank types are developed based on fluid dynamics and geometric relations. PID controllers are then designed using three different tuning methods: the Ziegler–Nichols (ZN) method, known for its simplicity and aggressive tuning; the Lambda (IMC-based) method, which provides robustness and smoother responses; and the Kappa–Tau (κ–τ) method, which utilizes process gain and time constant for model-based tuning. Each controller is applied to a linearized version of the nonlinear system around selected operating points. Simulation studies are performed to evaluate and compare the control strategies in terms of setpoint tracking, disturbance rejection, and performance indices such as settling time, overshoot, and integral error criteria (IAE, ISE,

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ITAE). The findings of this study offer practical insights into the suitability of these tuning methods for complex tank geometries and aim to guide control engineers in selecting effective PID strategies for nonlinear level control applications.

II. SYSTEM MODEL

The control of liquid levels in storage tanks is a critical function in many process industries. While cylindrical tanks have been extensively studied, non-standard geometries such as spherical and triangular vessels present additional challenges due to their nonlinear volume-to-level relationships. In spherical tanks, the rate of change of volume with respect to height is not constant, being more pronounced at the mid-level and slower near the top and bottom. Triangular or wedge-shaped tanks also exhibit nonlinear characteristics, particularly when arranged in a vertical orientation. These nonlinearities make classical linear control approaches, such as fixed-gain PID controllers, less effective unless they are specifically adapted or tuned for the changing dynamics.

PID controllers remain widely used due to their simplicity, but their performance depends heavily on the tuning method employed. One of the earliest and most widely applied methods is the Ziegler-Nichols (ZN) tuning technique, which determines controller parameters based on the system's ultimate gain and oscillation period. Although this method is straightforward and effective for linear systems, it often leads to aggressive controller behavior, resulting in overshoot and instability when applied to nonlinear processes such as those found in spherical and triangular tanks. Researchers such as Shamsuzzoha and Lee (2007) have noted the limitations of ZN tuning in nonlinear systems, emphasizing its tendency to produce poor performance under varying operating conditions.

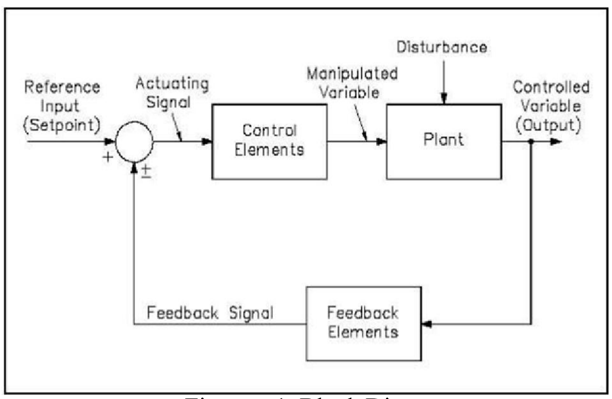


Figure – 1: Block Diagram

Mathematical Modelling of spherical tank:-

PID Consider non linear dynamics described by the first order differential equation with dead time depicted by following transfer function

$$\frac{dv}{dt} = Fin(t) - Fout(t)$$

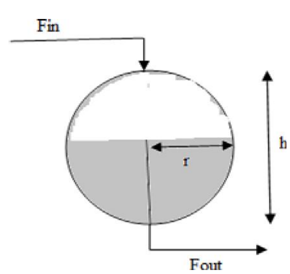


Figure 1 Schematic diagram of spherical tank system

$$V = \frac{4}{3}\pi h^3$$

Where,
V = Volume of spherical tank
F_{in} = Volumetric flow rate of spherical tank
F_{out} = Volumetric flow rate for outlet flow rate
h = Height of the tank in (cm)
Apply steady state values to system response from the plant and substituting the equations (1) and (2) to bring the process as a linear.

$$\frac{H(S)}{Q(S)} = \frac{Rt}{\tau s+1}$$

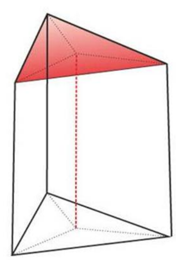
$$Rt = \frac{2hs}{Fout}$$

AND

$$\tau = 4\pi h_s Rt$$

Where,
h_s = height of the Tank at steady state τ = Time constant
Rt = Process gain
Transfer function is obtained by black box modeling method.

Mathematical Modelling of Spherical Tank



Assume a right-angled triangular cross-section (for simplicity).
Let:
b = base of the triangle (m)
h = height of the triangle (m)
L = length of the tank (m, into the page)
H(t) = liquid level at time t (m)
The cross-sectional area at level H is:

$$A(H) = \frac{b}{h} \times H \times H = b/h\, H^2$$

the volume of fluid at time t:

$$V(t) = A(H(t)).\, L = \frac{bL}{h} H^2(t)$$

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Dynamic System Equation

Let:

$q_{in}(t)$: inflow rate (m^3/s)

$q_{out}(t)$: outflow rate (m^3/s), often modeled as $C\sqrt{H(t)}$ (gravity drain)

Using mass conservation

$$\frac{dV}{dt} = q_{in}(t) - q_{out}(t)$$

from above

$$\frac{d}{dt}\left[\frac{bL\ H2}{h}\right](t) - C\sqrt{H(t)}$$

III. RESULTS AND DISCUSSION

Design Of Pid Controller

After deriving transfer function model the controller has to be designed for maintaining the system to the optimal set point. This can be achieved by properly selecting the tuning parameters Kp and for a PID controller. The tuning methods used in this paper are as follow,

ZEIGLER – NICHOLS (ZN) METHOD

A Step Response Method (Open-Loop)

Used for processes modeled as first-order plus dead time (FOPDT):

Process model:

$$G(s) = \frac{K}{\tau s + 1^{e^{-Ls}}}$$

where K = process gain, τ = time constant, L = dead time.

Lambda Method

Based on Internal Model Control (IMC).

For FOPDT process:

$$G(s) = \frac{K}{\tau s + 1^{e^{-Ls}}}$$

Choose desired closed-loop time constant λ (usually λ > L).

PID settings:

$$Kp = \frac{\tau}{(K\lambda + L)}$$

$$Ti = \tau$$

$$Td = 0 \text{ (for PI control)}$$

Kappa tau

Also derived from the FOPDT model:

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$$G(s) = \frac{K}{\tau s + 1^{e\text{-}Ls}}$$

Define kappa K= L/τ, a dimensionless number indicating process difficulty.
The tuning formulas depend on the value of \kappa. General form:

$$K_p = f(\kappa)\ .\ \frac{1}{K}$$

$$T\_i = g(\kappa)\ .\ \tau$$

$$T\_d = h(\kappa)\ .\ \tau$$

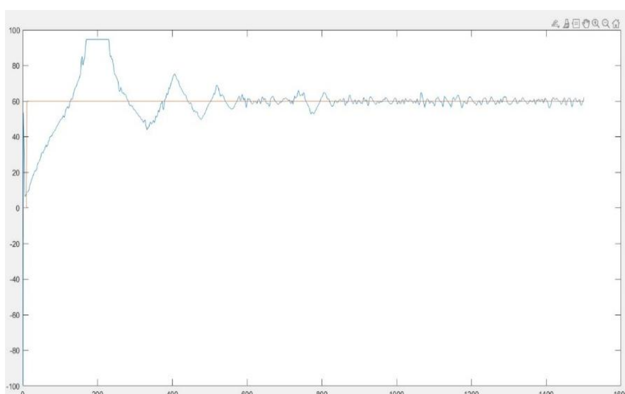
Tables or graphs are used for tuning depending on κ range.

Pid Settings Tuned By Various Methods for Spherical Tank

$$G_p(s) = \frac{1.13}{(216s+1)}e^{-4s}$$

Controller	Kp	Ki	Kd
ZN	57.14	7.143	114.28
Lambda	20.30	0.094	30.86
Kappa Tau	28.993	0.133	57.14

Table - 1: Time domain comparison



ZN Methods response of PID Controller

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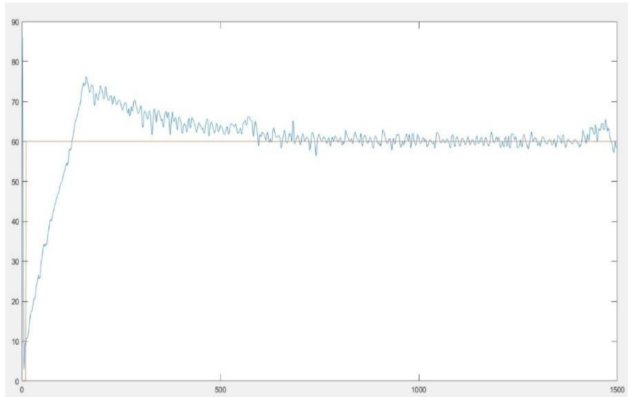
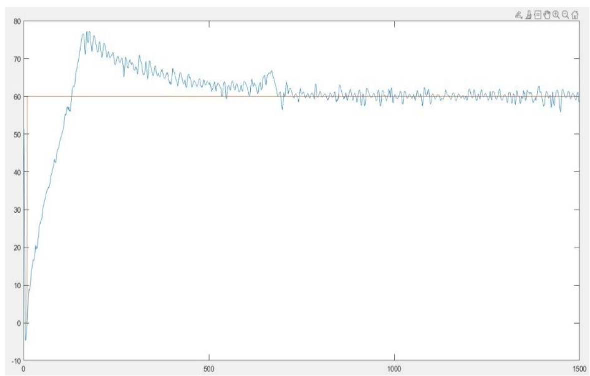


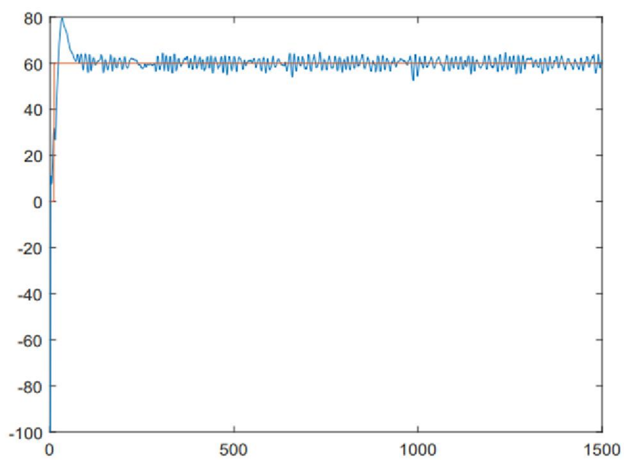
Lambda Methods response of PID Controller



Kappa Tau Methods response of PID Controller

Pid Settings Tuned By Various Methods for Triangular Tank

$$G_p(s)\frac{1.22}{(55.89s+1)}e^{-4.4s}$$



ZN Methods response of PID Controller

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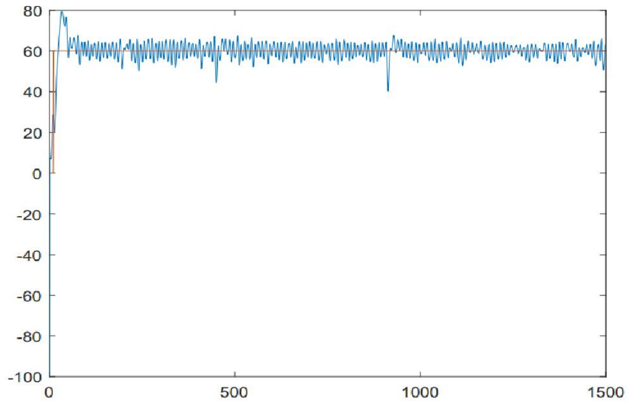
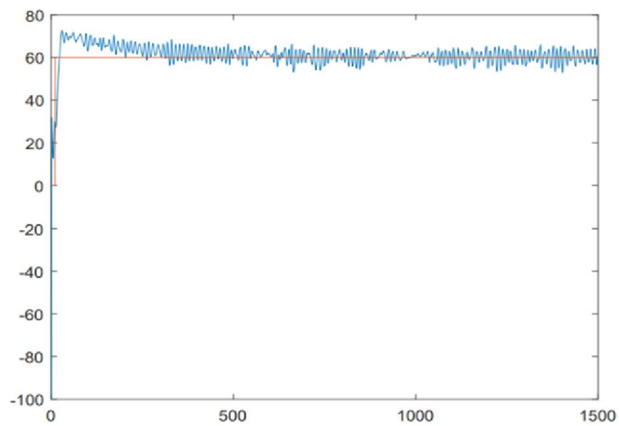


Lambda Methods response of PID Controller



Kappa Tau Methods response of PID Controller

IV. CONCLUSION

Controller	Kp	Ki	Kd
ZN	12.4	1.14	27.28
Lambda	4.33	0.0745	9.13
Kappa Tau	28.993	0.133	57.14

For a non-linear processes IMC based PID controller settings is obtained. Its performance is tested in real time spherical tank. Comparison with SIMC and various controller tuning methods indicates the effectiveness of the proposed method. Experimental results prove that the response smooth for both servo and regulatory changes. SIMC based PI controller shows better response than other tuning methods. This is validated by IAE and ISE values. It is concluded that the SIMC based pi controller outperforms well when compared to conventional controller tuning techniques in real time. By comparing the performance indices like the IAE and ISE values of different methods, we find that the SIMC is the best method with lesser values of IAE and ISE, among all other methods like ZN, IMC, Cohen-Coon and Chien.

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