

Study on Fuzzy Transportation Problem using Icosikaitetragonal Fuzzy Number

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Abstract: *The transportation problem represents a specific category within the applications of linear programming. Typically, this problem assumes that the decision-maker possesses precise knowledge of transportation costs, as well as the supply and demand for the product. However, there are instances where the decision-maker cannot define the objective with certainty and must instead rely on fuzzy concepts. In this study, we examine the fuzzy transportation problem (FTM) utilizing Icosikaitetragonal fuzzy numbers and their associated membership functions. We have established a ranking system for these fuzzy numbers to transform the fuzzy-valued transportation problem into a crisp-valued one. To demonstrate these methodologies, we have resolved a real-world issue using the Least Cost Method.*

Keywords: Fuzzy transportation problem (FTM), Icosikaitetragonal fuzzy number, membership function, least cost method

I. INTRODUCTION

The fuzzy set was offered to the world by Zadeh[1]. The fruits of fuzzy sets are extended to engineering and technology, management and economics by many researchers. The transportation problem originally developed by Hitchcock. The transportation problems are most useful to the industries and others to reduce the cost and maximize the profit. The transportation problem is a special case of linear programming problem. In a fuzzy transportation problem, costs, supply and demand values are fuzzy values. There are many approaches to solve the fuzzy transportation problem by different Authors. In many real life situations, it is not possible to determine both transportation unit cost and quantities, but the fuzzy numbers give best approximation of them. Raju and Jayagopal [2] introduced the Icosikaitetragonal fuzzy numbers and its membership function. Because there is always no possible to restrict the membership function in a particular form. Icosagonal fuzzy number is complex when it is compared to the triangular and trapezoidal fuzzy number both in form and computation. Using Icosikaitetragonal fuzzy numbers to solve the fuzzy transportation problem gives best optimal value when we compared with triangular and trapezoidal fuzzy numbers. Michael has proposed algorithm for solving transportation problem with fuzzy constraints and has investigated the relationship between the fuzzy algebraic structure of the optimum solution of the deterministic problem and its fuzzy equivalent. In this paper, we have solved fuzzy transportation problem using Icosikaioctagonal fuzzy number. We have illustrated fuzzy transportation problem using ranking technique with Icosikaitetragonal fuzzy numbers.

II. PRELIMINARIES

In this section, we give the preliminaries that are required for this study.

Definition 2.1. A fuzzy set A is defined by $A = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0, 1]\}$. Here x is crisp set A and $\mu_A(x)$ is membership function in the interval $[0, 1]$.

Definition 2.2. The fuzzy number A is a fuzzy set whose membership function must satisfy the following conditions.

- (i) A fuzzy set A of the universe of discourse X is convex
- (ii) A fuzzy set A of the universe of discourse X is a normal fuzzy set if $x \in X$ exists



(iii) $\mu_A(x)$ is piecewise continuous

Definition 2.3 An α -cut of fuzzy set A is classical set defined as ${}^\alpha[A] = \{x \in X | \mu_A(x) \geq \alpha\}$

Definition 2.4 A fuzzy set A is a convex fuzzy set iff each of its α -cut ${}^\alpha A$ is a convex set.

Definition 2.5 Mathematical formulation of a fuzzy transportation problem

The general form of Transportation problem is

$$\text{Minimize (total cost) } z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$$

$$\text{Subject to the constraints } \sum_{j=1}^n x_{ij} = a_i, i = 1, 2, 3, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, 3, \dots, n$$

$$x_{ij} \geq 0 \text{ For all } i \text{ and } j$$

2.6 Ranking of Icosikaitetragonal fuzzy number:

Let N be a normal Icosikaitetragonal fuzzy number. The value $V(M)$, called as measure of M is calculated as

$$M(P) = \frac{1}{2} \int_1^{k_1} (J_1 + J_2) dJ + \frac{1}{2} \int_{k_1}^{k_2} (A_1 + A_2) dA + \int_{k_2}^{k_3} (S_1 + S_2) dS + \int_{k_3}^{k_4} (T_1 + T_2) dT + \int_{k_4}^{k_5} (I_1 + I_2) dI + \int_{k_5}^{k_6} (N_1 + N_2) dN + \int_{k_6}^{k_7} (R_1 + R_2) dR$$

$$\text{where } 0 \leq k_1 \leq k_2 \leq k_3 \leq k_4 \leq k_5 \leq k_6 \leq k_7 \leq 1$$

$$M(P) = \frac{1}{4} \left[(r_1 + r_2 + r_{27} + r_{28})k_1 + (r_3 + r_4 + r_{25} + r_{26})(k_2 - k_1) + (r_5 + r_6 + r_{23} + r_{24})(k_3 - k_2) + (r_7 + r_8 + r_{21} + r_{22})(k_4 - k_3) + (r_9 + r_{10} + r_{19} + r_{20})(k_5 - k_4) + (r_{11} + r_{12} + r_{17} + r_{18})(k_6 - k_5) + (r_{13} + r_{14} + r_{15} + r_{16})(1 - k_6) \right]$$

$$\text{where } 0 \leq k_1 \leq k_2 \leq k_3 \leq k_4 \leq k_5 \leq k_6 \leq k_7 \leq 1$$

$$\text{we take the values for } k_1 = \frac{1}{7}, k_2 = \frac{2}{7}, k_3 = \frac{3}{7}, k_4 = \frac{4}{7}, k_5 = \frac{5}{7}, k_6 = \frac{6}{7}, k_7 = \frac{7}{7}$$

Definition 2.7 [2]

A fuzzy number $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, \dots, a_{24})$ is Icosikaitetragonal fuzzy number and its membership function is given by



$$\mu_A(x) = \begin{cases} 0, & \text{for } x < a_1 \\ k_1 \left(\frac{x - a_1}{a_2 - a_1} \right), & \text{for } a_1 \leq x \leq a_2 \\ k_1, & \text{for } a_2 \leq x \leq a_3 \\ k_1 + (k_2 - k_1) \left(\frac{x - a_3}{a_4 - a_3} \right), & \text{for } a_3 \leq x \leq a_4 \\ k_2, & \text{for } a_4 \leq x \leq a_5 \\ k_2 + (k_3 - k_2) \left(\frac{x - a_5}{a_6 - a_5} \right), & \text{for } a_5 \leq x \leq a_6 \\ k_3, & \text{for } a_6 \leq x \leq a_7 \\ k_3 + (k_4 - k_3) \left(\frac{x - a_7}{a_8 - a_7} \right), & \text{for } a_7 \leq x \leq a_8 \\ k_4, & \text{for } a_8 \leq x \leq a_9 \\ k_4 + (k_5 - k_4) \left(\frac{x - a_9}{a_{10} - a_9} \right), & \text{for } a_9 \leq x \leq a_{10} \\ k_5, & \text{for } a_{10} \leq x \leq a_{11} \\ k_5 + (1 - k_5) \left(\frac{x - a_{11}}{a_{12} - a_{11}} \right), & \text{for } a_{11} \leq x \leq a_{12} \\ 1, & \text{for } a_{12} \leq x \leq a_{13} \\ k_5 + (1 - k_5) \left(\frac{a_{14} - x}{a_{14} - a_{13}} \right), & \text{for } a_{13} \leq x \leq a_{14} \\ k_5, & \text{for } a_{14} \leq x \leq a_{15} \\ k_4 + (k_5 - k_4) \left(\frac{a_{16} - x}{a_{16} - a_{15}} \right), & \text{for } a_{15} \leq x \leq a_{16} \\ k_4, & \text{for } a_{16} \leq x \leq a_{17} \\ k_3 + (k_4 - k_3) \left(\frac{a_{18} - x}{a_{18} - a_{17}} \right), & \text{for } a_{17} \leq x \leq a_{18} \\ k_3, & \text{for } a_{18} \leq x \leq a_{19} \\ k_2 + (k_3 - k_2) \left(\frac{a_{20} - x}{a_{20} - a_{19}} \right), & \text{for } a_{19} \leq x \leq a_{20} \\ k_2, & \text{for } a_{20} \leq x \leq a_{21} \\ k_1 + (k_2 - k_1) \left(\frac{a_{22} - x}{a_{22} - a_{21}} \right), & \text{for } a_{21} \leq x \leq a_{22} \\ k_1, & \text{for } a_{22} \leq x \leq a_{23} \\ k_1 \left(\frac{a_{24} - x}{a_{24} - a_{23}} \right), & \text{for } a_{23} \leq x \leq a_{24} \\ 0, & \text{for } x > a_{24} \end{cases}$$

3.1 Balanced transportation problem:

The general form of Transportation problem is

$$\text{Minimize (total cost)} \quad z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$$



Subject to the constraints $\sum_{j=1}^n x_{ij} = a_i, i=1,2,3,\dots,m$

$$\sum_{i=1}^m x_{ij} = b_j, j=1,2,3,\dots,n$$

$$x_{ij} \geq 0 \text{ For all } i \text{ and } j$$

If total supply from all the sources is equal to the total demand in all destinations, then it is called as balanced transportation problem

3.2 Unbalanced transportation problem:

The general form of Transportation problem is

$$\text{Minimize (total cost) } z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$$

Subject to the constraints $\sum_{j=1}^n x_{ij} = a_i, i=1,2,3,\dots,m$

$$\sum_{i=1}^m x_{ij} = b_j, j=1,2,3,\dots,n$$

$$x_{ij} \geq 0 \text{ For all } i \text{ and } j$$

If in a transportation problem total supply from all the sources is not equal to the total demand in all destinations, then it is called as unbalanced transportation problem. But for a feasible solution to exist, total supply must be equal to the total demand thus it is necessary convert unbalanced problem into balanced problem.

3.3 Procedure for solving Least cost method (LCM)

Step 1: Select the cell having minimum unit cost C_{ij} and allocate as much as possible,

$$\text{i.e., } \min(s_i, d_j)$$

Step 2: Subtract this minimum value from supply S_i and demand d_j

Step 3: If the supply is S_i is zero then strike out that row and if the demand d_j is zero then strike that column

Step 4: If minimum unit cost cell is not unique, then select the cell where maximum allocation can be possible

Step 5: Repeat this steps for all uncrossed rows and columns until all supply and demand values are zero

IV. NUMERICAL EXAMPLE

Consider the following fuzzy transportation problem

	Destination			Supply
Origin	(-6,-5,-4,-3, -2,-1,0,1, 2,3,4,5, 6,7,8,9, 10,11,12,13, 14,15,16,17)	(-8,-7,-6,-5, -4,-3,-2,-1, 0,1,2,3, 4,5,6,7, 8,9,10,11, 12,13,14,15)	(1,2,3,4, 5,6,7,8, 9,10,11,12, 13,14,15,16, 17,18,19,20, 21,22,23,24)	(-4,-3,-2,-1, 0,1,3,5, 6,7,8,10, 12,13,15,17, 19,20,21,22, 24,25,26,27)



	(-6,-5,-4,-3, -2,-1,0,1, 2,3,4,5, 6,7,8,9, 10,11,12,13, 14,15,16,18)	(-11,-10,-9,-7, -6,-5,-4,-3, -2,-1,0,1, 2,3,4,5, 6,7,8,9, 10,11,12,13)	(0,1,2,4, 5,6,7,8, 9,10,11,12, 13,14,16,18, 20,22,24,26, 27,28,29,30)	(-10,-9,-8,-7, -6,-4,-2,-1, 0,1,2,3, 4,5,6,7, 9,10,11,12, 14,16,18,20)
	(0,1,2,3, 4,5,6,7, 9,10,11,13, 14,15,17,19, 21,22,24,25, 26,27,28,29)	(1,2,3,6, 8,9,10,12, 13,15,16,17, 19,20,22,23, 25,28,30,32, 34,35,37,39)	(-5,-4,-3,-2, -1,0,1,2, 3,4,5,6, 7,8,9,10, 11,12,13,14, 15,16,17,18)	(2,4,5,6, 8,10,12,13, 15,17,18,19, 20,22,23,24, 25,26,28,30, 31,32,33,34)
Demand	(2,3,4,6, 7,8,9,10, 11,12,13,14, 15,16,17,18, 19,20,21,22, 23,24,25,26)	(-8,-7,-6,-4, -3,-2,-1,0, 1,2,3,5, 6,7,8,9, 10,11,12,13, 14,15,16,17)	(-7,-6,-5,-4, -3,-2,-1,0, 1,2,3,4, 5,6,7,8, 9,10,11,12, 14,16,18,20)	

This problem is solved by taking the values for $k_1 = \frac{1}{7}, k_2 = \frac{2}{7}, k_3 = \frac{3}{7}, k_4 = \frac{4}{7}, k_5 = \frac{5}{7}, k_6 = \frac{6}{7}$. We obtain

the values of Measure of matrix A and is denoted by $\mu_{\text{lskoc}}(a_{ij})$

a_{11}	- 6,-5,-4,-3,-2,-1,0,1,2,3,4, 5,6,7,8,9,10,11,12,13,14,15,16,17	$\mu_{\text{lskoc}}(a_{11}) = 5.43$
a_{12}	- 8,-7,-6,-5,-4,-3,-2,-1,0,1, 2,3,4,5,6,7,8,9,10,11,12,13,14,15	$\mu_{\text{lskoc}}(a_{12}) = 2.36$
a_{13}	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16, 17,18,19,20,21,22,23,24	$\mu_{\text{lskoc}}(a_{13}) = 16.57$
a_{21}	- 6,-5,-4,-3,-2,-1,0,1,2,3,4,5, 6,7,8,9,10,11,12,13,14,15,16,18	$\mu_{\text{lskoc}}(a_{21}) = 5.67$
a_{22}	- 11,-10,-9,-7,-6,-5,-4,-3, - 2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13	$\mu_{\text{lskoc}}(a_{22}) = 9.25$
a_{23}	0,1,2,4,5,6,7,8,9,10,11,12,13,14,16,18, 20,22,24,26,27,28,29,30	$\mu_{\text{lskoc}}(a_{23}) = 22.43$
a_{31}	0,1,2,3,4,5,6,7,9,10,11,13,14,15,17,19, 21,22,24,25,26,27,28,29	$\mu_{\text{lskoc}}(a_{31}) = 14.5$



a_{32}	1,2,3,6,8,9,10,12,13,15,16,17,19,20,22,23, 25,28,30,32,34,35,37,39	$\mu_{fuzzy}(a_{32}) = 17.04$
a_{33}	-5,-4,-3,-2,-1,0,1,2,3,4,5,6 7,8,9,10,11,12,13,14,15,16,17,18	$\mu_{fuzzy}(a_{33}) = 6.5$

Fuzzy supplies are noted as follows

S_1	-4,-3,-2,-1,0,1,3,5,6,7,8,10,12,13, 15,17,19,20,21,22,24,25,26,27	11.32
S_2	-10,-9,-8,-7,-6,-4,-2,-1,0,1,2,3, 4,5,6,7,9,10,11,12,14,16, 18,20	4.07
S_3	2,4,5,6,8,10,12,13,15,17,18,19,20, 22,23,24,25,26,28,30,31,32,33,34	21.54

And Fuzzy demands are depicted as follows

D_1	2,3,4,6,7,8,9,10,11,12,13,14,15,16, 17,18,19,20,21,22,23,24,25,26	16.39
D_2	-8,-7,-6,-4,-3,-2,-1,0,1,2,3,5,6, 7,8,9,10,11,12,13,14,15,16,17	4.86
D_3	-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6, 7,8,9,10,11,12,14,16,18,20	5.25

And Total fuzzy supply and the total fuzzy demands are shown as follows

S	-12,-8,-5,-2,2,7,13,17,21,25,28,32,36,40,44,48,53,56,60,64,69,73,77,81	36.93
D	-13,-10,-7,-2,1,4,7,10,13,16,19,23,26,29,32,35,38,41,44,47,51,55,59,63	26.50

The crisp valued transportation is as follows

	Destination			Supply
Source	5.43	2.36	16.57	11.32
	5.67	9.25	22.43	4.07
	14.5	17.04	6.5	21.54
Demand	16.39	4.86	5.25	

Then the solution is explained in the following tables

	Destination						Supply
Source	5.43	2.36	0.89	16.57	0	10.43	11.32
	5.67	4.07	9.25	3.6	22.43	0	4.07
	14.5	12.32	17.04	3.97	6.5	5.25	21.54
Demand	16.39	4.86		5.25	10.43		36.93

The initial basic feasible solution is obtained by least cost method. We get the solution containing 8 non negative independent allocations equal to $m+n-1$

Origin	5.43	2.36	0.89	16.57	0.89 0
	5.67	9.25		22.43	4.07
	14.5	17.04		6.5	21.54
Demand	16.39	4.86 3.97		5.25	

	Destination				Supply
Origin	5.67	4.07	9.25	22.43	4.07 0
	14.5		17.04	6.5	21.54
Demand	16.39 12.32		3.97	5.25	

	Destination				Supply
Origin	14.5	17.04	6.5	5.25	21.54 16.29
Demand	12.32	3.97	5.25 0		

	Destination			Supply
Origin	14.5	17.04	3.97	16.29 12.32
Demand	12.32	3.97 0		

	Destination		Supply
Origin	14.5	12.32	12.32
Demand	12.32		

The transportation cost is

$$(2.36 \times 0.89) + (0 \times 10.43) + (5.67 \times 4.07) + (14.5 \times 12.32) + (17.04 \times 3.97) + (6.5 \times 5.25)$$

$$= 2.1004 + 0 + 23.0769 + 178.64 + 67.6488 + 34.125$$

Total cost = 305.5911

V. CONCLUSION:

In this paper, unbalanced transportation problem has been solved with Icosikaitetragonal fuzzy number. The use of Icosikaitetragonal fuzzy number in fuzzy transportation problem is given. Fuzzy transportation problem converted to



crisp valued problem and is illustrated by an example Optimal value obtained using Icosikaitetragonal fuzzy numbers are more optimal than the solution obtained by using hexadecagonal and Icosagonal fuzzy numbers

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