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Family of Operators that Cannot be Subspace Hypercyclic

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Abstract: *A bounded linear operator on a Banach space* $\boldsymbol{X}$ *is said to be subspace hypercyclic, if for some closed subset* $\boldsymbol{M} \subseteq \boldsymbol{X}$*, there exists* $\boldsymbol{x} \in \boldsymbol{X}$ *such that* $\boldsymbol{orb}(\boldsymbol{x}, \boldsymbol{T}) \cap \boldsymbol{M}$ *is dense in* $\boldsymbol{M}$*. Here we are going to show that no power bounded operators and hence a few other operators cannot be subspace hypercyclic*

Keywords: Hypercyclic, subspace hypercyclic, power bounded operators

I. INTRODUCTION

An operator T on a Banach space X is said to be hypercyclic if some point in X has a dense orbit. The study of hypercyclic and chaotic operators has been an interesting area of research. It is somewhat surprising that hypercyclic operators exist since they do not exist on finite-dimensional Banach spaces. The study of hypercyclicity goes back a long way, and has been studied in more general settings, for example in topological vector spaces.Classifications of variety of family of hypercyclic operators has been done. In this paper, we will show a few operators that cannot be subspace hypercyclic.

II. BASIC DEFINITIONS AND RESULTS

Let $\mathcal{H}$ be a separable Hilbert space and $\mathcal{B}(\mathcal{H})$ denote the set of all bounded linear operators on $\mathcal{H}$.

Definition 1: Let $\mathrm{T} \in \mathrm{B(H)}$ and let M be a nonzero subspace of$\mathcal{H}$. We say that T is subspace-hypercyclic for M if there exists $x \in \mathcal{H}$ such that $\mathrm{Orb(T, x)} \cap \mathrm{M}$ is dense in M. We call x a subspace-hypercyclic vector.

Definition 2: An operator T is called power bounded if $sup_{n\geq 0} \parallel T^n \parallel < \infty$.

An operator T is called a contraction if $\parallel T \parallel < 1$ and quasinilpotent if$\lim_{n\to\infty} \parallel T^n \parallel^{\frac{1}{n}} < 0$.

It is proved that no power bounded operators can be hypercyclic and hence contractions and quasinilpotent operator is hypercyclic. Here we prove that power bounded operators can never be subspace hpercyclic and also finite operators are not subspace hypercyclic.

III. MAIN RESULTS

Proposition 1: No power bounded operator$T \in \mathcal{B}(\mathcal{H})$ is subspace hypercyclic.

Proof: Assume, for contradiction T is subspace hypercyclic for some closed subspace M of $\mathcal{H}$. Then there exists $x \in H$ such that $orb(T, x) \cap M$ is dense in M. Since T is power bounded there exists $C > 0$ such that $\parallel T\text{^}n \parallel < c\ \forall n \epsilon \mathbb{N}$. Then $\|T^n x\| \leq \parallel T^n \parallel \parallel x \parallel \leq C \parallel x \parallel$. Thus the orbit of x is a countable bounded set. By Baire's Category Theorem, a countable bounded set cannot be dense in an infinite-dimensional Banach space or any of its infinite-dimensional closed subspaces. Each singleton $\{T^n x\}$ is nowhere dense, and a countable union of nowhere dense sets cannot be dense in a complete metric space. If M were finite-dimensional, its unit ball would be compact. But a countable (and hence discrete) set cannot be dense in a compact set. Thus, we reach a contradiction in both cases.

Every contraction and quasinilpotent operator is power bounded and thus we have the following corollary.

Corollary: No contraction and quasinilpotent operator is hypercyclic

There are no hypercyclic operators on finite dimensional spaces and also operators whose range has finite dimension cannot be hypercyclic. It is proved that bounded operator on finite dimensional space cannot be subspace hypercyclic

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and also compact operators can never be subspace hypercyclic. Next result proves finite rank operators cannot be subspace hypercyclic for any closed subspace M.

Proposition: Let $T \in \mathcal{B}(\mathcal{H})$ be a finite rank operator. Then T is not subspace hypercyclic for any closed subspace M of H.

Proof: Suppose that T is subspace hypercyclic for some closed subspace M. Then M is infinite dimensional. Since T is subspace hypercyclic, there exists $x \in H$ such that $orb(T,x) \cap M$ is dense in M. But since range of T is finite dimensional, this is impossible and T cannot be subspace hypercyclic.

IV. CONCLUSION

Subspace hypercyclic properties on various types of operators were studied and we have shown that power bounded operators and finite rank operators can never be subspace hypercyclic for any closed subspace.

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