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Algorithmic Approach and an Application for Algebraic Equations

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Abstract*: Iteration is the repetition of a process to solve a problem or defining a set of processes to called repeated with different values. The method mentioned in this survey article, we will find the roots of equations which are described. This method is called bisection, RegulaFalsi and Newton Raphson Method. The uses of these methods are implemented on an electrical circuit element. The solution of the problems is finding the only real roots of the equation. In different types of applications, sometimes the real roots cannot be finding. In this situation, the complex roots of the equation are determined. On the other side, the finding of the complex roots is needed to make again numerical analysis. The numeric analysis is except for this article. Drawn the flow chart of the method can be present different approach for this method with using C, Matlab programming language.*

Keywords: RegulaFalsi and Newton Raphson

I. INTRODUCTION

The construction of numerical solutions for non-linear equations is essential in many branches of science and engineering. Most of the time, non-linear problems do not have analytic solutions; the researchers resort to numerical methods. New efficient methods for solving nonlinear equations are evolving frequently, and are ubiquitously explored and exploited in applications. Their purpose is to improve the existing methods, such as classical Bisection, False Position, Newton–Raphson, and their variant methods for efficiency, simplicity, and approximation reliability in the solution. There are various ways to approach a problem; some methods are based on only continuous functions while others take advantage of the differentiability of functions. The algorithms such as Bisection using midpoint, False Position using secant line intersection, and Newton–Raphson using tangent line intersection are ubiquitous. There are improvements of these algorithms for speedup; more elegant and efficient implementations are emerging. The variations of continuity-based Bisection and False Position methods are due to Dekker [1,2], Brent [3], Press [4], and several variants of False Position including the reverse quadratic interpolation (RQI) method. The variations of derivative based quadratic order Newton–Raphsonmethod are 3rd, 4th, 5th, 6th order methods. From these algorithms, the researcher has to find a suitable algorithm that works best for every function [5,6]. For example, the bisection method for a simple equation $x - 2 = 0$, on interval [0, 4], will get the solution in one iteration. However, if the same algorithm is used on a different, smaller interval [0, 3], iterations will go on forever to get to 2; it will need some tolerance on error or on the number of iterations to terminate the algorithm. For derivative based methods, several predictor–corrector solutions have been proposed for extending the Newton–Raphson method with the support of Midpoint (mean of endpoints of domain interval), Trapezoidal (mean of the function values at the endpoint of the domain), Simpsons (quadratic approximation of the function) quadrature formula [7], undetermined coefficients [8], a third order Newton-type method to solve a system of nonlinear equations [9], Newton's method with accelerated convergence [10] using trapezoidal quadrature, fourth order method of undetermined coefficients [11], one derivative and two function evaluations [12], Newton's Method using fifth order quadrature formulas [13], using Midpoint, Trapezoidal, and Simpson quadrature sixth order rule [14]. Newton–Raphson and its variants use the derivative of the function; the derivative of the function is computed from integrations using quadrature-based methods. The new three-way hybrid algorithm presented here does not use any variations or quadrature or method of undetermined coefficient, and still, it competes with all these algorithms. For these reasons, a two-way blended algorithm was designed and

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implemented that is a blend of the bisection algorithm and regulafalsi algorithm. It does not take advantage of the differentiability of the function [15]. Most of the time, the equations involve differentiable functions. To take advantage of differentiability, we design a threeway hybrid algorithm that is a hybrid of three algorithms: Bisection, False Position, and Newton–Raphson. The hybrid algorithm is a new single pass iterative approach. The method does not use predictor–corrector technique, but predicts a better estimate in each iteration. The new algorithm is promising in outperforming the False Position algorithm and the Newton–Raphson algorithm. Table 1 is a listing of all the methods used for test cases. It is also confirmed [Tables 2–5] that it outperforms the quadrature based [7,13,14], undetermined coefficients methods [8], and decomposition based [16] algorithms in terms of number of function evaluations per iteration as well as overall number of iterations, computational order of convergence (COC), and efficiency index (EFF). This hybrid root finding algorithm performs fewer or at most that many iterations as these cited methods or functions. The bisection and regulafalsi algorithms require only continuity and no derivatives. This algorithm is guaranteed to converge to a root. The hybrid algorithm utilizes the best of the three techniques. The theoretical and empirical evidence shows that the computational complexity of the hybrid algorithm is considerably less than that of the classical algorithms. Several functions cited in the literature are used to confirm the simplicity, efficiency, and performance of the proposed method. The resulting iteration counts are compared with the existing iterative methods in Table 2. Even though the classical methods have been developed and used for decades, enhancements are made to improve the performance of these methods. A method may perform better than other methods on one dataset, and may produce inferior results on another dataset. We have seen an example just above that different methods have their own strengths/weaknesses. A dynamic new hybrid algorithm is presented here by taking advantage of the best in the Bisection, False Position, and Newton–Raphson methods to locate the roots independent of Dekker, Brent, RQI, and 3rd–6th order methods. Its computational efficiency is validated by comparing it with the existing methods via complexity analysis and empirical evidence. This new hybrid algorithm outperforms all the existing algorithms; see Section 4 for empirical outcomes validating the performance of the new algorithm. This paper is organized as follows. Section 2 describes background methods to support the new algorithm. Section 3 is the new algorithm. Section 4 is experimental analysis using a multitude of examples used by researchers in the literature, results, and comparison with their findings. Section 5 is on the complexity of computations. Section 6 is the conclusion

II. BACKGROUND DEFINITIONS

In order to review the literature, we briefly describe methods for (1) root approximation, (2) error calculation and error tolerance, and (3) algorithm termination criteria. There is no single optimal algorithm for root approximation. Normally we look for the solution in the worst-case scenario. The order of complexity does not tell the complete detailed outcome. The computational outcome may depend on implementation details, the domain, tolerance, and the function. No matter what, for comparison with different algorithms accomplishing the same task, we use the same function, same tolerance, same Eng 2021, 2 82 termination criteria to justify the superiority of one algorithm over the other. When we are faced with competing choices, normally the simplest one is the accurate one. This is particularly true in this case. We provide a new algorithm that is simpler and outperforms all these methods. For background, we will briefly refer to two types of equations: (1) those requiring only continuous functions, and (2) those requiring differentiable functions along with the desired order of derivative in their formulations. There are two types of problems: (1) continuity based with no derivative requirement, such as Bisection, False Position, and their extensions; and (2) derivative based, such as Newton–Raphson and its variations. For simulations, we use error Tolerance $\varepsilon = 0.0000001$, tolerance coupled with iterations termination criteria as $(|x_k - x_{k-1}| + |f(x_k)|) < \varepsilon$, and upper bound on iterations as 100: for function $f:[a, b] \to R$, such that (1) $f(x)$ is continuous on the interval $[a, b]$, where R is the set of all real numbers, and (2) $f(a)$ and $f(b)$ are of opposite signs, i.e., $f(a)\cdot f(b) < 0$, then there exists a root $r \in [a, b]$ such that $f(r) = 0$, or (20) the function $f(x)$ is differentiable with $g(x) = x - f(x)/f0 (x)$ and $|g0 (x) < 1$, then there exists a root $r \in [a, b]$ such that $f(r) = 0$. Since the solution is obtained by iterative methods, the definition of convergence is as follows:

Definition (Convergence) [10,12,17]. Let x_n, and $\alpha \in R$, $n \geq 0$. Then, the sequence $\{x_n\}$ is said to converge to α if $\lim_{n\to\infty} |x_n - \alpha| = 1$ Definition (Order of Convergence). Let x_n, and $\alpha \in R$, $n \geq 0$, sequence $\{x_n\}$ converge to α. In addition, [12,18] if there exists a constant $C > 0$ ($C 6= 0$, $C 6= \infty$) and an integer $p \geq 1$ such that $\lim_{n\to\infty} |x_{n+1} - \alpha|\ |x_n - \alpha|\ p = C$ then the sequence $\{x_n\}$ is said to converge to α with convergence order p, and C is the asymptotic error

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constant [10,12,18,19]. Since xn = α + en, the error equation becomes en+1 = C en p + O(en p+1) If p = 1, 2, or 3, the convergence is called linear, quadratic, or cubic convergence, respectively. These theoretical criteria do not take into consideration how much computation the function values perform in each step. The order of convergence p can be approximated by the Computational Order of Convergence (COC) that takes into account the combinatorial cost of the method. Definition (Computational Order of Convergence). Suppose three iterations xn−1, xn, xn+1 are closer to the root α; then, the order of convergence is approximated by [10,12,14] COC = limn→∞ □ □ log | |xn+1−α| |xn−α| | log | |xn−α| |xn−1−α| | □ □ Additionally, since α is not known a priori, there are other ways to compute COC, namely, using four iterations [8] instead of three iterations [10,12,17]. COC = limn→∞ □ □ log | |xn+1−xn| |xn−xn−1 | | log | |xn−xn−1 | |xn−1−xn−2| | □ □

In mechanical, electrical, construction as well as during the implementation of the different engineering disciplines which are created for the solution of the problem of mathematical modeling some of the systems are nonlinear equations, or equations [1]. Non-linear or non-linear equations are including two or more higher order polynomials equations or exponential, logarithmic, trigonometric called mathematical equations that contain terms such as non-linear. Graphical representation of the non-linear equations, linear expressions can be found. For example; x 4 +3x3 - 2x2 +5cosx=0 (1.1) or x+2cotx=4e-x (1.2) These statements are only unknowns mathematical expressions. Nonlinear expressions, f(x)=0 (1.3) is defined like this. Similarly, more than one variable in the expressions,

III. CONCLUSION

The initial value is incorrect repetitions faulty condition is selected, so the root will be reflected in the process of obtaining. Function which will be in the process of settlement of the range set for the number of repetitions is not connected, i.e the sensitivity of the value depends on the sensitivity of the equation. Convergence of bisection method is slower than other methods, and the number of repetitions are more than the other methods. The biggest drawback is that the number of repetitions compared to other methods. Be determined for the maximum number of repetitions by the person who analyzes the problem.