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Solutions of the Genralities for the Plane Wave in the Symmetrical Space Time of Four Dimension

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Abstract: *For the plane wave* $\left(\frac{\sqrt{t_1^2+t_2^2}}{z}\right)$ *we obtain the solutions of the generalities* $R_{ij} = 0$ *by taking the symmetrical space time as*

$$ds^2 = -Ady^2 - \emptyset_2^2 Bdz^2 + \emptyset_3^2 2Bdt_1^2 + 2Bt_2^2$$

and the solutions are as

$$P = \frac{1}{2}\left(\frac{\overline{\overline{M}}}{M} - \frac{\overline{M^2}}{2M} - \frac{\overline{M}\,\overline{B}}{M\,B}\right) - \frac{1}{2}\left(\frac{\overline{B}}{B} - \frac{3\overline{B^2}}{2B^2}\right).$$

By calculating the curvature tensor & Ricci tensor the similar solution is obtained

Keywords: Plane Wave solutions , Review of literature, Research Methodology Generalities, Mathematical Formulation, four dimensional space-time, General theory of relativity , curvature tensor & Ricci tensor.

I. INTRODUCTION

Warade in the paper [1] have obtained pws g_{ij} of the generalities $R_{ij} = 0$ in V_4 as

$$\overline{\overline{w}}\rho_{\alpha\beta} + \overline{w}\sigma_{\alpha\beta} = \overline{\overline{\emptyset_2}}\rho_{\alpha\beta} + \overline{\emptyset_2}\sigma_{\alpha\beta} = \overline{\overline{\emptyset_3}}\rho_{\alpha\beta} + \overline{\emptyset_3}\sigma_{\alpha\beta} = 0 \quad (1.1)$$

where

$$\emptyset_2 = \frac{Z_{,2}}{Z_{,4}} \quad \emptyset_3 = \frac{Z_{,3}}{Z_{,4}}$$

$$W = \emptyset_\alpha x^\alpha = \emptyset_2 z + \emptyset_3 t_1 + t_1$$

Also we have established the existence of $\left(\frac{\sqrt{t_1^2+t_2^2}}{z}\right)$ plane waves with the reference to [2] & the solution (1.1) reduced to $\overline{L_2} - \overline{\rho_4} + \frac{\rho_4^2}{4} - L_2\rho_4 + \frac{L_1}{4} = 0$ (1.2)

II. REVIEW OF LITERATURE

The Techno H chapter one, the purely gravitational waves is reviewed for this paper. The development and application of Einstein's field equation has been studied by many researchers. The few names are of Sir Arthurs Eddington led (1919) an astronomical expedition that confirmed the gravitational deflection of light by the sun in Pound-Rebka [1959] Hafele-Keating 1971, gravitational time dilation predicted by general relativity, where the red shifting of gamma rays was measured in a laboratory a Thorne, Kip (2003). "Warping spacetime". The future of theoretical physics and

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cosmology: Fulvio Melia (2009) Cracking the Einstein Code, Castelvecchio, Davide; Witze, Witze (11 February 2016). "Einstein's gravitational waves found at last". Over bye, Dennis (10 April 2019). "Black Hole Picture Revealed for the First Time –Astronomers at last have captured an image of the darkest entities in the cosmos". General relativity has developed into an essential tool in modern astrophysics. It provides the foundation for the current understanding of blackholes, regions of space where the gravitational effect is strong enough that even light cannot escape.

III. RESEARCH METHODOLOGY

Investigation analysis will be performed to find out or investigate in the mathematical formulation and solution technique. The result of proposed study will be first compared with those available literature for few guidelines of applying formulation and solvable technique in some Special cases. Action research is a method that has proven to be valuable as a problem-solving tool. It can provide opportunities for the study of universe, how to plane wave solution in general relativity, to applying geometrical theory in gravitation as well as different types of plane waves. The C20th theories of general relativity have limited scope. GR1916/GR 1960 doesn't successfully match with particle physics, because an attempt to model particles as small gravitational sources leads to an acoustic metric and a different set of equations of motion to special relativity, which GR1916/60 assumes to be correct. although C20th GR has been extended to deal with an expanding universe, it cannot cope with large scales consistently, because the causal structure of a cosmological horizon is acoustic and does not correspond to the theory's black hole horizons - the Hubble shift law is also not the shift law of special relativity and the behavior of black holes under C20th GR, if we believe quantum mechanics, is qualitatively wrong. In the present research work we shall study more deeply into the solution of FEs and obtained in the concrete from of the solution for this present type of wave

3.1 MATHEMATICAL FORMULATION

H Takeno solved the field equations in GR,

$$k_{ij}=-8\pi E_{ij},(i,j,k=1,2,3\ldots 4)$$

$$F_{ij,k}+F_{jk,i}+F_{ki,j}=0$$

$$F_j^{ij}=0$$

and obtain some exact pws& investigated their properties. In these equations,

k_{ij} is the Ricci tensor of the space time,

F_{ij} is the antisymmetric tensor describing the electromagnetic filed,

E_{ij} is theelectromagnetic energy tensor,

G_{ij} is the fundamental tensor of the space time,

and semicolon denotes the covariant derivative.

Takeno obtained the concrete form of the filed equation as under

$$\overline{L_2}-\overline{\rho_4}+\frac{\rho_4^2}{2}-L_2\rho_4+\frac{L_1}{4}=0 \quad 3.2$$

The results of the equation (1.2) for $\left(\frac{\sqrt{t_1^2+t_2^2}}{z}\right)$ plane wave in space time of V_4 :

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For this plane wave the line element given in the abstract becomes

$$ds^2 = -Ady^2 - \left(\frac{t_1^2+t_2^2}{zt_2}\right)^2 Bdz^2 + \left(\frac{t_1}{t_2}\right)^2 2Bdt_1^2 + Bdt_1^2 \quad (3.1)$$

where A&B are functions of

$$Z = \frac{\sqrt{t_1^2+t_2^2}}{z}$$

$$\emptyset_2 = \frac{z_{,2}}{z_{,4}} = -\left(\frac{t_1^2+t_1^2}{zt_2}\right)$$

$$\emptyset_3 = \frac{z_{,3}}{z_{,4}} = -\left(\frac{t_1}{t_2}\right)$$

Then we have,

$$v^i = \phi_\alpha g^{\alpha i} = [v^1, v^2, v^3, v^4]$$

$$= \left[0, \left(\frac{zt_2}{t_1^2+t_2^2}\right)\frac{1}{B}, \left(\frac{t_2}{t_1}\right)\frac{1}{2\beta}\right]$$

$$= \left[0, \left(\frac{zt_2}{t_1^2+t_2^2}\right), \frac{1}{2\beta t_1}, \frac{1}{2B}\right] \quad (3.2)$$

$$\rho_i = \overline{g_{ij}}\, v^1 = [\rho_1, \rho_2, \rho_3, \rho_4]$$

$$= \left[0, -B\left(\frac{t_1^2+t_2^2}{Bzt_2}\right), \frac{\bar{B}t_1}{Bt_2}, \frac{\bar{B}}{B}\right] \quad (3.3)$$

$$L_2 = \frac{\bar{m}}{2m} + \frac{\overline{3B}}{2B} \;\&\; L_1 = \frac{\bar{m}^2}{2m^2}\frac{\overline{3B^2}}{B^2} \quad (3.4)$$

According to all these values the equation (1.2) gives that,

$$P = \frac{1}{2}\left(\frac{\bar{\bar{m}}}{m} - \frac{\overline{m}}{2m^2} - \frac{\bar{m}\bar{B}}{mB}\right) - \frac{1}{2}\left(\frac{\bar{\bar{B}}}{B} - \frac{3\bar{B}^2}{2B^2}\right) \quad (3.5)$$

We obtain this solution by our wave $\left(\frac{\sqrt{t_1^2+t_2^2}}{z}\right)$ by using nonvanishing components of curvature and Ricci tensors.

3.3 The components of Christoffel symbol:

For the given wave the Christoffel symbol from the line element (1.3) are obtained as under,

$$\Gamma_{12}^1 = \left(\frac{-\bar{A}}{2A}\right)\frac{\sqrt{t_1^2+t_2^2}}{z^2}, \Gamma_{13}^1 = \left(\frac{-\bar{A}}{2A}\right)\frac{t_1}{\sqrt{t_1^2+t_2^2}}, \Gamma_{14}^1 = \left(\frac{-\bar{A}}{2A}\right)\frac{t_1}{\sqrt{t_1^2+t_2^2}},$$

$$\Gamma_{11}^2 = \left(\frac{\bar{A}}{2A}\right)\frac{t_2^2\sqrt{t_1^2+t_2^2}}{(t_1^2+t_2^2)^2}, \Gamma_{11}^3 = \left(\frac{\bar{A}}{4B}\right)\frac{t_1}{Z\sqrt{t_1^2+t_2^2}}, \Gamma_{11}^4 = \left(\frac{\bar{A}}{4B}\right)\frac{t_1}{Z\sqrt{t_1^2+t_2^2}},$$

$$\Gamma_{22}^2 = \left(\frac{-\bar{B}}{2B}\right)\frac{\sqrt{t_1^2+t_2^2}}{z^2}\; \Gamma_{12}^3 = \left(\frac{-\bar{B}}{4B}\right)\frac{(t_1^2+t_2^2)^2}{z^3 t_1\sqrt{t_1^2+t_2^2}}\; \Gamma_{22}^4 = \left(\frac{-\bar{B}}{4B}\right)\frac{(t_1^2+t_2^2)^2}{z^3 t_2\sqrt{t_1^2+t_2^2}}$$

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$$\Gamma_{33}^{2} = \left(\frac{-\bar{B}}{B}\right)\frac{t_1^2\sqrt{t_1^2+t_2^2}}{(t_1^2+t_2^2)^2}\ \Gamma_{33}^{3} = \left(\frac{\bar{B}}{2B}\right)\frac{t_1}{z^3 t_1\sqrt{t_1^2+t_2^2}}\ \Gamma_{33}^{4} = \left(\frac{-\bar{B}}{2B}\right)\frac{t_1^2}{z\, t_2\sqrt{t_1^2+t_2^2}}$$

$$\Gamma_{44}^{2} = \left(\frac{-\bar{B}}{B}\right)\frac{t_2^2\sqrt{t_1^2+t_2^2}}{(t_1^2+t_2^2)^2}\ \Gamma_{44}^{3} = \left(\frac{-\bar{B}}{2B}\right)\frac{t_2^2}{z^3 t_1\sqrt{t_1^2+t_2^2}}\ \Gamma_{44}^{4} = \left(\frac{-\bar{B}}{2B}\right)\frac{t_2}{z\sqrt{t_1^2+t_2^2}}$$

$$\Gamma_{23}^{2} = \left(\frac{-\bar{B}}{2B}\right)\frac{t_1}{z\sqrt{t_1^2+t_2^2}}\ \Gamma_{23}^{3} = \left(\frac{-\bar{B}}{2B}\right)\frac{\sqrt{t_1^2+t_2^2}}{z}\ \Gamma_{24}^{4} = \left(\frac{\bar{B}}{2B}\right)\frac{t_2}{z\sqrt{t_1^2+t_2^2}}$$

$$\Gamma_{24}^{2} = \left(\frac{-\bar{B}}{2B}\right)\frac{\sqrt{t_1^2+t_2^2}}{z^2}\ \Gamma_{34}^{3} = \left(\frac{-\bar{B}}{4B}\right)\frac{t_2}{z\sqrt{t_1^2+t_2^2}}\ \Gamma_{34}^{4} = \left(\frac{-\bar{B}}{4B}\right)$$

3.4 The 12 components of curvature tensor are obtained as:

$$R_{1212} = \left(\frac{(t_1^2+t_2^2)}{z^4}\right)G,\ R_{1214} = \left(\frac{-t_2}{z^3}\right)G,$$

$$R_{1213} = \left(\frac{-t_2}{z^4}\right)G,\ R_{1313} = \left(\frac{t_1^2}{z^2(t_1^2+t_2^2)}\right)G,$$

$$R_{1314} = \left(\frac{t_1 t_2}{z^2(t_1^2+t_2^2)}\right)G,\ R_{1414} = \left(\frac{t_2^2}{z^2(t_1^2+t_2^2)}\right)G,$$

$$R_{2323} = \left(\frac{-t_1^2\,(t_1^2+t_2^2)}{z^4 t_2^2}\right)H,\ R_{2324} = \left(\frac{t_1\,(t_1^2+t_2^2)}{z^4 t_2}\right)H,$$

$$R_{2344} = \left(\frac{-2t_1^2}{z^3 t_2^2}\right)H,\ R_{2424} = -\left(\frac{t_1^2+t_2^2}{z^4}\right)H,$$

$$R_{2434} = \left(\frac{2t_1}{z^3}\right)H,\ R_{3434} = \left(\frac{4t_1^2}{z^2(t_1^2+t_2^2)}\right)H,$$

where $G = \left[\frac{\bar{\bar{A}}}{2} - \frac{\bar{A}^2}{4A} - \frac{\bar{A}\bar{B}}{2B}\right]$, and $H = \left[\frac{\bar{\bar{B}}}{2} - \frac{3\bar{B}^2}{4B}\right]$,

These components are related as,

$$\left(\frac{z^4}{t_1^2+t_2^2}\right)R_{1212} = \left(\frac{-z^3}{t_1}\right)R_{1213} = \frac{z^2(t_1^2+t_2^2)}{t_1 t_2}\ R_{1314}$$

$$= \frac{z^2(t_1^2+t_2^2)}{{t_1}^2}\ R_{1313} = \left(\frac{-z^3}{t_2}\right)R_{1214} = \frac{z^2(t_1^2+t_2^2)}{t_2^2}\ R_{1414} = \mathrm{G}$$

And $\frac{-z^4 t_2^2}{{t_1}^2(t_1^2+t_2^2)}R_{2323} = \frac{z^4 t_2}{t_1\,(t_1^2+t_2^2)}R_{2324} = \left(\frac{-z^3 t_2}{2t_1^2}\right)\ R_{2334}$

$$= \left(\frac{-z^4}{t_1^2+t_2^2}\right)R_{2424} = \left(\frac{z^3}{2t_1}\right)R_{2434} = \frac{-z^2(t_1^2+t_2^2)}{4t_1^2}\ R_{3434} = \mathrm{H}$$

where $G = \left[\frac{\bar{\bar{A}}}{2} - \frac{\bar{A}^2}{4A} - \frac{\bar{A}\bar{B}}{2B}\right]$, and $H = \left[\frac{\bar{\bar{B}}}{2} - \frac{3\bar{B}^2}{4B}\right]$,

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3.5 The components of Ricci tensor are calculated as under,

$$R_{22} = \left(\frac{t_1^2+t_2^2}{z^4}\right)\left[\frac{G}{A}+\frac{H}{B}\right]$$

$$R_{23} = \left(\frac{-t_1}{z^3}\right)\left[\frac{G}{A}+\frac{H}{B}\right]$$

$$R_{24} = \left(\frac{-t_2}{z^3}\right)\left[\frac{G}{A}+\frac{H}{B}\right]$$

$$R_{33} = \left(\frac{t_1^2}{z^2(t_1^2+t_2^2)}\right)\left[\frac{G}{A}+\frac{H}{B}\right]$$

$$R_{34} = \left(\frac{t_1 t_2}{z^2(t_1^2+t_2^2)}\right)\left[\frac{G}{A}+\frac{H}{B}\right]$$

$$R_{44} = \left(\frac{t_2^2}{z^2(t_1^2+t_2^2)}\right)\left[\frac{G}{A}+\frac{H}{B}\right]$$

From these components we get the relation as;

$$\left(\frac{z^4}{t_1^2+t_2^2}\right)R_{22} = \left(\frac{-z^3}{t_1}\right)R_{23} = \left(\frac{-z^3}{t_2}\right)R_{24}$$

$$= \frac{z^2\,(t_1^2+t_2^2)}{t_1^2}R_{33} = \frac{z^2\,(t_1^2+t_2^2)}{t_1 t_2}R_{34} = \frac{z^2\,(t_1^2+t_2^2)}{t_2^2}R_{44} = P$$

where , $P = \left[\frac{G}{A}+\frac{H}{B}\right]$

Hence the field equations (1.2) reduces to the form, $\left[\frac{G}{A}+\frac{H}{B}\right] = 0$, which is equivalent to equation (1.7)

$$P = \frac{1}{2}\left(\frac{\bar{\bar{m}}}{m} - \frac{\overline{m}}{2m^2} - \frac{\bar{m}\bar{B}}{mB}\right) - \frac{1}{2}\left(\frac{\bar{\bar{B}}}{B} - \frac{3\bar{B}^2}{2B^2}\right)$$

IV. CONCLUSION

The plane wave solutions of generalities $R_{ij} = 0$ in four dimensional space-times V_4 for $[\sqrt{t_1^2+t_2^2/z}]$ – type plane waves can be obtained by using the concept of curvature tensor as well as without using the concept of curvature tensor and found that both the results are equivalent to each other.

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