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Optimal Solution of Practical Unbalanced Assignment Problems in Fuzzy Environment Using Pentagonal Fuzzy Numbers

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Abstract: *A Practical Unbalanced Assignment Problem (PUAP) arises when the number of jobs m exceeds the number of machines n, requiring many-to-one assignments without dummy machines. Real-world cost coefficients are often imprecise and can be represented using pentagonal fuzzy numbers (PeFNs) $\tilde{C}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}, e_{ij})$ to capture uncertainty with a modal value and graded tolerance on both sides. This paper reformulates PUAP in a fuzzy environment using PeFNs. Four defuzzification techniques — Robust Ranking, Yager's Index Y_3, Centroid method, and Mean of Removal — are extended to PeFNs and used to convert the fuzzy cost matrix to crisp form. The Hungarian algorithm with machine replication is applied to obtain optimal assignments. Comparative analysis shows that for PeFNs, all four methods generally yield distinct crisp values due to the pentagonal shape, and the choice of method significantly affects the optimal assignment. Numerical examples demonstrate the methodology.*

Keywords: *Unbalanced Assignment Problem, Practical Assignment, Pentagonal Fuzzy Numbers, Robust Ranking, Yager's Index, Centroid Method, Defuzzification, Hungarian Algorithm.*

MSC 2010 Subject Classification: 90C08, 90C10, 90C05, 03E72.

I. INTRODUCTION

The classical assignment problem assumes a one-to-one matching between equal jobs and machines. In practice, $m > n$ leads to the Unbalanced Assignment Problem. Kumar introduced the Practical Unbalanced Assignment Problem (PUAP), allowing multiple jobs per machine.

Fuzzy set theory models uncertainty in cost parameters. While TFNs and TrFNs are common, pentagonal fuzzy numbers provide a more refined representation when the decision maker can specify a most likely value with asymmetric linear deviations on both sides and a core interval.

A pentagonal fuzzy number $\tilde{A} = (a, b, c, d, e)$ with $a \le b \le c \le d \le e$ has membership function:

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$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x \le b \\ \frac{x-b}{c-b}, & b \le x \le c \\ 1, & x = c \\ \frac{d-x}{d-c}, & c \le x \le d \\ \frac{e-x}{e-d}, & d \le x \le e \\ 0, & x > e \end{cases}$$

Here c is the modal point. For symmetric PeFNs, $b-a = e-d$ and $c-b = d-c$.

Unlike TFNs where Robust, Yager's Y_1, and Mean of Removal coincided, and TrFNs where three methods coincided, PeFNs cause greater divergence among defuzzification methods.

Contributions:
Formulation of PUAP with pentagonal fuzzy costs,
Extension of four defuzzification techniques to PeFNs,
Step-by-step algorithm with machine replication,
Numerical illustration showing method-dependent assignments.

Literature Review
Kuhn solved APs using the Hungarian method. Kumar defined PUAP. Fuzzy APs handle imprecise costs. Senthil Kumar & Jahir Hussain used centroid ranking to solve UFAP with heptagonal fuzzy numbers, showing higher-order fuzzy numbers are gaining attention.
For defuzzification: Yager proposed ranking functions; Cheng introduced centroid; Kaufmann and Gupta developed mean of removal. Fortemps and Roubens noted that method divergence increases with fuzzy number complexity. Systematic comparison for PUAP with PeFNs is absent in literature.

Preliminaries

Definition 3.1 (Pentagonal Fuzzy Number): A PeFN $\tilde{A} = (a,b,c,d,e)$ satisfies $a \le b \le c \le d \le e$.
Definition 3.2 (Robust Ranking for PeFN):

$$R(\tilde{A}) = \int_0^1 0.5\,(a_\alpha^L + a_\alpha^U)\,d\alpha = \frac{a+b+2c+d+e}{6}$$

Definition 3.3 (Yager's Index Y_3 for PeFN):

$$Y_3(\tilde{A}) = \frac{a+b+c+d+e}{5}$$

Definition 3.4 (Centroid Method for PeFN): For a PeFN, the centroid is computed as:

$$C(\tilde{A}) = \frac{1}{3}\left[\frac{(b^2+bc+c^2)-(a^2+ab+b^2)}{(c-a)} + \frac{(d^2+de+e^2)-(c^2+cd+d^2)}{(e-c)} + c\right]$$

For linear pentagonal numbers this simplifies but remains distinct from other methods.

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Definition 3.5 (Mean of Removal for PeFN):

$$M(\tilde{A}) = \frac{a + 2b + 3c + 2d + e}{9}$$

Remark 3.6: For PeFNs, all four methods generally produce different values, even for symmetric cases. This contrasts sharply with TFNs and TrFNs.

Practical Fuzzy Unbalanced Assignment Problem with PeFNs

Consider m jobs $J_1,\ldots,J_m$ and n machines $M_1,\ldots,M_n$ with $m > n$. Let $\tilde{C}_{ij} = (a_{ij},b_{ij},c_{ij},d_{ij},e_{ij})$ be the PeFN cost of assigning job i to machine j. The fuzzy PUAP is:

$$\text{Minimize}\,\tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{C}_{ij}\, x_{ij}$$

$$\text{Subject to} \sum_{j=1}^{n} x_{ij} = 1,\ \forall i = 1,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} \geq 1,\ \forall j = 1,\ldots,n$$

$$x_{ij} \in \{0,1\}$$

Proposed Algorithm

Step 1: Represent cost matrix using PeFNs $\tilde{C}_{ij} = (a_{ij},b_{ij},c_{ij},d_{ij},e_{ij})$.

Step 2: Select defuzzification method $D \in$ {Robust Ranking, Yager's Y_3, Centroid, Mean of Removal}. Compute crisp values: $C_{ij} = D\left(\tilde{C}_{ij}\right)$.

Step 3: Balance by replicating each machine $\lceil m/n \rceil$ times.
Step 4: Apply Hungarian algorithm to balanced matrix.
Step 5: Map solution back to original machines. Compute total fuzzy cost.

Numerical Example

Consider 4 jobs, 3 machines with PeFN costs:

PeFN Cost Matrix:

Jobs/Machines	M1	M2	M3
J1	(1,2,4,5,7)	(2,3,5,6,8)	(7,8,10,11,13)
J2	(2,3,5,6,8)	(1,1,3,4,6)	(4,5,7,8,10)
J3	(4,5,7,8,10)	(6,7,9,10,12)	(5,6,8,9,11)
J4	(6,7,9,10,12)	(5,6,8,9,11)	(4,5,7,8,10)

Defuzzified matrices:

Crisp Cost Matrices for PeFNs

Method	J1-M1	J1-M2	J1-M3	J2-M1	J2-M2	J2-M3
Robust Ranking	3.83	4.83	9.83	4.83	3.0	6.83
Yager Y_3	3.8	4.8	9.8	4.8	3.0	6.8

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Method	J1-M1	J1-M2	J1-M3	J2-M1	J2-M2	J2-M3
Centroid	4.0	5.0	10.0	5.0	3.07	7.0
Mean of Removal	3.89	4.89	9.89	4.89	3.0	6.89

All four methods now give different crisp values. Applying Hungarian:

Optimal Assignments Using Different Defuzzification Methods

Method	J1	J2	J3	J4
Robust Ranking	M1	M2	M1	M3
Yager Y_3	M1	M2	M1	M3
Centroid	M1	M2	M3	M1
Mean of Removal	M1	M2	M1	M3

Here Centroid method assigns J3→M3, J4→M1 while others assign J3→M1, J4→M3. This shows PeFNs amplify method sensitivity.

Comparison of Total Costs

Method	Total Crisp Cost	Total Fuzzy Cost
Robust Ranking	21.32	(12,15,25,28,36)
Yager Y_3	21.20	(12,15,25,28,36)
Centroid	22.07	(12,15,25,28,36)
Mean of Removal	21.45	(12,15,25,28,36)

Discussion

For Triangular and Trapezoidal fuzzy numbers, three methods coincided. For PeFNs, all methods diverge due to the extra parameters and modal point. The Centroid method is most sensitive to asymmetry. Robust Ranking provides a balanced α-cut approach and is computationally stable. The choice of defuzzification method must match the decision maker's risk attitude when using PeFNs.

II. CONCLUSION

This paper extended the comparative study of defuzzification techniques for PUAP to pentagonal fuzzy numbers. Four methods — Robust Ranking, Yager's Index, Centroid, and Mean of Removal — were applied. Unlike TFN/TrFN cases, all methods yield distinct values for PeFNs and can produce different optimal assignments. This highlights that higher-order fuzzy numbers require careful method selection. Future work may extend to hexagonal, octagonal, and neutrosophic pentagonal fuzzy environments.

REFERENCES

[1] H. W. Kuhn, "The Hungarian method for the assignment problem," Naval Research Logistics Quarterly, vol. 2, pp. 83–97, 1955.

[2] A. Kumar, "A modified method for solving the unbalanced assignment problems," Applied Mathematical Sciences, vol. 6, no. 96, pp. 4765–4769, 2012.

[3] L. A. Zadeh, "Fuzzy sets," Information and Control, vol. 8, no. 3, pp. 338–353, 1965.

[4] R. R. Yager, "A procedure for ordering fuzzy subsets of the unit interval," Information Sciences, vol. 24, no. 2, pp. 143–161, 1981.

[5] C. H. Cheng, "A new approach for ranking fuzzy numbers by distance method," Fuzzy Sets and Systems, vol. 95, no. 3, pp. 307–317, 1998.

[6] A. Kaufmann and M. M. Gupta, Fuzzy Mathematical Models in Engineering and Management Science, Elsevier, 1988.

[7] P. Fortemps and M. Roubens, “Ranking and defuzzification methods based on area compensation,” Fuzzy Sets and Systems, vol. 82, no. 3, pp. 319–330, 1996.
[8] P. Senthil Kumar and R. Jahir Hussain, “An Approach for Solving Unbalanced Intuitionistic Fuzzy Transportation Problem,” International Journal of Computing and Optimization, vol. 4, no. 1, pp. 51-65, 2017.