



Principle of Vedic Mathematics Sutras and Tricks

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Abstract: *An Ancient system of Indian mathematics is known as Vedic mathematics. It is a gift given to this world by ancient stages of India. The Vedic mathematics was constructed using 16 formulas called sutras and 13 upa-sutras. The Vedic mathematics is designed in a way that calculations are carried out mentally. The main fact about Vedic mathematics was, for any difficult problem the answer should be calculated in one line. Vedic sutras are applicable in all fields and this can be applied directly to plain, spherical geometry, trigonometry, conics, calculus and applied mathematics of various kind. Vedic mathematics is a part of four Vedas. Sthapathya-veda, which covers the concepts of civil engineering and architectures this is an upa- Veda of atharva Veda. Sutra is given a very short formula to carry out difficult mathematical calculations in a very easy and simple manner and to executing them mentally. These sutras are used for multiplication, division, factorization, recurring decimals and solutions of simple equations. Quadratic equation, system of equations, integration by partial fraction and differential calculus are used. Some topics of geometry such as Pythagorean Theorem, and some theorems of Apollonius and it also covers more advanced mathematics such as analytical expressions of straight line and analytical conics.*

Keywords: -Vedic Mathematics, Sutras, Multiplication, Division and Equations.

I. INTRODUCTION

Vedic mathematics was rediscovered from Vedas in between 1911 and 1918 by Sri Bharathi Krishna tirthagi. He is better known by gurudeva or jagadguruji. The reconstruction of 16 sutras mainly gathered from the AtharvaVeda. Vedic mathematics sutras are also applicable even in astrology. The mathematics can also be used to check out the answer whether it is correct or not. The term "Vedas" is a Sanskrit word means divinely revealed and it is the store house of all knowledge. It is not only related to the spiritual matters but also to the humanity. The sutras in the Vedic mathematics are designed in a way that naturally how human mind will work. Vedic mathematics is a kind of magic. Vedic mathematical technique will reduce the time, area and power consumption. This mathematics is unique and makes it easy and enjoyable for learning. The nine important features of Vedic mathematics are coherence, flexible and it improves mental power creatively. It is applicable for everyone and it increases the mental eligibility, efficient and fast. Vedic are the ancient record of human experience and knowledge and it was written about 5000 years ago. By using this technique it will increase the step up to 15 times faster than actual technique. The main beauty of Vedic mathematics is to reduce complex calculations in to simple one. MAC, ALU are some of the basic application of the Vedic mathematics sutras. In these sutras urdhva-triyagbhyam is best for the multiplication because it is applicable to all cases and it consumes less power and works at high speed. Generally Vedic mathematics sutras consume less power, silicon area and it is high speed when comparing to various other multipliers.

The Sixteen sutras and up-sutras "13" are as follows:

1. EKADHIKENA PURVENA
2. EKANYUNENA PURVENA
3. ANURUPYE SUNAYAMANYAT
4. CHALANA KALANABHYAM
5. SANKALANA VYAVAKALANABHYAM
6. GUNAK SAMUCCAYAH

7. SESANYANKENA CARAMENA
8. PURANAPURANABHYAM
9. GUNITASAMUCCAYAH
10. URDHVA TIRYAKBHYAM
11. YAVADUNAM
12. VYASTISAMASTIH
13. SOPANTYADVAYAMANTYAM
14. SUNYAM SAMYASAMUCCAYE
15. NIKHILAM NAVATASCARAMAM DASATAH
16. PARAVARTYA YOJAYET

The **fourteen** sub –sutras are:

1. ANURUPYENA
2. SISYATE SESASMJNAH
3. ADYAMADYENANTYAMANTYENA
4. KEVALAIH SAPTAKAM GUNYAT
5. VESTANAM
6. YAVADUNAM TAVADUNAM
7. YAVADUNAM TAVADUNIKRTYA VARGANCA YOJAYET
8. ANTYAYORDASAKE PI
9. ANTYAYOREVA
10. SAMUCCAYAGUNITAH
11. LOPANASTHAPANABHYAM
12. VILOKANAM
13. GUNITASMUCCAYAH SAMUCCAYAGUNITAH

II. SUTRAS

2.1 EKADHIKENA PURVENA

It means “by one more than one before”. This sutra is mainly used to square the numbers and also used for the multiplication of two numbers under certain condition. The condition is

1. The sum of the last digits should be equal to 10.
2. The previous digit should be same for the two numbers.

The steps involved in the multiplication of two numbers are...

$1. \quad 35 \times 35 = 3 \times 4 = 12$ $5 \times 5 = 25$ $35 \times 35 = 1225$
$2. \quad 46 \times 44 = 2024$

2.2 EKANYUNENAP URVENA

It means “by one less than the previous one”. This sutra is applicable only when the multiplier is 9 or array of 9. The steps involved in the multiplication of two numbers are..

- **Step 1:** The multiplier should be 9 or array of 9, and then decrease the multiplicand value by one
- **Step 2:** The result from the previous step should be subtracted from the multipliers.

EXAMPLE 1: Multiply 3 and 9

$$3 \times 9 =$$

$$-1$$

$$27$$

Subtract one from 3 and remaining number need to make nine (i.e. $3-1=2$, 2 need to make 9, 7. then write answer 27, this criteria same for any number multiplied by 9,99,999,9999 etc .

Step 1: The multiplier should be 9 or array of 9 then, $3-1=2$.

Step 2: Subtract the multiplier from 2 i.e. $9-2=7$.The result is 27.

1.

$$45 \times 99 = 4455$$

$$45 - 1 = 44, \text{ we need to complete } 99, 55.$$

Subtract one from 5 and make the number 9. $5-1=4$. How many no needs to make 4 nine that is 5

EXAMPLE 2: Multiply 523 and 99

Step 1: Check for the number of digits in multiplier in the example is 2 so split multiplicand by last two digit and first digit i.e. split as 5 and 23. Then increment 4 by 1 i.e. $5+1=6$, and subtract multiplicand by 6, $523-6=517$.

Step 2: Subtract 99 by 23 and the result is increment by 1, i.e. $99-23=76$ then $76+1=77$.The result is 51777.

2.3 ANURUPYESUN YAMANYA

It means “if one is the ratio other is zero”, this sutra is mainly used for the values of unknowns in the two linear algebraic equations. According to this sutra the steps involved in calculating values of unknown variable are

Step 1: Taking the co-efficient of the variables as well as the constant.

Step 2: Taking y-coefficient ratio i.e. in $A_0:A_1$ and constant values ratio as $A_0:A_1$, if the ratio of coefficient of y and constant are equal then take x value as zero by using the value of x find y.

EXAMPLE: Solve $5x + 9y = 4$ and $3x + 36y = 16$.

Step 1: The co-efficient of y are in the ratio of $9:36 = 1:4$, the ratio of constant as $4:16 = 1:4$.

Step 2: The ratio of y co-efficient and constant is same then let's take $x=0$ and substitute x in any one of the equations and solve it for y. i.e. taking equation $1, 9y=4$ then $y=4/9$ (Or) equation $2, 36y=16$ then $y=16/36$ i.e. $y=4/9$. The values of variable x and y are 0 and $4/9$ respectively.

2.4 CHALANA-KALANBHYAM

It means “differential calculus”. Every quadratic equation can be broken down in to two binomial factors; this sutra is mainly used to find the roots of the quadratic equation by using the formula,

EXAMPLE: Solve $X^2 + 6X + 2 = 0$ by using the formula, here $a=1$, $b=6$, $c=2$.

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= -6 \pm \sqrt{36-8}$$

$$2X = -6 \pm (5.29)$$

SOLUTION: $X = -3 \pm (2.645)$.

The roots of quadratic equation are $X = -0.355$, $X = -5.645$.

2.5. SANKALANA VYA KALANABHYAM

It means “by addition and by subtraction”, this sutra is same as like ANURUPYE SUNYAMANYA sutra. This sutra is used to solve the linear equations but it should have the co-efficient of X in equation 1 and Y in equation 2 respectively and vice versa for balance X and Y term in equation 1 and 2.

EXAMPLE: $aX + bY = C_1$.

$bX + aY = C_2$.

The steps involved in solving equations are...

Step 1: If the above condition is satisfied then add equation 1 and equation 2, the addition of the two equations will give another equation named as equation 3.

Step 2: Subtract the equation 1 and 2; the resultant equation in step 2 is named as equation 4.

Step 3: Solve the equation 3 and 4, it result either X or Y value, substitute that X and Y value in either equation 3 or 4 we get Y or X respectively.

EXAMPLE:

$$20X + 40Y = 60 \quad (1)$$

$$40X - 20Y = 80 \quad (2)$$

Step 1: Addition of equation (1) and (2),

$$(20X + 40Y) + (40X - 20Y) = 60 + 80.$$

$$60X + 20Y = 140.$$

$$3X + Y = 7 \quad (3)$$

Step 2: Subtraction of equation 1 and 2,

$$(20X + 40Y) - (40X - 20Y) = 60 - 80, -20X + 60Y = -20$$

$$X - 3Y = 1 \quad (4)$$

Step 3: Solve equation (3) and (4).

$$3X + Y = 7$$

$$X - 3Y = 1$$

$$9X = 22$$

$X = 22/9$. Then substitute x value in equation (4), so $22/9 - 3Y = 1$

$Y = 13/27$. The result is $X = 22/9$ and $Y = 13/27$.

2.6 GUNAK SAMUCCAYAH

It means “the factors of the sum is equal to the sum of the factors”, this sutra is same as CHALANA-KALANBHYAM. It is used to find the roots of quadratic equation using the formula .

$$X = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}, \quad 2ax + b = \pm \sqrt{(b)^2 - 4ac}$$

Step 1: Find the factors of the given quadratic equation.

Step 2: Add the factors of the quadratic equation and that must be equal to the $2aX + b$, is this condition satisfies then the founded roots are the roots of quadratic equation.

EXAMPLE:

Solve: $X^2 + 5X + 2 = 0$.

From the given equation $a=1, b=5, c=2$.

Step 1: The roots of quadratic equation are $(X+2)(X+3)$.

Step 2: Check if $2aX + b = (X+d)(X+e)$

This sutra mainly used for the verification of factors with the given quadratic equation.

Steps involved in this sutra are...

Step 1: Find the factors of the given quadratic equation.

Step 2: Add the factors of the quadratic equation and that must be equal to the $2aX + b$, is this condition satisfies then the founded roots are the roots of quadratic equation.

2.7. SESANYANKENA CARMENA

It means “the remainders by the last digit”, this sutra is used to express fraction numbers in to decimal numbers up to required decimal places under the condition that numerator should be less than the denominator . The steps involved in calculation are...



Step 1: Check that the numerator should be less than the denominator if so the multiply numerator by 10 and then divide by denominator, if remainder value is not equal 1, then again multiply remainder by 10 and divide that value by denominator and the process should repeat until remainder is equal to 1 .

Step 2: Multiply the entire remainder with denominator respectively from step 1.

Step 3: Taking the last digits of the output from step 2 and written the output from top to bottom i.e. 0.X₁ X₂X₃.....etc up to required decimal places...

EXAMPLE: Solve: 2/7

Step 1: Numerator is less than denominator,(2 x 10)/7 =6.

$$(6 \times 10)/7=4.$$

$$(4 \times 10)/7 =5.$$

$$(5 \times 10)/7 =1.$$

Step 2: Multiply all the remainder with denominator, (6 x 7) =42.

$$(4 \times 7)=28.$$

$$(5 \times 7)=35.$$

$$(1 \times 7)=7.$$

Step 3: the output is 0.2857....

2.8. PURANAPURANABHYAM

It means “the by completion or non-completion”, this sutra is used to find the roots of the cubic equation. The steps involved in the calculation of roots are...

Step 1: Check for the nearest cubic formula i.e. $(X + A)^3$, for the given cubic equation.

Step 2: Subtract the nearest cubic equation and the given cubic equation and then add the result from the subtraction on both side of the given cubic equation.

Step 3: The resultant equation contain common terms on both side of the equation , so replace that common term with another variable, and check for the values which satisfies the condition .

Step 4: The values which satisfies the condition are taken as roots of the equation.

EXAMPLE:

$$\text{Solve: } X^3 + 6X^2 + 11X + 6 = 0 \rightarrow 1$$

Step 1: The given cubic equation is nearer to $(X+2)^3$.

$$\text{Step 2: } [(X^3 + 6X^2 + 11X + 6) - (X^3 + 6X^2 + 312X + 8)] = X + 2.$$

Step 3: Add $(X+2)$ on both sides of equation 1, then the equation becomes, $[X^3 + 6X^2 + 11X + 6] + (X+2) = (X+2).X^3 + 6X^2 + 12X + 8 = X + 2$

$$(X+2)^3 = (X+2).$$

Let assume $Y=(X+2)$, now the equation becomes $Y^3 = Y$.

The value that satisfies the condition is 0,1 and -1.

Substitute the values in $Y=X+2$, if $Y=0$, then $X=-2$.

If $Y=1$, then $X=-1$.

If $Y=-1$, then $X= -3$.The roots of the given cubic equation are $(X+2)$, $(X+1)$ and $(X+3)$.

2.9. GUNITASAMUCCAYAH

It means “the product of sum is equal to the sum of the product”, this sutra is used to check whether the given equation is correct or not using the roots of the equation.

The steps involved in this sutra are...

Step 1: Equating the roots of the equation and the quadratic equation, then equate the roots and the quadratic equation with their co-efficient.

Step 2: If both the values are same then the quadratic equation and their roots are correct.



EXAMPLE: Solve: $X^2 + 5X + 6 = 0$. The roots of the quadratic equation are $(X+2)$ and $(X+3)$

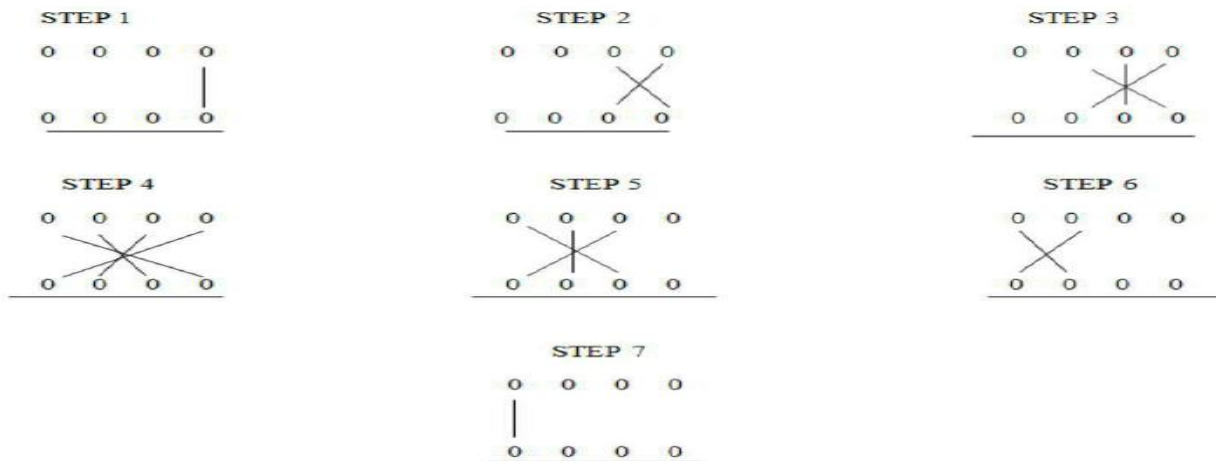
Step 1: $(X+2)(X+3) = X^2 + 5X + 6$

Step 2: $(1+2)(1+3) = (1+5+6)$

$12=12$. It satisfies the condition so the given equation is correct.

2.10. URDHVA TIRYAKBHYAM (Cros- Criss)

It means “vertically cross wise”, this sutra is used for the multiplication of two numbers up to N number of digits, and the procedure for the multiplication of numbers is same for decimal number system as well as in binary number system. The steps involved in the calculation for two digit decimal number are....



2.11. YAAVADUNAM

It means “by the deficiency”, this sutra is used to calculate the square of the numbers using the deficiency of the number from the nearest numbers in powers of 10 .steps involved in the calculation of the square of numbers are..

Step 1: Find the nearest number in powers of 10, and subtract this number from the given number and find the deficiency and represent as A_0 .

Step 2: Subtract the deficiency i.e. A_0 from the given number and keep it as left side part of the final answer and represent as B_0 .

Step 3: The squared value of the deficiency will give the right side part of the final answer and represented as C_0 . The final answer of square of the given number is represented as $B_0 C_0$.

EXAMPLE:

Find the square of 998.

Step 1: The given number is nearest to 1000, the deficiency = $(1000-998)$, here the deficiency is 2.

Step 2: $998-2=996$, this is the left side part of final answer.

Step 3: Square of deficiency = $(2)^2 = 004$, this is the right side part of the final answer. The square of 998 is **996004**.

2.12.VYASTISAMANSTIH

It means “part and whole”, i.e. this sutra is used to find the part of any particles from the mixture. This sutra is like calculating the probability of getting particles from the mixture. The steps in calculating the part whole ratio of the contents from the mixture are...

Step 1: Find the total count of particles present in the mixture and the components and the individual count of the particles.

Step 2: Divide in the count of particle from the total count in mixture and calculate the same for all the partials in the mixture.



EXAMPLE:

A box contains 4 white balls, 3 red balls, 6 blue balls and 5 yellow balls.

Step 1: Total count of balls in the box is i.e. $(4+3+6+5) = 18$.

Step 2: The part and whole ratio of white balls $= 4/18$.

The part and whole ratio of red balls $= 3/18$.

The part and whole ratio of blue balls $= 6/18$.

The part and whole ratio of yellow balls $= 5/18$.

2.13 SOPAANTY ADVAYAMANTYAM

It means “ultimate and pen ultimate”, this another method used for the multiplication of two numbers ,where the multiplicand can be any number but the multiplier should be 12, then only this sutra is applicable. The steps involved in multiplication of two numbers are.

Step 1: Insert zero at first and last place of the multiplicand.

Step 2: Add the last digit with the second last digit multiplied by 2, and the same process is carried for all the digits in the multiplicand respectively.

Step 3: The resultant value are written from bottom to top.

EXAMPLE 1: Multiply 1321×12 .

Step 1: The new multiplicand is 013210.

Step 2: $0 + (2 \times 1) = 2$

$1 + (2 \times 2) = 5$

$2 + (2 \times 3) = 8$

$3 + (2 \times 1) = 5$.

$1 + (2 \times 0) = 1$. Start from Here to up

Step 3: The answer is 15852.

EXAMPLE 2: Multiply 196×12 .

Step 1: The new multiplicand is 01960.

Step 2: $0 + (2 \times 1) = 2$

$6 + (9 \times 2) = 24$

$9 + (2 \times 1) = 11$

$1 + (2 \times 0) = 1$.

Step 3: The above results are written from bottom to top respectively 1 11 24 12, the result is calculated as $(1+1) (1+2) (4+1) (2)$, multiplication of 196 and 12 is 2352.

2.14. SUNYAM SAMYASAMUCCAYE

It means “when sum is same that sum is zero”, this sutra is used when same sum occurs equate that sum to zero. This sutra is applicable under six cases .the six cases are...

Step 1: When the common factor occurs, and then equates that common to zero.

Step 2: If the numerator are same then sum of the denominator is equate to zero.

Step 3: Sum of denominator on left side equal to the right side, then equate that sum to zero.

Step 4: When sum of the numerator and denominator is equal on both sides then equate that sum to zero.

Step 5: If it having any common term on either the numerator or denominator on both sides of the equation, then equate the sum of numerator and denominator on either side equal to zero.

Step 6: (I) If sum of the numerator and sum of the denominator are equal then equate either the sum of numerator or denominator to zero.

(II) If the difference of the numerator or denominator is same then equate the difference of either the numerator or denominator to zero.



EXAMPLE:

Step 1: $2(X+2) = 5(X+5)$, then equate $(X+5)=0$.

Step 2: $5/(X+2) = 5/(X+5)$ numerators are same so sum of denominator $(X+7)=0$.

Step 3: $1/(X+2) + 1/(X-7) = 1/(X+9) + 1/(X-4)$, sum of the denominator are same so ,sum of denominator $(X+5)=0$.

Step 4: $(X+2)/(X+5) = (X+5)/(X+2)$ sum of numerator and denominator are same so, sum $(2X+7)=0$.

Step 5: $(2X+3)/(4X+5) = (X+1)/(2X+3)$ common factor on either numerator or denominator of either side so sum, $(3X+4)=0$.

Step 6: $(2X+5)/(X+2) = (X+2) / (2X+5)$

1. $(2X+5) + (X+2) = (X+2) + (2X+5)$ are same so, $(3X+7)=0$.

2. $(2X+5) - (X+2) = (X+2) - (2X+5)$ are same so, $(X+3)=0$.

2.15. NIKHILAM NAVATASCARAMAM DASATAH

It means “All from 9 and the last from 10”[17]. It subtracts from nearest power of 10 or 10n.

This type of sutra can be applied:

When number slightly less than power of 10(10,100,1000...)

Formulas used $(x-a)(x-b) = x(x-a-b) + ab$

When number slightly greater than power of 10(10,100,1000...)

Formulas used $(x+a)(x+b) = x(x+a+b) + ab$

EXAMPLE: 6 X 8:

6 -4(subtract the base 10 from 6)

8 -2(subtract the base 10 from 8)

48

Step 1: Left Hand Side: $6 + (-2) = 4$ (add top to bottom right)

Step 1: Left Hand Side: $6 + (-2) = 4$ (add top to bottom right)

$8 + (-4) = 4$ (add bottom to top right)

$6 + 8 - 10 = 4$ (add number in left column and sub from base)

$10 + (-4) + (-2) = 4$ (add number in right column and the base)

Step 2: Right Hand Side:

$(-4) \times (-2) = 4$ (right column multiply)

Step 3: Answer: $6 \times 8 = 48$.

2.16. PARAVARTYA YOJAYET

It means “transpose and apply”, this sutra is applicable when divisor should be greater than power of 10.the steps involved in the calculation are ...

Step 1: Divisor should be greater than power of 10.

Step 2: Write the divisor leaving the second digit, and write below the divisor in negative sign i.e. (x) from left to right.

Step 3: Write the dividend to the right leaving the last digit and mention it as remainder, write the first digit as it and multiply that with $-x$ and add with the second digit, and continue the process until last digit.

Step 4: Finally the last digit is remainder and the digits before last digits are mentioned as quotient.

EXAMPLE: 1220/12

Step 1: Divisor is greater than power of 10 so proceed with the next steps.

Step 2: 12

-2

Step 3: 12 122 0

-2

10

Step 4: 12 122 0

$$\begin{array}{r} 12\ 122\ 0 \\ -2\ -20\ -2\ -20\ 0 \\ \hline 10\ 102\ 10\ 102\ 0 \end{array}$$

Step 5: quotient =102 remainder =0.

III. CONCLUSION

From the above discussion of Vedic mathematics sutras, for multiplication urdhva-triyagbhyam sutra is best in case of speed, area and power consumption when compared to the nihilism navatasaraman sutra and other sutras which are applicable for multiplication. Nikhil am navatasaraman sutra is also best but it is having certain conditions for multiplication. It is the biggest drawback in this sutra. Gunaksamuccayah and puranapurabhyam are used to find the roots of quadratic and cubic equation respectively. Sesankenacarmena is used for recurring decimal places. Anurupyesunyamanya and sankalanavyakalanabhyam is used to find the solution of the equation but in which anurupyesunyamanya is best in this to find the solution of the equation. Paravartya yojayet and Nikhil am navatasaraman sutras are used for division and they are applicable when the divisor is less than power of 10 and the divisor is greater than power of ten respectively. Gunitasamuccayah sutra is used to find the roots of quadratic equation using calculus formula. Really if it come back in syllabus of schooling. Students enjoy and learn mathematics.

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