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The Role of Persistent Homology in Big Data Analytics

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Abstract*:* *In some areas of computer science, computational topology analyzes data and resolves issues by fusing effective algorithms with theoretical topological techniques. With reference to the domains of artificial intelligence, robotics, machine learning, and computer graphics or image processing, we examine the many applications of computational or applied topology in computer science in this article. We discovered that the use of topological data analysis has rapidly increased in the aforementioned fields. The goal of this article is to compile and synthesize the most current research relating to the usage of topological data analysis in computer science as well as the many approaches used to integrate topological data analysis tools into different computer science applications..*

Keywords: Persistent homology, robotics, artificial intelligence, machine learning, and topological data analysis

I. INTRODUCTION

It is well known that topology is an area of mathematics that has its roots in geometry. The study of both local and global characteristics of forms or objects under constant deformation is known as topology. When topology first emerged, it was thought of as rubber sheet geometry because forms and objects could be bent without losing their fundamental characteristics. The idea of topology provides a clear illustration of the in-depth comprehension of surface geometry. A topologist claims that a donut and a torus are identical due to some invariant characteristics they have in common. Figure 1 shows identical object instances that vary geometrically in length, angle of measure, and curvature.

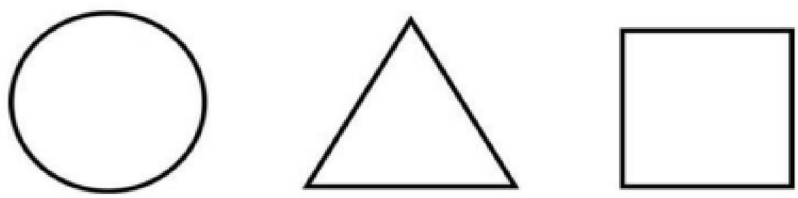

Fig. 1 These shapes are topologically equivalent but geometrically non-identical

On the other hand, geometric objects are identical via congruence, while things are identical through continuous deformation. Congruent objects having the same volume, perimeter, curvature, angles, and equivalent side lengths.

Definition 1 Zomorodian (2005) A subset τ of 2 X that satisfies the following axioms is said to have a topology in a set X.

I. X and the empty set are elements of τ.

II. Finite intersections of τ elements are elements of τ.

III. Any element of τ may be arbitrarily united to form an element of τ. Note: A topological space is the pair (X,τ) comprising the set X and its topology τ.

Definition 2 Kinsey (1997) On a set of objects, an equivalence relation ≃ is a relation such that

I. For every an in the set, a ≃ a (reflexivity property)

II. The symmetric condition states that if a ≃ b, then b ≃ a.

III. The transitivity property states that if a ≃ b and b ≃ c, then a ≃ c.

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However, in recent years, topological concepts have been used in many scientific domains, particularly computer science. Despite being distinct disciplines, topology and computer science are intimately related. Mathematical models for computer-related things are often created in theoretical computer science; these models typically represent the semantic meanings of programs or data stored on a computer (Reed et al., 1991). Since it was first developed, the measure of data form has undergone significant advancements in the rapidly expanding area of computational topology. Data shape is measured using computational topology, and the concept of homology is used to calculate the number of n-dimensional holes (Obeng-Denteh and Adjei, 2022). We start this work by describing some of the computer science applications of computational topology. Therefore, we examine the idea of persistent homology to artificial intelligence, robotics, and machine learning. Additionally, we provide an overview of the use of TDA techniques in text recognition, clustering, classification, and many other fields.

II. COMPUTATIONAL TOPOLOGY APPLICATION TO MACHINE LEARNING

More recently, the use of Topological Data Analysis (TDA) to assess the general form of data has increased (Adams and Moy, 2021). Persistent homology, a tool in topological data analysis, has been integrated into machine learning through measuring shape, such as clustering (Xu and Wunsch, 2005), nonlinear dimensionality reduction (Roweis and Saul, 2000), (Tenenbaum et al., 2000), Kohonen (2012), (McInnes et al., 2018), transformation of time series (Chung et al., 2020), analysis of physical signals (Chung et al., 2021), (Wang et al., 2018), coverage in sensor networks (De Silva and Ghrist, 2007), and tumor diagnosis (Dunaeva et al., 2016). Persistent homology is a complex TDA technique that was initially presented in (Edelsbrunner et al., 2000). It uses persistent diagrams and barcodes to show these characteristics and records changes in homology over filtering (Zomorodian and Carlsson, 2005). An expanding series of spaces is a filtering of space Y. $\cdots \subset Y_n = Y$ $\emptyset = Y_0 \subset Y_1$ To fully exploit persistent homology in machine learning, a system developed by Tauzin et al. (2021) allows a variety of data points to be transformed into forms suitable for calculating persistent homology.

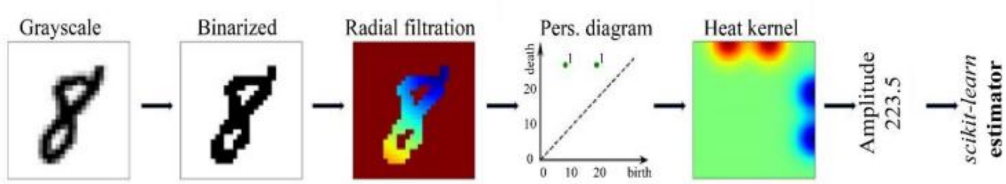

Fig. 2 Example of giotta-tda. Figure is from (Tauzin et al., 2021)

Over the last two decades, persistent homology has been used to study machine learning. After examining the data, new algorithms are created and persistent homology is used to regularization, a machine learning approach that penalizes complicated models to avoid overfitting (Chen et al., 2019). Recent machine learning research includes time series data analysis (Umeda et al., 2019), RNA hairpin folding modeling (Singh et al., 2007), repeated measurement (Riihim¨aki et al., 2020), chatter detection (Khasawneh et al., 2018), and morphism (Cawi et al. Some of these domains use TDA-derived approaches such persistent homology, which suits a dataset's machine learning goal. Others allow direct TDA tool installation for weather forecasting (Muszynski et al., 2019). TDA solved multi-class classification problems and improved results without machine learning (Kindelan et al., 2021). This method computes persistent homology and directs complicated structures by clustering massive data sets. After moving away from categorization, topological data analysis has been used to evaluate generative adversarial networks (Goodfellow et al., 2014), visual cortex population activity (Singh et al., 1991), and power system contingency (Bush et al., 2021). Machine learning improves performance by learning from the past (Das and Behera, 2017). This offers data-driven modeling and categorization (Carleo et al., 2019) and topological materials (Yun et al., 2022; Rodriguez-Nieva and Scheurer, 2019; Che et al., 2020; Scheurer and Slager, 2020). Deep learning is effective for pattern solving with complicated data sets (Goodfellow et al., 2016). Mapper diagrams are TDA tools for convolutional neural networks (Carlsson and Gabrielsson, 2020). The concept that neural networks are similar to mammalian brain networks led to the creation of neural networks in (Carlsson and Gabrielsson, 2020). The datasets were analyzed and grouped using a mapper technique and variance-normalized Euclidean metric. It supported connectedness and loops by replicating the data set's structure. Neural

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networks performed well in several studies. Optimization makes its convolutional neural network computing techniques difficult to optimize (Carlsson and Gabrielsson, 2020).

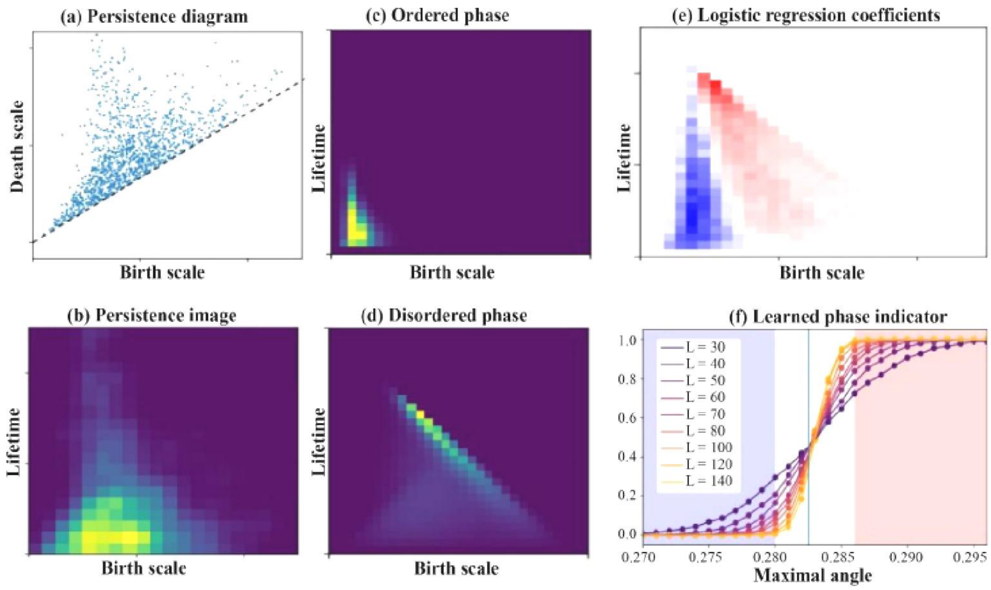

Fig. 3 Topological Data Analysis machine learning of phase transition in XY dimensional model (a)Persistence diagram computed using filtration (b)persistence diagram vectorised into landscape. (c)Persistence images obtained from spin configurations. (e) Persistent image of a coefficients of a trained logistic model. (f)Finite size scaling of the logistic regression prediction used to estimate the phase transition point. Figure is from (Sale et al., 2022)

III. COMPUTATIONAL TOPOLOGY APPLICATION TO ARTIFICIAL INTELLIGENCE

Topological data analysis has advanced faster since its late 20th century debut due to applications in several scientific domains. Despite (Edelsbrunner et al., 2000), (Zomorodian and Carlsson, 2005) introduced persistence. TDA has used Topological Data Analysis to AI. This variable resolution topological framework has huge promise for creative data analysis (Robins et al., 2004), (Robins, 1999), (Robins, 1998), (Robins, 2000). Researchers have also used persistent homology for aviation data (Li et al., 2019), neural network analysis (Liu et al., 2016), text recognition (Makarenko et al., 2016), sports (Goldfarb, 2014), and persistence diagrams for neural networks (Ballester et al., 2022). Topological data analysis in AI is the most relevant study. These tools provide a generic description of data shape, but AI algorithms must still use it to analyze data. Research in precision medicine, imagery, music, fraud detection, anomaly detection, denoising, discourse structure, and motion recognition (Iniesta et al., 2022; Al-Jaberi et al., 2020; Hu et al., 2019; Asaad and Jassim, 2017; Klaren, 2018; Tymochko et al., 2021; Davies, 2022; Al-Jaberi and Hameed, 2021; Savle et al., 2019; Zelawski and Hachaj˙, 2021) Big Data Analytics is a TDA study topic that includes AI. Big Data analytics removes particular data properties. Russom et al. (2011) found 38% of organizations employed sophisticated analytics. Big Data is the latest IT that simplifies large dataset analysis. Over the last decade, this Technology has garnered attention (Bi and Cochran, 2014). Big data analytics extracts vast volumes of data for business insights (Cárdenas et al., 2013). The following technologies were developed: Apache Flume, Sqoop, Piq, Zookeeper, and others analyze algorithms. Large companies like Amazon utilize big data analytics to manage billions of items (Zakir et al., 2015). Computational algorithms are used for clustering, sequential patterns, classification, social media data mining, and healthcare intelligence (Zhang et al., 1996; ESTER et al., 1996; Ester and Wittmann, 1998; Chiu et al., 2004; Han et al., 2001; Yan et al., 2003; Mic´o et al., 1996; Djouadi and Bouktache, 1997; Mehta et al., 1996; Almgren et al., 2017; Joshi and Joshi, 2019).

IV. COMPUTATIONAL TOPOLOGY APPLICATION TO ROBOTICS

Since Asimov's books laid the groundwork for robotics, humans have been fascinated by them. Solutions are found by robots without human interaction. Robotics has affected several scientific fields (Garcia et al., 2007). However, industrial robots didn't appear until the early 20th century, and robotics research dominated the tech sector in the late 20th century. In the early 20th century, the automobile industry defined and constructed robots, but now implicit and explicit algorithmic techniques dominate robotic research (Khatib, 1986). Implicit algorithms use future time, whereas explicit methods use earliest time. Industrial, mobile, medical, submersible, humanoid, and other robots exist. Topology has become a new topic of study as robotics has advanced. Topology and robotics grow along many interrelated routes that challenge and inform each other (Farber et al., 2007a). Topological configuration spaces, where complex systems may be represented by topological objects, include robotics. Many papers, including (Hausmann et al., 2007) and (Fadell and Neuwirth, 1962), cover configuration space basics. The topological difficulty of motion planning has been the focus of recent topological robot research. Topological complexity research includes hyperplane, formal spaces, and collisionfree motion (Yuzvinsky, 2007; Lechuga and Murillo, 2007; Farber et al., 2007b). Configuration space homology and cohomology prepare for persistent homology and robotics. Topology of configuration spaces at n-dimensional points is defined in (Arnold, 1969) and (Cohen et al., 2007), whereas persistent homology and its exploration are addressed in (Alpert and Manin, 2021). Robotic persistent homology applications have improved rapidly in the recent decade. Persistent homology is utilized in robotics applications including trajectory, Internet of Things, mapping, manipulation, and motion planning. However, Vasudevan et al. (2013) demonstrated persistent homology's value in robotic bipedal walking. A simplicial complex is created by thresholding an image across its intensity and adding each pixel as a node to compute persistent homology. This strategy made it easy to monitor betti numbers as a function of the threshold value and fill in data gaps to deliver a consistent result across time.

V. COMPUTATIONAL TOPOLOGY APPLICATION TO COMPUTER GRAPHICS OR IMAGE PROCESSING

Computer graphics is used to identify objects and determine their relationships (Bernstein et al., 2020), (Kovalevsky, 1989), but most modern research uses high-resolution digital images. This section evaluates Topological Data Analysis methods for image processing and computer graphics (Mumford and Desolneux, 2010; Chan and Shen, 2005; Kimmel et al., 2005; Weickert and Hagen, 2005). Topological data analysis for computer graphics and image processing was created from algebraic and computational topology (Edelsbrunner and Harer, 2022; Kaczynski et al., 2004). Various studies have explored various methods for solving object data, including shape analysis, space planning, scientific visualization, medical imaging, and image classification (Patrangenaru et al., 2018; Carriere et al., 2015; Feng et al., 2018; Medjdoub and Yannou, 2000; Tierny, 2016; Koseki et al., 2020; Vandaele et al., 2020; Shen, 2021; Saggar et al., 2018; Jazayeri et al., 2022; Dey et al., 2017; Yang Steganography research has strengthened Topological Data Analysis methods for picture processing. Rashid et al. (2018) created biometric stego pictures from 1000 randomly selected natural photographs. 3D printing is another prominent image processing technique. Topological Data Analysis techniques discover 3D printing problems using persistent homology. Rosen et al. (2018) evaluate 3D-printed things using persistent homology. One recent advancement in image processing employing topological data analysis is applying it to black hole photos. Black holes are inaccessible from infinity (Bieri, 2018). The Event Horizon Telescope has taken horizon-scale pictures of an astrophysical black hole in the interior of the M87 galaxy (Collaboration et al., 2019; Akiyama et al., 2019). A topological signature of the black hole picture is seen in the EHT observation (Collaboration et al., 2019; Akiyama et al., 2019): a brilliant ring around a dull area. Christian et al. (2022) deconstructed synthetic black hole pictures using motorization and persistent homology.

VI. CONCLUSION

This paper reviews some of the contributions on the use of topological data analysis in computer science. We started by going over the fundamentals of topology, including persistent homology, topological spaces, and topology. We examined the approach that applies topological data analysis methods to real-world scenarios in the application section. We spoke about machine learning applications, computer graphics or image processing, robotics, and artificial intelligence. Given the great quality of TDA in all of these domains and the advancement of technology in these domains, it is plausible that TDA will gain popularity as a study topic at the intersection of topological data analysis and computer science.

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