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Stability Analysis of Hermite Collocation Method for Pulp Washing Models

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Abstract: *Pulp washing is concerned with detaching cellulose fibres from black liquor with the use of a minimal amount of wash liquor. An efficient numerical technique of hermite collocation method is used for the solution of mathematical models related to pulp washing. The linear and non-linear models are solved using Quintic Hermite collocation method with Dirichlet's and mixed Robin's boundary conditions. Numerical solution of the models are derived using MATLAB ode15s. This study deals with the justification of accuracy of the method with stability analysis. The present method is more convenient, simple and elegant for solving the two point boundary value problems and the results found are very much stable from numerical point of view.*

Keywords: Pulp washing, model, eigen values, stability.

I. INTRODUCTION

In a paper industry, pulp washing is the foremost process which has to be performed with an eco-friendly and efficient manner. In this practice, weak wash liquor is introduced to remove solute (black liquor) residing in the irregular void places of the packed bed. The inside present solute is expelled out when the bulk fluid is introduced. Washing performance of packed bed is proposed as three types of modelling viz. macroscopic, semi-quantitative and microscopic by different investigators. These models are established with material balance equations.

These equations involve different variables and their partial derivatives. These equations along with various linear and nonlinear adsorption isotherms describe the relation between concentration of the solute adsorbed on fibers and with the concentration of the solute in the flowing liquor. The linear and nonlinear diffusion dispersion models are solved with proposed numerical method and results can be specified in terms of exit solute concentration [5]

II. BACKGROUND

A complete review of the numerous process models used to describe pulp washing has been presented by [12]. Besides this [1,3,5,9,11,13,] have studied these washing models in the form of boundary value problems (BVPs) with different conditions and found the effect of parameters on wash liquor. A great deal of effort has been applied to compute efficiently the solution of BVPs using analytically and numerically.

III. DESCRIPTION OF THE METHOD

OCFE is a technique which is a combination of OCM and FEM proposed by [2]. In this method the trial function is expressed in terms of Lagrangian interpolation polynomial. The accuracy is increased in comparison to OCM but this method requires a subsidiary condition of continuity of function and its first derivative[7]. But in CHCM Hermite polynomial are used as a trial function which automatically have continuous first derivative. Cubic Hermite collocation method has high order of accuracy [4].

In the present study,quintic Hermite polynomials are used as basis function to solve the BVP described in the earlier paragraphs because it is C^2 continuous[8]. The continuity conditions of trial function and its derivatives at the grid points are satisfied. It reduces the number of equations due to continuity condition. Numerical stability plays a major role in checking the applicability of the method. The paper comprises of the solution of washing model with QHCM(

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quintichermite collocation method) and stability analysis of the method. The pulp washing models (dimentionless form) alongwith initial and boundary conditions and different isotherms are expressed in table-1.

Table-1: Linear and nonlinear models of pulp washing

	Model Equation(Dimensionless)	Boundary Condition	Initial Condition	Adsorption Isotherm
Linear Model-1	$\frac{\partial C}{\partial T}=\frac{1}{Pe}\frac{\partial^2 C}{\partial Z^2}-\frac{\partial C}{\partial Z}$	$\left.\begin{array}{l} C=0 \quad at \;\; Z=0 \\ \frac{\partial C}{\partial Z}=0 \; at \;\; Z=1 \end{array}\right\}$ for all $T\geq 0$	$C(Z,0)=1$	-
Linear Model-2	$\frac{\partial C}{\partial T}=\frac{1}{Pe}\frac{\partial^2 C}{\partial Z^2}-\frac{\partial C}{\partial Z}$	$\left.\begin{array}{l} PeC=\frac{\partial C}{\partial Z} at \;\; Z=0 \\ \frac{\partial C}{\partial Z}=0 \; at \;\; Z=1 \end{array}\right\}$ for all $T\geq 0$	$C(Z,0)=1$	-
Linear Model-3	$\frac{\partial C}{\partial T}=\frac{1}{Pe}\frac{\partial^2 C}{\partial Z^2}-\frac{\partial C}{\partial Z}-\frac{\mu Bi}{Pe}C$	$\left.\begin{array}{l} PeC=\frac{\partial C}{\partial Z} at \;\; Z=0 \\ \frac{\partial C}{\partial Z}=0 \; at \;\; Z=1 \end{array}\right\}$ For all $T\geq 0$	$C(Z,0)=1$	-
Linear Model-4	$R_d\frac{\partial C}{\partial T}=\frac{1}{Pe}\frac{\partial^2 C}{\partial Z^2}-\frac{\partial C}{\partial Z}$	$\left.\begin{array}{l} C=0 \quad at \;\; Z=0 \\ \frac{\partial C}{\partial Z}=0 \; at \;\; Z=1 \end{array}\right\}$ for all $T\geq 0$	$C(Z,0)=1$	

IV. STABILITY ANALYSIS

Stability analysis is a study of how well a numerical solution behaves when applied to a linearized system. The numerical method is said to be stable if a small perturbation does not cause divergence from the solution [10].Further, an algorithm for solving a linear partial differentialequation is said to be stable when the total variation of the numerical solution at a fixed time remains bounded as the step size goes to zero. Also, by analyzing the error and stability of a numerical technique, one can control the step size adaptively.

The stability of a time-dependent equation:

$$\frac{\partial C}{\partial t}=d(C), \tag{1.1}$$

where C is a function of both x and t and d is a spatial differential operator, is dependent upon the eigenvalues of the coefficient matrix of space discretization Eq. (1.1). Using the quintic Hermite collocation method, Eq. (1.1) is reduced into a set of ordinary differential equations in time:

$$\frac{dC}{dt}=[A]C+B \;. \tag{1.2}$$

where C is an unknown vector of the functional values at the interior grid points, *B*contains boundary values and a non-homogeneous part, and [*A*] is the coefficient matrixobtained after discretization. The detail of the same has already been explained in section 4.9 of chapter 4:

Linear model-1 and 2, $[A]=\frac{1}{Peh^2}(H_q^k)''(u_r)-\frac{1}{h}(H_q^k)'(u_r)$. (1.3)

Linear model-3, $[A]=\frac{1}{Peh^2}(H_q^k)''(u_r)-\frac{1}{h}(H_q^k)'(u_r)-\frac{\mu B_i}{Pe}(H_q^k)(u_r)$. (1.4)

Linear model - 4, $[A]=\frac{1}{R_d Peh^2}(H_q^k)''(u_r)-\frac{1}{R_d h}(H_q^k)'(u_r)$. (1.5)

+where $q=1,2,...,6, k=1,2,...,N$ and $r=1,2,3,4$.

It is well-known that the stability of Eq. (1.2) depends upon the stability of the numerical technique that is adopted to solve it. Any stable numerical technique for time discretization may not attain convergent solutions if the corresponding system of ordinary differential Eq. (1.2) is unstable. Mittal and Rohila (2016) explained in their study that the stability of Eq. (1.2) depends upon the Eigenvalues of the coefficient matrix A. Suppose λ_i to be the Eigenvalues of the coefficient matrix A.

According to Korkmaz and Dag (2011), when t approaches infinity, for the stable solution X one must have:

(a) If all the Eigen values are real, $-2.78<\Delta t\lambda_i<0$;

(b) If all the Eigenvalues have only complex components, $-2\sqrt{2}<\Delta t\lambda_i<2\sqrt{2}$; and

(c) If all the Eigen values are complex, $\Delta t\lambda_i$ should be in the region Figure 1.1.

The eigenvalues of linear model-1 and 2, linear model-3, and linear model-4 for $N=51, \Delta t=0.0001$, where *N* is the number of partions of domain and *t* is time, are plotted in Figures 1.2 to 1.4 respectively.

Here the results of concenrations are derived for different values of *Pe* (peclet number). From Figures 1.2 to 1.4, all the eigenvalues are complex and lie in the stability region. Hence, the system in Eq. (1.3) to Eq. (1.5) is stable.

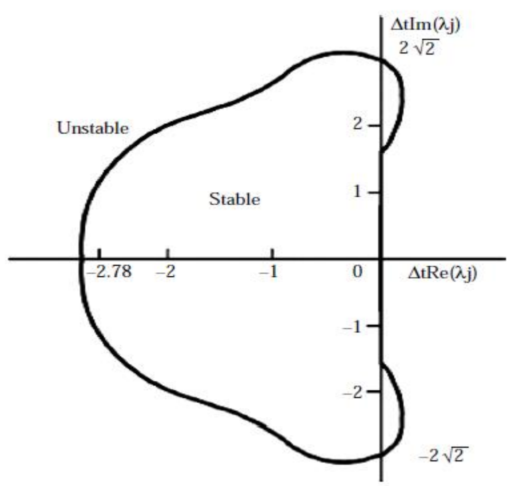

Figure 1.1. Stability region when eigenvalues are complex.

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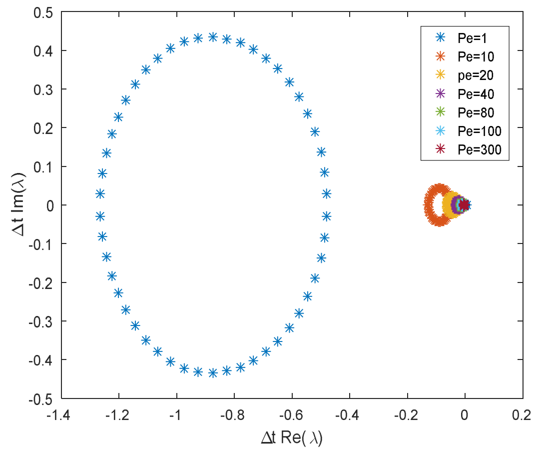

Figure 1.2. Eigenvalues of matrix A for linear model-1and2 with N=51, Δt=0.0001

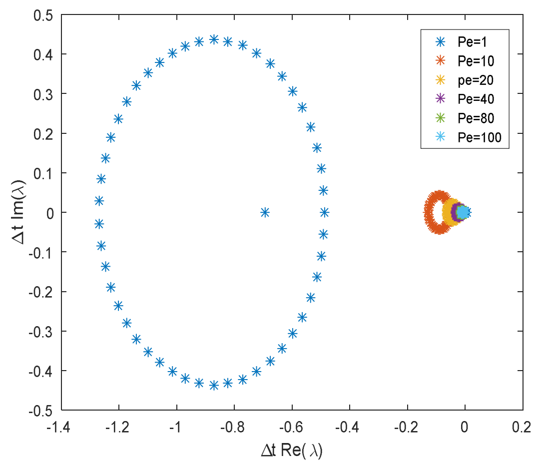

Figure 1.3. Eigenvalues of matrix A for linear model-3 with N=51, Δt=0.0001

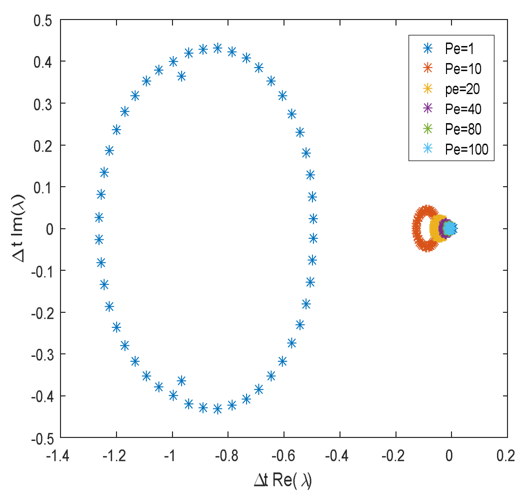

Figure 1.4. Eigenvalues of matrix A for linear model-4 with N=51, Δt=0.0001

V. CONCLUSION

In this paper, the stability of the numerical technique based on eigenvalues is explained. The linear and nonlinear models of pulp washing associated with diffusion dispersion phenomena are solved using QHCM for different values of peclet numbers. The eigenvalues for linear and nonlinear models are complex and lie in the stability region. This result support the stability of numerical scheme. So, It can be concluded that the QHCM is stable and it can be used to solve other mathematical models of real life situations.

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