IJARSCT
ISSN (Online) 2581-9429
International Journal of Advanced Research in Science, Communication and Technology (IJARSCT)
International Open-Access, Double-Blind, Peer-Reviewed, Refereed, Multidisciplinary Online Journal
Impact Factor: 7.53
Volume 4, Issue 1, September 2024

Applications of Laplace Transform in Solving Problems on Electrical Circuits

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Abstract*: Electrical engineering is a subject that depends heavily on the study of electrical circuits, and this paper shows how effective the Laplace Transform is at solving circuit-related problems. Laplace transforms are more important in many fields than electrical circuits, such as science, technology, electrical and communication engineering, and quantum physics. This work aims to provide a solid foundational grasp of Laplace transforms by highlighting important ideas and fundamental uses. The Laplace transform is a multifaceted mathematical method that comes in very handy when solving linear differential equations, particularly when dealing with initial value issues. Laplace transforms ordinary linear differential equations into algebraic ones, making problem-solving more streamlined and effective. This is one of the reasons why it is widely used in many scientific and technical fields.*

Keywords: Laplace transform, Electrical Circuits, Linear differential equations, Multifaceted Mathematical Method

I. INTRODUCTION

When functions are being moved from the time domain to the frequency domain, Laplace transforms must be used to create a flexible and strong mathematical structure. This conversion makes it easier to analyse and depict dynamic systems thoroughly, which leads to a more in-depth look at their traits and behavior.Laplace transforms, frequently used to solve linear differential equations, provide a more efficient method by turning regular linear differential equations into algebraic equations. This is particularly useful for addressing initial value difficulties. This transformative method is especially useful when working with differential equations with boundary values since it eliminates the need to find the values of arbitrary constants and the general solution. Because of their efficiency and efficacy, Laplace transforms are extremely useful in many science, engineering, and mathematics applications. This essay explores the fundamental ideas of Laplace transforms, highlighting their use in problem-solving and demonstrating theirrelevance in various analytical contexts.

Laplace Transform:

Examining a continuous function f(t)defined for all positive t values will help. This function's Laplace transform, represented as L{f(t)}, is a mathematical procedure that converts the function's time domain representation to the complex frequency domain. f(t)'s Laplace transform can be found using the integral below:

$$L\{f(t)\} = \bar{f}(s) = \int_0^\infty e^{-st} f(t)dt \text{ , ………… (1)}$$

The integral in this formula covers all positive values of t, and s is a complex variable. Because it makes solving linear differential equations easier, the Laplace transform is a useful tool in mathematical analysis and engineering applications.

The Laplace transform is especially helpful for solving linear time-invariant systems since it encodes a function f(t) in complex exponentials. The product of f(t) and the exponential term e $^{-st}$, weighted by the complex variable s, makes up the integral. The original function f(t) from the time domain is represented in the frequency domain by the converted function F(s), where's' is a complex variable. This intricate function offers a potent mathematical characterization that

DOI: 10.48175/IJARSCT-19501

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captures phase and amplitude data, enabling a more thorough and detailed examination of the behavior of the system at various frequencies. One reliable and comprehensive method for resolving differential equations and understanding dynamic systems is the Laplace transform. Its extensive use in various fields, including circuit analysis, signal processing, and control systems, attests to its adaptability and efficiency in handling challenging issues. The Laplace transform, which converts functions from the time domain to the frequency domain, offers a comprehensive and efficient way to streamline the analysis and solution of linear time-invariant systems. This mathematical instrument, which offers a strong foundation for modeling and comprehending the behavior of systems in numerous disciplines of study, has become essential in the domains of engineering, physics, and applied mathematics.

I] Linearity Property of Laplace Transform:

A key feature of the Laplace Transform that makes studying complicated systems easier is its linearity attribute, which permits the transformation of linear combinations of functions. To represent the linearit property for a given constant a and functions f(t) and g(t), use the following formula:

$$L\{X \cdot f(t) + Y \cdot g(t)\} = X \cdot L\{f(t)\} + Y \cdot L\{g(t)\}$$

This feature makes the Laplace Transform technique considerably easier to employ by reducing large functions down into smaller, more manageable components for easy examination. It is a crucial component that is used in many various contexts, including as the analysis of dynamic systems, the study of electrical circuits, and the solving of linear differential equations. The Laplace Transform's linearity feature highlights how adaptable and effective it is at managing mathematical tasks involving linear combinations of functions.

First Translation Property or First Shifting Property:

One of the essential elements of the Laplace Transform is the first translation property, sometimes referred to as the first shifting property. It explains how an exponential term multiplied simply in the Laplace domain translates to a shift in time in the original function.

If $L\{f(t)\} = \bar{f}(s)$, then $L\{e^{at} f(t)\} = \bar{f}(s + a)$.

In Laplace Transform applications, this trait is quite helpful, especially when working with time-shifted functions. By using a simple adjustment in the Laplace domain, it enables practitioners to evaluate and solve differential equations with temporal delays or advances.

Laplace Transform of Derivatives:

The derivative property of Laplace transforms is a special property of Laplace transforms that has to do with derivatives. This characteristic offers a practical method for calculating the Laplace transform of a function's derivatives.

If $L\{f(t)\} = f\overline{(s)}$ and $f(t)$ is a continuous function for $t \geq 0$, is of exponential order k and if it is a function of class A, then for $s > k$

$L\{f'(t)\} = sf\overline{(s)} - f(0)$, where $f(0) = f(t)$at $t = 0$

$L\{f''(t)\} = s^2\bar{f}(s) - s\,f(0) - f'(0)$, where $f'(0) = f'(t)$ at $t = 0$

$L\{f'''(t)\} = s^3\bar{f}(s) - s^2 f(0) - sf'(0) - f''(0)$, where $f''(0) = f''(t)$at $= 0$.

When solving differential equations containing derivatives, this characteristic comes in quite handy. In the Laplace domain, it permits differential equations to be transformed into algebraic equations, making it easier to solve them using conventional algebraic techniques. One important tool for Laplace transform applications in engineering and mathematical analysis is the Derivative Property of Laplace Transforms.

Inverse Laplace Transform of Some Elementary Functions:

The inverse Laplace transform is the procedure that takes a function out of its Laplace transform. The inverse Laplace transform can be calculated analytically for some simple functions.

The inverse Laplace transform of$\bar{f}(s)$is $f(t)$and it represented by the notation

$L\{f(t)\} = \bar{f}(s)$, if $L^{-1}\{\bar{f}(s)\} = f(t)$, ... (2.1)

DOI: 10.48175/IJARSCT-19501
ISSN 2581-9429 IJARSCT

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whereL^{-1} is the inverse Laplace transform operator and is such that$LL^{-1} = L^{-1}L = 1$.

Linearity Property of Inverse Laplace Transform:

The linearity property of the Laplace Transform and the Inverse Laplace Transform are the same. Like how the Laplace Transform handles linear combinations of functions, the Inverse Laplace Transform allows us to handle linear combinations in the time domain.

With the help of this feature, it is possible to divide the inverse Laplace transform of a linear combination of Laplace-transformed functions into its component parts, each of which is scaled by a constant, and add them up.

The linearity property of the inverse Laplace transform provides a convenient way to handle complicated expressions comprising linear combinations of Laplace-transformed functions, which is helpful in solving differential equations and understanding the time-domain behavior of systems.

If $L^{-1}\{\bar{f}(s)\} = f(t)$and $L^{-1}\{\bar{g}(s)\} = g(t)$, then $L^{-1}\{c_1\bar{f}(s) \pm c_2\bar{g}(s)\} = c_1L^{-1}\{\bar{f}(s)\} \pm c_2L^{-1}\{\bar{g}(s)\}$

Some Standard Functions $f(t)$ and Their Laplace Transforms $L\{f(t)\} = \bar{f}(s)$

Function f(t)	Laplace transform L{f(t)}=$\overline{f(s)}$	Explanation
A	$\frac{A}{s}$	Constant function representing a fixed value A.
u(t)	$\frac{1}{s}$	Unit step function, a step change at *t*=0.
e^{at}	$\frac{1}{s-a}$	Exponential function, common in dynamic systems
sin(kt)	$\frac{k}{s^2+k^2}$	Sine function, sinusoidal component in circuits
cos(kt)	$\frac{s}{s^2+k^2}$	Cosine function, similar to sine but with a phase shift.
tu(t)	$\frac{1}{s2}$	Unit ramp function, a linear increase starting at *t*=0.
d(t)	1	Impulse function (Dirac Delta), concentrated at *t*=0.

Some Standard Functions $\bar{f}(s)$ and Their Inverse Laplace Transforms$L^{-1}\{\bar{f}(s)\} = f(t)$

Function f(t)	**Inverse Laplace Transform f(t)**
Constant(a)	$L^{-1}\{\frac{a}{s}\} = a$
Unit Step(μc(t))	$L^{-1}\{\frac{e^{-cs}}{s}\}$=μc(t)
Impulse(d(t))	$L^{-1}\{1\} = d(t)$
Exponential(e^{at})	$L^{-1}\{\frac{1}{s-a}\} = e^{at}$
Sinusoidal(sin(wt))	$L^{-1}\{\frac{w}{s^2-w^2}\} = sin(wt)$
Sinusoidal(cos(wt))	$L^{-1}\{\frac{s}{s^2+w^2}\} = cos(wt)$

Partial Fraction Method:

A useful method in mathematics, especially in calculus and algebra, is the partial fraction method, which breaks down a rational function into smaller fractions called partial fractions. When working with inverse Laplace transforms and integrating complex expressions, this approach is very helpful

It is simpler to work with the inversion term by term when a rational function is expressed as the sum of partial fractions, especially when the Laplace transform is involved. A complicated rational function is decomposed into smaller fractions, each of which is equivalent to a term in the partial fraction decomposition.

Application to Electrical Circuits

Game theory is used in the field of electrical circuits to simulate interactions between entities or components that aim to achieve optimal results. Examples of these interactions include resource sharing, network protocols, and strategic power allocation, where the dynamics of cooperation can affect the behavior and performance of the system.

Laplace Transform in Simple Electric Circuits:

The Laplace Transform is an effective mathematical tool for analyzing basic electric circuits. The Laplace Transform is useful in describing the behavior of a circuit that consists of a series connection of a resistance (R), inductance (L), capacitor (C), and an electromotive voltage source (E), all of which are controlled by a switch. The Laplace Transform provides a methodical way to analyze and solve dynamic electrical circuit reactions by allowing the evaluation of voltage decreases across the resistance, inductance, and capacitance as a result of the circuit being closed and creating a current flow (i) via these components.

Between the resistance R, the voltage drop V_R is equal to Ri.

An inductance L has a voltage drop V_L, which is equal to $L\frac{di}{dt}$.

$V_c=\frac{q}{C}$ is the formula for the voltage drop V_c across a capacitor C.

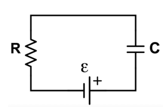
Keep in mind that the current, i, equals the time rate of charge change, or $i = \frac{dq}{dt}$ or $q = \int i\,dt$.

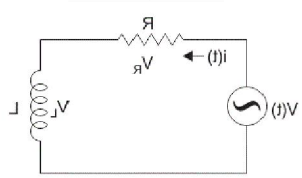
Kirchhoff's law therefore gives us

Kirchhoff's Law of Voltage: The electromotive force (EMF) that arises from the algebraic sum of the voltage drops across each element of a closed electrical circuit is equal to the EMF, according to Kirchhoff's Law of Voltage, often known as Kirchhoff's Voltage Law (KVL). This equation is a cornerstone of circuit analysis and is predicated on the idea of energy conservation.

R –C Circuit $Ri + \frac{q}{C} = E$ Or $R\frac{dq}{dt} + \frac{q}{C} = E$

L – R Circuit $L\frac{di}{dt} + Ri = E$





Formulation

The Laplace Transform provides a systematic approach to solving differential equations through a four-step process. Initially, the Laplace Transform of the given differential equation is obtained by using the derivative property as needed. The modified equation is then integrated using the initial conditions. The output variable is then determined by solving the modified equation. After the solution, the outcome can be found by examining the Laplace Transform tables. The solution to the original differential equation is ultimately obtained in the time domain by using the Inverse Laplace Transform. This method is particularly useful when working with linear time-invariant systems, and it is widely applied in many mathematical and engineering applications.

Applications of Laplace and Inverse Laplace Transformations:

As we know Laplace Transforms of differential function is obtain as follows:

$(a)\ L\{f'(t)\} = s\bar{f}(s) - f(0)$, where $f(0) = f(t)$ at $t = 0$.

$(b)\ L\{f''(t)\} = s^2\bar{f}(s) - s\,f(0) - f'(0)$, where $f'(0) = f'(t)$ at $t = 0$

$(c)\ L\{f'''(t)\} = s^3\bar{f}(s) - s^2 f(0) - sf'(0) - f''(0)$, where $f''(0) = f''(t)$ at $= 0$.

In a circuit with a battery E, resistance R, and inductance L, when a switch is closed, the current i increases at the rate shown by

$$\boldsymbol{L\frac{di}{dt}+Ri=E}.$$

Determine I in relation to t. If E=6 volts, R=100 ohms, and L=0.1 henry, how long will it take for the current to reach half of its maximum value?

Solution: The given differential equation is

$$L\frac{di}{dt}+Ri=E$$

Which is $\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$

Laplace transform applied on both sides

$$L\left\{\frac{di}{dt}+\frac{R}{L}i\right\}=L\left\{\frac{E}{L}\right\}$$

or,

$$s\bar{\imath}(s)-i(0)+\frac{R}{L}\bar{\imath}(s) = \frac{E}{L}\frac{1}{s}$$

As at t = 0 , i = 0

$$\left(s+\frac{R}{L}\right)\bar{\imath}(s) = \frac{E}{L}\frac{1}{s}$$

$$\bar{\imath}(s) = \frac{E}{L}\left[\frac{1}{s\,\left(s+\frac{R}{L}\right)}\right]$$

$$\bar{\imath}(s) = \frac{E}{L}\times\frac{L}{R}\left[\frac{1}{s}-\frac{1}{\left(s+\frac{R}{L}\right)}\right]$$

Taking Inverse Laplace transform on both sides

$$i(t)=\frac{E}{R}\left(1-e^{-\frac{R}{L}t}\right)$$

Hence, $i_{\max}=\frac{E}{R}$.

Let the current in the circuit be half its maximum value in T seconds, then from (2)

$$\frac{E}{2R}=\frac{E}{R}\left(1-e^{-\frac{RT}{L}}\right)$$

i.e. $\frac{1}{2}=1-e^{-\frac{RT}{L}}$ or,

$$-\frac{RT}{L}=\log\frac{1}{2}=-\log 2$$

or,

$$T=+\frac{L}{R}\log 2$$

$$=+\frac{0.1}{100}(0.69315).$$

$$=0.00069315\text{ seconds}$$

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When a 0.01 farad capacitor and a 20 ohm resistor are connected in series, the electromotive force E volts is calculated using the formula $40e^{-3t} + 20e^{-6t}$. Prove that the capacitor has a maximum charge of 0.25 coulombs if q=0 at t=0.

By Kirchhoff's law of voltage drops, we have

$$Ri + \frac{q}{c} = E(t)$$

$$\Rightarrow R\frac{dq}{dt} + \frac{q}{c} = E(t)$$

$$\therefore\ 20\frac{dq}{dt} + \frac{q}{0.01} = 40\,e^{-3t} + 20\,e^{-6t}$$

or,

$$\frac{dq}{dt} + 5q = 2\,e^{-3t} + e^{-6t} \ldots\ldots\ldots (1)$$

Laplace transform applied on both sides

$$L\left\{\frac{dq}{dt} + 5q\right\} = L\{2\,e^{-3t} + e^{-6t}\}$$

$$s\bar{q}(s) - q(0) + 5\bar{q}(s) = \left\{\frac{2}{(s+3)} + \frac{1}{(s+6)}\right\}$$

Now $q = 0$ at $t = 0$ gives

$$\bar{q}(s) = \left\{\frac{2}{(s+3)(s+5)} + \frac{1}{(s+6)(s+5)}\right\}$$

$$\bar{q}(s) = \left\{\frac{1}{(s+3)} - \frac{1}{(s+5)} + \frac{1}{(s+5)} - \frac{1}{(s+6)}\right\}$$

$$\bar{q}(s) = \left\{\frac{1}{(s+3)} - \frac{1}{(s+6)}\right\}$$

Applying both sides of the Inverse Laplace transform

$$q = e^{-3t} - e^{-6t}$$

For maximum charge q, we have $\frac{dq}{dt} = 0$, giving from (3),

$$\frac{dq}{dt} = 0 = -3\,e^{-3t} + 6\,e^{-6t}$$

$$\Rightarrow 6\,e^{-6t} = 3\,e^{-3t}$$

$$\Rightarrow 2 = e^{3t}$$

$$\Rightarrow e^{-3t} = \frac{1}{2}$$

From (3), we have therefore

$$q_{\max} = e^{-3t} - (e^{-3t})^2 = \frac{1}{2} - \left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4} = 0.25 \text{ coulombs.}$$

II. CONCLUSION

In summary, this essay has demonstrated the importance of the Laplace Transform in the field of electrical circuits and how well it works to simplify challenging issues, especially those involving stability and control. The Laplace Transform's adaptability goes beyond electrical circuits, showcasing its strength as an effective instrument in numerous fields of mathematics, physics, and engineering. The direct simulation of Laplace transformable equations has been made possible by the development of research software and its convenience of use, which has greatly advanced the field of study. The study also emphasizes the effective use of Laplace transformation in the resolution of fractional ordinary differential equations with both constant and variable coefficients and initial boundary value issues, highlighting the transformation's wide use and influence in a variety of scientific fields.

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