

# Sensitivity Analysis Nusselt Number Correlation to Thermo Physical and Rheological Properties

Shubhangi S. Patil<sup>1</sup>, K. Muthamizhi<sup>2</sup>, P. Kalaichelvi<sup>3</sup>

Assistant Professor, Department of Chemical Engineering

Dr. D.Y. Patil Institute of Engineering, Management and Research, Pune, India<sup>1</sup>

PhD Scholar, National Institute of Technology, Tiruchirappalli, Tamil Nadu, India<sup>2</sup>

Associate Professor, National Institute of Technology, Tiruchirappalli, Tamil Nadu, India<sup>3</sup>

**Abstract:** *We present the results of sensitivity analysis of thermo physical and rheological properties to temperature and concentration and hence sensitivity of Nusselt number to thermo physical and rheological properties. Sodium alginate which is a power law fluid has been used. Thermo physical and rheological properties were first measured experimentally and using RSM models has been developed as a function of temperature and concentration. Sensitivity of these developed models has been studied for temperature and concentration. From dimensional analysis it was found that Nusselt number depends on the thermo physical and rheological properties of power law fluids. Hence sensitivity of Nusselt number model has been studied for these properties. Studied thermo physical properties include density, surface tension, specific heat and thermal conductivity. Rheological properties like consistency index and power law index have been studied. The sensitivity analysis indicates that properties, density, surface tension and specific heat are sensitive to temperature than concentration. Result for Nusselt number shows that model is very sensitive to specific heat, thermal conductivity and power law index.*

**Keywords:** Thermo physical properties, Rheological properties, Nusselt number, sensitivity analysis

## I. INTRODUCTION

Sensitivity analysis' aims to describe how much model output values are affected by changes in model input values. Various techniques have been developed to determine how sensitive model outputs are to changes in model inputs. Most approaches examine the effects of changes in a single parameter value or input variable assuming no changes in all the other inputs. Sensitivity analyses can be extended to examine the combined effects of multiple sources of error, as well. Changes in particular model input values can affect model output values in different ways. It is generally true that only a relatively few input variables dominate or substantially influence the values of a particular output variable or performance indicator at a particular location and time. If the range of uncertainty of only some of the output data is of interest, then undoubtedly only those input data that significantly affect the values of those output data need be included in the sensitivity analysis. Sensitivity analysis can be useful for a range of purposes, including 1. Testing the robustness of the results of a model or system in the presence of uncertainty, 2. Increased understanding of the relationships between input and output variables in a system or model, 3. Uncertainty reduction: identifying model inputs that cause significant uncertainty in the output and should therefore be the focus of attention if the robustness is to be increased (perhaps by further research), 4. Searching for errors in the model (by encountering unexpected relationships between inputs and outputs), 5. Model simplification – fixing model inputs that have no effect on the output, or identifying and removing redundant parts of the model structure, 6. Enhancing communication from modelers to decision makers (e.g. by making recommendations more credible, understandable, compelling or persuasive), 7. Finding regions in the space of input factors for which the model output is either maximum or minimum or meets some optimum criterion.

There are many previous studies on the sensitivity analysis. Several investigators have used these sensitivity analysis techniques for different purposes. Nicolai et al. (1995) has presented new method to compute the sensitivity of the temperature course inside conduction heated foods with respect to the surface heat transfer coefficient. Sensitivity charts are constructed which relate the dimensionless centre temperature with the Biot and Fourier number for slab,

cylinder and sphere shapes. Processes with low surface heat transfer coefficient shows large deviation in the core temperature of the food for small deviation in surface heat transfer coefficient. It was observed that the evaluation of the dimensionless temperature is particularly sensitive to variations in the Biot number. Aberham et al 2012 introduced Uncertainty analysis and global sensitivity analysis with variation of 15% or a variation of 30% (more uncertain parameters) around the nominal value was assumed to a coupled heat and mass transfer model of a contact baking process. The method was implemented in the framework COMSOL-MATLAB version 3.5. For the sensitivity analysis both the scatter plot and the standardized regression coefficients method (SRC) were used. The global sensitivity analysis results with the SRC method provided a ranking of the parameters, from influential to non-influential. Ranking was significantly different for the different model outputs ( $T$ ,  $X_i$ , and  $X_v$ ). Overall, some of the parameters ( $k_i$ ,  $T_{air}$ , and  $h_{top}$ ) have little or no impact on all the state variables of the model, whereas some of the input parameters have a strong impact:  $k$  and  $k_{evp}$  on the temperature predictions and  $D_l$  on the prediction of the liquid water concentration at the bottom layer of the product. Hikari et al. 2013 performed sensitivity analysis to optimize the design of the double-layer ground-source heat pump. The results of the sensitivity study on installation depth showed that the optimum depth of the upper layer was 1.5 m in case that the depth of the lower layer was fixed at 2.0 m. Also, the influence of the reflectance of the land surface was investigated by changing the albedo of the land surface as 0.1, 0.3 and 0.6. The numerical simulations showed that lower albedo is preferable in heating operations, while higher albedo is favorable in cooling operations. Wong et al. 2003 presented sensitivity analysis of heat transfer formulation for insulated structural steel component. Results of the analysis show that, for certain insulation materials, the temperatures predicted by the Eurocode may differ substantially from those by exact analytical solution. Fesanghary et al. 2008 used global sensitivity analysis (GSA) and harmony search algorithm for design optimization of shell and tube heat exchangers (STHXs) from the economic viewpoint. The aim of the sensitivity analysis is to identify geometrical parameters that have the largest impact on total cost of STHXs and GSA could successfully found the most important parameters. Clarke et al. 2000 have used Monte Carlo method to study the sensitivity and uncertainty of heat exchanger design to physical properties estimation. It was reported that uncertainty of 3% in critical constants and acentric factors and uncertainty of 5% in physical properties such as  $C_p$ ,  $\rho$ ,  $n$  and  $k$  can affect significantly the heat exchanger design and predicted performance.

As we could see that there is limited study available on sensitivity analysis in heat transfer area and no one has taken care the sensitivity of heat transfer to rheological properties of power law fluids. Even though now days power law fluids are very interesting area in heat transfer augmentation. Hence purpose of this study here is (i) to study sensitivity of thermo physical and rheological properties to temperature and concentration. And as these properties affect the rate of heat transfer (ii) to study sensitivity of Nusselt number to thermo physical properties nad rheological properties.

## II. MATERIALS AND METHOD

Sodium alginate pure (food grade) was provided by LOBACHemie, Merck & Himedia. The aqueous solution of sodium alginate was prepared by dissolving powder in water very gently using an agitator to avoid agglomeration of powder.

2.1 Experimental methods used for measuring properties are tabulated below. Shubhangi has done experiments to find the values of density, surface tension, specific heat and thermal conductivity and rheological properties by as a part of M. Tech. project for concentration range of 0.1-0.6%w/w and temperature range of 293.15-333.15 K. Methods listed in following table have been used.

Table 1. Experimental methods used for measuring properties.

Property	Method
Density	Specific Gravity Bottle
Surface tension	Drop weight method
Specific heat	Joule's calorimeter method
Thermal conductivity	Thermal conductivity of liquid apparatus (HT-08)
Rheological properties	Brookfield viscometer (LV DV-II)

**2.2 Modeling of Properties.**

In the same project work Shubhangi has developed the models for each property by using experimental data for each property Response Surface Methodology (RSM). A two factor, five level central composite rotatable designs (CCRD) was employed using Design-Expert 8.0.6 software to model equations. Independent variables, i.e. temperature and concentration were coded at five levels between -2 and +2, where the temperature (*T*) in the range of 293.15 - 333.15 K and concentration in the range of 0.1 - 0.6 %w/w respectively as shown in table 2.

Factors	Unit	Levels and range				
		-2	-1	0	1	2
Temperature	K	284.86	293.15	313.15	333.15	341.43
Concentration	%	0	0.1	0.35	0.6	0.7

Developed models are tabulated in table 3 below which have been used for sensitivity analysis.

Table 3. Developed models for properties.

Property	Model	Eq. No.
Density	$\rho = 2161.85 - 125.142T - 381.763C + 0.5TC + 0.18875T^2 + 416C^2$	1
Specific heat	$C_p = 98.78575 - 15.9955C + 0.06TC + 0.001109T^2 - 0.5C^2$	2
Surface tension	$\sigma = 0.311495 - 0.00147T + 0.045C - 0.00013TC - 0.000002T^2 + 0.0018C^2$	3
Thermal conductivity	$K = -2.11 + 0.0145T - 0.0053TC - 0.0000198T^2 + 0.0271C^2$	4
Consistency index	$k = 1.31 - 0.0037T - 0.836C + 0.003125TC - 7.5E - 7T^2 - 0.027C^2$	5
Power law index	$n = -1.24 + 0.0053T + 3.45C - 0.0068TC + 5.5E - 19T^2 - 0.621C^2$	6

**2.3 Development of model for Nusselt number.**

Muthamizhi et al.(2014) has developed the Nusselt number correlation for a counterflow six-channel corrugated type PHE with CMC as a working fluid [14]. Dimensional analysis has been used to develop the Nusselt number correlation which is the mathematical technique of deriving relations between physical quantities by identifying their dimensions. Developed correlation also take care the rheological properties of CMC which affect the heat transfer. The constants and powers of the parameters involved in the correlation developed were estimated using least square method. An excel tool function has been used to determine the effect of each variable.

The following expression of heat transfer coefficient in term of Nusselt number was reported by Muthamizhi et al. (2014).

$$Nu_x = 0.415834 \left( \left( \frac{U}{D^{(n-2)} \rho^{1/(n-2)}} \right)^{0.651139} \left( \frac{cp}{D^{(n-2)} \rho^{(n-2)} K} \right)^{2.267862} \left( \frac{\Delta T \beta g}{D^{(n-2)} \rho^{(n-2)}} \right)^{-0.15291} \right)$$

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Developed model have been also used by Shubhangi for sodium alginate which is also a power law fluid after studying its thermal performance experimentally in Plate Heat Exchanger of same geometry. Nusselt number Eq. 7 has been used for sodium alginate solution which also gives good results with little more error compared to CMC solution.

**2.4 Sensitivity analysis**

The first step in sensitivity analysis is to determine the uncertainty range of input parameter. For this purpose the so-called expert review procedure is used. On the basis of the literature and expert knowledge, a variation of + 1% and a variation of - 1% around the nominal value was assumed. Then excel was used to study how much output deviates from its actual value. For studying sensitivity of thermo physical and rheological properties input parameters temperature and concentration was changed. Sensitivity of Nusselt number was studied by changing properties which affect Nusselt number.

**III. RESULT AND DISCUSSION**

The experimental measurements are often unavailable, expensive and time consuming; model has been developed for predicting properties and Nusselt number. Also sensitivity have been carried out for testing the robustness of the results of a model or system in the presence of uncertainty. Results of sensitivity have been discussed in this section.

**3.1 Sensitivity analysis for thermo physical and rheological properties.**

Sensitivity analysis have been carried out for thermo physical properties and rheological properties (i) to determine how much will these properties will change if measurement of temperature and concentration is wrong by + and - 1%, (ii) Which variable has more effect on model prediction so that one can measure that variable with more precision. In the present study temperature and concentration have been changed by +-1% to observe the deviation in thermo physical and rheological properties. Model equations tabulated in Table 3 were used to study sensitivity analysis. Results are tabulated in Table 4 to represent the error in model prediction when there is positive 1% and negative 1% error in measurement of concentration and temperature

Table 4. %Error in thermo physical and rheological properties for change in concentration and temperature.

Property	%Error in Properties			
	For +1% error in Conc.	For -1% error in Conc.	For +1% error in Temp.	For -1% error in Temp.
Density	0.012	-0.012	-0.303	0.331
Specific Heat	0.159	-0.159	4.223	-4.125
Surface Tension	-0.489	-0.145	-1.804	1.104
Thermal conductivity	-0.715	0.693	0.143	-0.267
Consistency Index	0.418	-0.416	-11.712	8.651
Power law index	0.331	-0.332	1.307	-1.341

From Table 4 it is observed that % error in density for studied uncertainty in input parameter i.e. temperature and concentration is very low which can be neglected. Error is more for change in temperature than concentration so we can say that density is more sensitive for temperature than concentration. Error in specific heat is also more for change in temperature than concentration, hence we can say that specific heat is more sensitive to temperature. So to get accurate results for specific heat we have to measure temperature more accurately than temperature. Results for surface tension also show the similar results. Surface tension is more sensitive for temperature as %error is more for change in temperature than concentration. So to get accurate surface tension temperature has to measure accurately. Thermal conductivity is the only property which shows more deviation for concentration than temperature. Hence thermal conductivity is more sensitive for concentration than temperature.

Sensitivity analysis for rheological properties like consistency index and power law index shows that both these properties are more sensitive to temperature than concentration. Among these two consistency index is more sensitive to temperature than power law index as it deviates more for same change in temperature. Hence to get correct result for consistency index temperature have to be measure very accurately.

Negative error in properties shows that there is inverses proportionality in property and respective input parameter.

### 3.2 Sensitivity analysis for nusselt number

Muthamizi et al 2014 have reported that developed model for Nusselt number using CMC gives RSM deviation of 14.61 which is lowest compared to other literature models. Also the developed model has been checked for sodium alginate solution with same PHE configuration by Shubhangi Power in her project work. The RSM deviation was found 16.87. It shows that developed model can be used for predicting Nusselt number of power law fluids with little error. Hence sensitivity analysis was carried out for developed Nusselt number to properties like velocity, density, specific heat, thermal conductivity, temperature difference and power law index.

To study sensitivity of Nusselt number for these properties uncertainty of + and - 1% was introduced in the input parameters. The %error was tabulated in table 5.

Table 5.% error in Nusselt number.

	%Error in Nu for +1% error in input properties					
properties	Velocity	N	$\rho$	K	$C_p$	$\Delta T$
% Error in Nu	0.652	2.045	0.832	-2.253	2.253	-0.154
	% Error in Nu for -1% error in input properties					
properties	Velocity	N	$\rho$	K	$C_p$	$\Delta T$
% Error in Nu	-0.656	-2.047	-0.839	2.305	-2.305	0.153

Table 5 represents the error in the Nusselt number prediction when thermophysical and rheological properties have been increased by 1%. It is observed that maximum error observed is 2 for 1% change in power law index, specific heat and thermal conductivity which can be neglected or we can measure these three variables with more precautions. Error observed in the model prediction is very less for change in velocity, density and temperature difference. Hence developed Nusselt number model is more sensitive for change in velocity, density and temperature difference.

### IV. CONCLUSION

From sensitivity analysis it is concluded that developed models for thermo physical and rheological properties are more sensitive towards temperature than change in concentration. Hence temperature has to be measure with more precaution to get accurate results. Exception is thermal conductivity model which is more sensitive towards change in concentration. Sensitivity analysis for Nusselt number correlation shows change in power law index, specific heat and thermal conductivity give more deviation in Nusselt number values. Hence to get accurate results for Nusselt number by using Eq. 7 these three properties has to be measure accurately.

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