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Generalized Synchronization of Different Dimensional Chaotic Dynamical Systems

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Abstract: *In this project, we investigate the generalized synchronization behavior between two different dimensional chaotic dynamical systems. In this project we deal for two different dimensional chaotic system one Shimizu Morioka (3D) and another is Lorenz stenflo system (4D).*

Keywords: Lorenz stenflo system

I. INTRODUCTION

Synchronization is an essential occurrence that entitle compatible dealing in coupled systems. In 1990, a method to synchronize two equivalent chaotic systems with different initial conditions is introduced by Pecora and Carroll[1]. Since then, there are many possible but not yet actual applications in impervious communication[2], biological science[3], chemical reaction[4], social science[5], and many other fields, the synchronization of coupled chaotic dynamical systems has been one of the most interesting topics in nonlinear science and many theoretical and experimental results[6] have been obtained. We get a meaningful result by finding a variety of different synchronization occurrence, such as complete synchronization[7] or anticipated synchronization (AS)[8] and generalized synchronization (GS)[9] etc.

Now most of theoretical results about synchronization phenomena concentrate on structurally equivalent systems (identical systems or non identical systems whose nonidentity is resulted from a rather small parameter mismatch). Sometimes, we used to investigate compatible behavior of strictly different dynamical systems (including different dimensional systems),particularly the systems in biological science[10] and social science[11]. For example, we cannotice that both circulatory and respiratory systems behave in contemporary way, but their models are materially different and they have different dimensions. So, the theoretical research of synchronization for strictly different dynamical systems is very significant. Whereas this kind of research is just at starting stage. Apparently of our knowledge, there are few theoretical results about generalized synchronization of different dimensional chaotic dynamical systems. In this paper, we accumulate an approach to realize the globally generalized synchronization between two different dimensional dynamical systems.

The rest of the paper is organized as follows. In Section 2, theoretical results are discussed. Based on the Lyapunov stability theory, we propose a systematic and powerful approach to realize the generalized synchronization between two different dimensional dynamical systems. In Section 3, numerical simulations results are proposed for generalized synchronization between two different dimensional chaotic dynamical systems Shimizu Morioka system and Lorenz stenflo system. Finally, brief conclusion is drawn in section 4.

II. THEORETICAL RESULTS

Consider the following coupled chaotic systems:

$$\begin{cases} \dot{x} = f(x) \\ \dot{y} = g(y) + u(y,x) \end{cases} \quad (1)$$

where $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in R^n$ is the n-dimensional state vector, $f: R^n \to R^n$ is a continuous vector function, $y = (y_1, y_2, \ldots, y_m)^{\mathrm{T}} \in R^m$ is the m-dimensional state vector, $g: R^m \to R^m$ is a continuous vector function, and $u(y,x) \in R^m$ is a controller.

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Let $x(t, x_0)$ and $y(t, y_0, x_0)$ be the solutions of drive system (the first subsystem of systems (1)) and response system (the second subsystem of systems (1)), where x_0 and y_0 are initial values. For simplicity, we denote $x(t, x_0)$ and $y(t, y_0, x_0)$ by $x(t)$ and $y(t)$, respectively. $\|\cdot\|$ denotes a vector norm.

Definition. We say systems (1) are generalized synchronous with respect to the vector map ϕ, if there exist a controller $u(y,x) \in R^m$ and a given map $\phi: R^n \to R^m$ such that the solutions of systems (1) satisfy the following property:

$$\lim_{t\to\infty} \| y(t) - \phi(x(t)) \| = 0.$$

Remark. There are many exact classes of GS based on the mathematical property of the map ϕ[16,17]. Here, the map ϕ is an arbitrary continuously differentiable function. So, with this definition, our study about GS has more extensive meaning than before.

It is well known that many systems, such as Lorenz system, Chen system, Chua's circuit, Rössler system, hyperchaotic Rössler system, and Lü system, can be written as the form

$$\dot{y} = By + G(y), \quad (2)$$

where $B = (b_{ij})_{m\times m}$ is a constant matrix, $G(y)$ is the nonlinear part. So, without loss of generality, we can describe systems (1) as

$$\begin{cases} \dot{x} = f(x) \\ \dot{y} = By + G(y) + u(y,x). \end{cases} \quad (3)$$

In order to study the generalized synchronization of systems (3), we define the generalized synchronization error $e = y - \phi(x)$, where $\phi: R^n \to R^m$ is a continuously differentiable map. The error system e is given by

$$\dot{e} = \dot{y} - \dot{\phi}(x) = By + G(y) + u(y,x) - D\phi(x)f(x), \quad (4)$$

where $D\phi(x)$ is the Jacobian matrix of the map $\phi(x)$:

$$D\phi(x) = \begin{pmatrix} \frac{\partial\phi_1(x)}{\partial x_1} & \frac{\partial\phi_1(x)}{\partial x_2} & \cdots & \frac{\partial\phi_1(x)}{\partial x_n} \\ \frac{\partial\phi_2(x)}{\partial x_1} & \frac{\partial\phi_2(x)}{\partial x_2} & \cdots & \frac{\partial\phi_2(x)}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial\phi_m(x)}{\partial x_1} & \frac{\partial\phi_m(x)}{\partial x_2} & \cdots & \frac{\partial\phi_m(x)}{\partial x_n} \end{pmatrix}$$

From the definition of generalized synchronization, we know that the study of the generalized synchronization between two coupled systems can be translated into the analysis of the asymptotical stability of zero solution of error system. So, we discuss the asymptotical stability of zero solution of system (4) and draw the following conclusion.

Theorem 1. In the systems (3), for any continuously differentiable map $\phi: R^n \to R^m$, let us design the controller

$$u(x,y) = \varepsilon(\phi(x) - y) + D\phi(x)f(x) - B\phi(x) - G(y),$$

Where $\varepsilon = (\epsilon_{ij})_{m\times m}$ is a constant matrix. If $(B-\varepsilon)^{\mathrm{T}} + (B-\varepsilon)$ is a negative definite matrix, then systems (3) are globally generalized synchronous with respect to the vector map ϕ.

Proof. Substituting the controller

$$u(x,y) = \varepsilon(\phi(x) - y) + D\phi(x)f(x) - B\phi(x) - G(y)$$

into the error system (4), we get

$\dot{e} = \dot{y} - \dot{\phi}(x) = By + G(y) - D\phi(x)f(x) + \varepsilon(\phi(x) - y) + D\phi(x)f(x) - B\phi(x) - G(y) = (B - \varepsilon)e.$

(5)

Construct a Lyapunov function in the form of $V(t) = e^{\mathrm{T}}(t)e(t)$. If $(B-\varepsilon)^{\mathrm{T}} + (B-\varepsilon)$ is a negative definite matrix, then the derivative of V along the solution of (5) is

$\dot{V} = \dot{e}^{\mathrm{T}}e + e^{\mathrm{T}}\dot{e} = e^{\mathrm{T}}[(B-\varepsilon)^{\mathrm{T}} + (B-\varepsilon)]e < 0$. From the Lyapunov stability theory, the zero solution of error system (4) is globally asymptotically stable, i.e., systems (3) are globally generalized synchronous with respect to the vector map ϕ.

For a given map ϕ, linear controller usually can not be found to achieve generalized synchronization between two different dimensional chaotic dynamical systems. The controller should contain the information of f, G and ϕ. From the view of mechanical control, our controller seems a little bit complicated, but it is appropriate for biological and social systems, and we also note that similar controller has been adopted . Following examples will show that our approach is effective.

III. NUMERICAL RESULTS

In order to show the effectiveness of our approach, two examples are used to discuss two kinds of cases: $n < m$ and $n > m$, respectively.

(1) $n < m$

We choose Shimizu Morioka system[12] as drive system and Lorenz stenflo system[13] as response system. The Shimizu-Morioka system can be described by following nonlinear ODE:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_1 - \lambda x_2 - x_1 x_3 \\ \alpha x_3 + x_1^2 \end{pmatrix} \quad (6)$$

which has a chaotic attractor when $\lambda = 0.799, \alpha = 0.54$ The Shemozu-Morioka chaotic attractor is shown in Fig. 1 .

The s Lorenz stenflo system can be described by following nonlinear ODE:

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$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{pmatrix} = \begin{pmatrix} \sigma(y_2 - y_1) + \gamma y_4 \\ y_1(\mathrm{r} - y_3) - y_2 \\ y_1 y_2 - b y_3 \\ -y_1 - \sigma y_4 \end{pmatrix} \quad (7)$$

which has a chaotic when $\sigma = 1, \gamma = 1.5, b = 0.7, r = 26$. The projections of chaotic attractor in (y_1, y_2, y_3) and (y_4, y_2, y_3) are shown in Figs. 2 and 3 .

System (7) can be rewritten as the form $\dot{y} = By + G(y)$, where

$$B = \begin{pmatrix} -\sigma & \sigma & 0 & \gamma \\ \mathrm{r} & -1 & 0 & 0 \\ 0 & 0 & -\mathrm{b} & 0 \\ -1 & 0 & 0 & -\sigma \end{pmatrix}, \quad G(y) = \begin{pmatrix} 0 \\ -y_1 y_3 \\ y_1 y_2 \\ 0 \end{pmatrix}$$

With the parameters above, let $u(x,y) = \varepsilon(\phi(x) - y) - B\varphi(x) - G(y) + D\phi(x)f(x)$, where

$\phi(x_1, x_2, x_3,) = (x_1, x_2, x_3, x_1 + x_3)^{\mathrm{T}}$

$$D\phi(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\varepsilon = \begin{pmatrix} 1 & 1 & 0 & 0.54 \\ 26 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

So,

$$B - \varepsilon = \begin{pmatrix} -2 & 0 & 0 & 0 \\ 0 & -1.5 & 0 & 0 \\ 0 & 0 & -1.7 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

It is easy to know that $(B-\varepsilon)^T + (B-\varepsilon)$ is a negative definite matrix. Then the conditions of Theorem 1 are satisfied. With the help of Matlab, we get the numeric results that are showed in Fig.

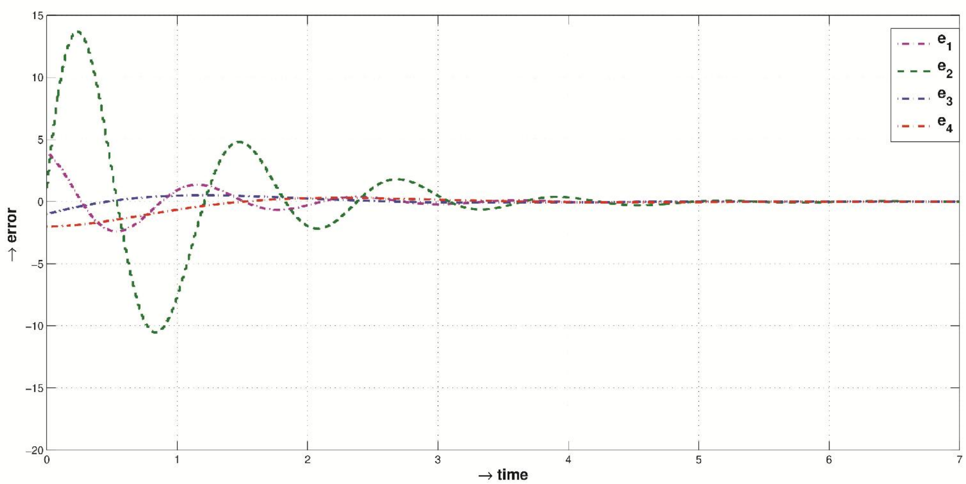

FIG.1. Generalized synchronization error for the system (6 & 7)

where$x(0) = (10,10,10), y(0) = (1,1,1,1)$, and $e_i = y_i - \phi_i(x(t)), i = 1,2,3,4$. 2) $n > m$

We choose Lorenz stenflo system as drive system and Shimizu-Morioka system as response system.

$$\dot{x} = f(x) = \begin{pmatrix} \sigma(x_2 - x_1) + \gamma x_4 \\ x_1(\mathrm{r} - x_3) - x_2 \\ x_1 x_2 - b x_3 \\ -x_1 - \sigma x_4 \end{pmatrix} \qquad (8)$$

$$\dot{y} = g(y) = \begin{pmatrix} y_2 \\ y_1 - \lambda y_2 - y_1 y_2 \\ \alpha y_3 + y_1^2 \end{pmatrix} \qquad (9)$$

System (9) can be rewritten as the form $\dot{y} = By + G(y)$, where

$$B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & \alpha \end{pmatrix}, \quad G(y) = \begin{pmatrix} 0 \\ -y_1 y_2 \\ y_1^2 \end{pmatrix}$$

With the parameters above, let $u(x,y) = \varepsilon(y - \phi(x)) - B\phi(x) - G(y) + D\phi(x)f(x)$, where

$$\phi(x_1, x_2, x_3, x_4) = (2x_1, x_2, x_1 + x_3 + x_4)^T$$

$$D\phi(x) = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\varepsilon = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0.201 & 0 \\ 0 & 0 & 2.54 \end{pmatrix}$$

$$B - \varepsilon = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

We can see $(B-\varepsilon)^T + (B-\varepsilon)$ is a negative definite matrix, and the conditions of Theorem 1 are satisfied. With the help of Matlab, we get the numeric results that are showed in Fig. 2. In fig 2 we observed that the error systems converges to zero which confirms that the two coupled systems are in generalized synchronization.

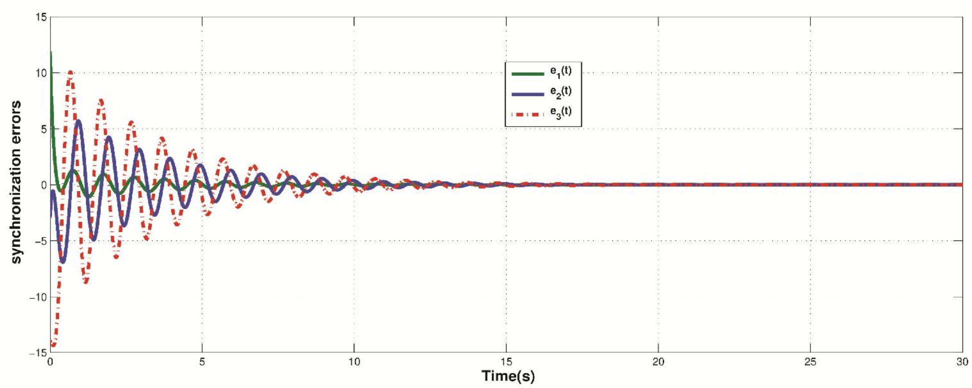

FIG.2. generalized synchronization. Error for the system (8 & 9)

where $x(0) = (-50,1,30,30), y(0) = (10,10,10)$, and $e_i = y_i - \phi_i(x(t)), \quad i = 1,2,3.$

IV. CONCLUSION

We have studied the generalized synchronization of chaotic dynamical systems with different dimensions. Based on the Lyapunov stability theory, a method to realize generalized synchronization for two different dimensional chaotic systems is proposed. The validity of this approach is verified theoretically and numerically. We also believe that our propose scheme may be helpful for secure communication.

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